

*Ex Dono Gulielmi Forbes*  
A 1608/304  
COMPENDIOUS SYSTEM

OF

# Natural Philosophy.

*Liber Societatis Med. Abridon.*  
With NOTES

Containing the

MATHEMATICAL DEMONSTRATIONS,

AND

Some Occasional REMARKS.

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In Four PARTS.

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VOL. I.

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T H E

# P R E F A C E.

*I* T being an usual Complaint with those who are unacquainted with Geometry, that they are discouraged by the Mathematical Demonstrations, from perusing Books of Natural Philosophy; I apprehended that some Papers I had drawn up for the Use of my Pupils in the University, would not be altogether unacceptable, if published in such Form, that the Propositions, or Substance of the Book, might be read without Interruption from the Mathematical Demonstrations. And therefore after a large Explanation, and sometimes an Illustration also, of the Particulars, as occasion seemed most to require, I have endeavoured to shew the Truth of them, in a familiar and popular Manner, without Geometry, by way of Text:

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And

*And for the sake of those who are skilled in Geometry, have added the Demonstrations, with some occasional Remarks, by way of Notes. And whereas the Writers on this Subject have appropriated to themselves a Stile too technical for Beginners, I have, in hopes of being more easily understood, sometimes chose a different Method of Expression, though perhaps not so accurate.*

*In the Introduction to the first Part, Notice is taken of the Method of Philosophising made Use of by Des Cartes, and others before him, so far as the Design of this Compendium required. I shall add here a few Considerations relating to the Method which prevails at this Time.*

*In the present Method of Philosophising, all Matter is considered (with respect to its Substance) as homogeneous, or of the same Kind; and no other Cause or Principle of Action in Matter is allowed*



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*lowed of, but what is well established by Facts.*

*Some Philosophers admit Elementary Fire, as they call it, among their Principles; or which comes to the same thing, they consider Fire as endowed with active Powers distinct from those of other Matter. Keill, in his Letter to Dr. Cockburn, De Legibus Attractionis, aliisque Physices Principiis, makes use of three Principles, viz. 1. Empty Space. 2. The infinite Divisibility of Quantity. 3. The Attraction of Matter. And affirms, that all Physics depends thereon.*

*The first of his Principles the Reader may perhaps think ridiculous; but he may consider, that at that time of Day, the Notion of a Plenum was not wholly exploded: The laying down Empty Space as a first Principle, was only calling out for Elbow-Room and a clear Stage ----- But not to trouble the Reader with what others have done, I have*  
A 2
chosen

*chosen and every where stuck to three; and as oft as a Phænomenon occurred, which I could not account for by them, I have given it up as a Difficulty; not despairing, but that when all the Circumstances of the Phænomenon shall be thoroughly known, they alone may be found sufficient. It seems not consistent with the Regard a Philosopher should have to the Uniformity of Nature, every where observable, to call in a new Principle at every knotty Point. Those which I make use of are,*

*First, Attraction of Gravitation. That is, a Disposition in Bodies to move towards each other, even when at great Distances asunder.*

*Secondly, Attraction of Cohesion. That is, a like Disposition in Bodies to move towards each other, but distinct from the former, in as much as it is observed to take Place only when the Bodies are very near together.*

*Thirdly, Repulsion, or a Disposition*  
*in*

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in Bodies, whereby in some Cases they endeavour to avoid, or fly from each other.

The first of these is Matter of daily Observation. Thus, a Ball let go from the Hand falls to the Ground.

The second may be seen in the following Instance. A small Portion of a Fluid forms itself into a Sphere or Drop: Which can only happen from a Disposition in the Particles of which it consists, to come as near as possible to each other.

An instance of the third is this. If Air inclosed in a Bladder, be squeez'd into a less Compass, the Air within, when the Pressure is taken off, restores the Bladder to its former Size: A plain Indication that the Particles of which the Air consists, endeavour to avoid or fly from each other (a).

These Dispositions in Bodies are not the Result of any Mechanical Cause

(a) See another Instance of this Disposition, in Part III. Page 161. Note m of this Compendium.

whatever;



*whatever; that is, such as may arise from the Effluvia of Bodies, or the Action of any other material Substance (b): They are therefore the Act of an immaterial Cause, in Virtue of which inactive Matter performs the Offices for which it was designed.*

*From*

(b) Demonstration. In the first Place it is well known, that if Gravity acts upon Bodies with the same Degree of Intensity, whether they be in Motion or at Rest; it may be demonstrated that Bodies, when projected, will describe Parabola's; and that when vibrating in Cycloids, their Vibrations will be isochronous, &c. In the next Place it is as well known, that Bodies when projected do describe Parabola's, and that when vibrating in Cycloids, their Vibrations are isochronous, &c. From which two Propositions it demonstratively follows, that if Gravity be the cause of the abovementioned Effects, it must act upon Bodies with the same Force, whether they be in Motion or at Rest.

Again, it is well known, that if Attraction of Cohesion acts upon Rays of Light, with the same Degree of Intensity, whatever be the Velocity they move with; it may be demonstrated, that the Ratio of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction, will be given. But in Refraction of Light, the Ratio of those Sines is given in Fact; if therefore Attraction of Cohesion be the Cause of the Refraction of Light, it must act upon Rays of Light with the same Intensity, whatever Velocity they move with.

But no Effluvia of Bodies, no material Substance, and in short no material Cause whatever, can act with the same



*From the first and third Principle (c), together with the Properties of Matter enumerated in the first Chapter of this Compendium, which Properties must be always understood, the Elasticity or Spring of the Air, and from thence the Nature and Propagation of Sound are accounted for. And from the Spring of the Air considered as being augmented by Heat, and diminished by Cold, as*

*same Intenseness, or have the same Effect upon a Body in Motion, as upon the same Body at Rest; because as it is very well known to Mathematicians, to whom I address this Note, Body can only act upon Body, according to the Sum or Difference of their Motions. It remains therefore, that the two Dispositions herein mentioned, are not the Result of any material Cause whatever: Which is one Part of the Proposition to be demonstrated.*

*As to the other Disposition in Bodies, their Repulsion, since Rays of Light are also affected by it, as it appears they are by an Experiment of Sir Isaac Newton's, referred to in the foregoing Note, it may very reasonably be supposed, though we don't at present know the exact Law of its Action, to affect Bodies in Motion after the same Manner that it would do the same at Rest, and that it therefore is also the Result of no material Cause whatever.*

*(c) The Law or Manner wherein these Principles are observed to act in different Circumstances, are determined from Facts, in Part I. Chap. 3. The Law of the third, so far as it relates to the Air, will be found in Part II. Chap. 3. of this Compendium.*

*it*

*it is observed to be, and the Air's being at the same time affected by the first Principle, the Phænomena of the Winds are explained. By the second Principle, the Cohesion of Matter, the various Degrees of Hardness observable in it, the Dissolution of Bodies by Fluids, with other chemical Operations; and in particular the Phænomena of Fermentation, and consequently the Causes of Thunder and Lightning, &c. By this Principle also the rising of Fluids in small Tubes, and from thence the ascent of Sap in Vegetables are accounted for; all which Particulars, except the two first, are treated of in the second Part of this Compendium: as also the Refraction of Light, and consequently all that Train of Phænomena depending thereon, which is the Subject of the third Part. By the first Principle, the several Circumstances relating to falling Bodies, and to the Motion of Projectiles, together with the Doctrine of*  
Pendulums,

# P R E F A C E. ix

Pendulums, (*treated of in the first*) and likewise all those which relate to the Pressure of Fluids, (*treated of in the second Part*) are determined. And above all is deduced that most curious Doctrine of central Forces, just touched upon in the first, but largely and fully explained in the fourth Part, by which (assuming that the heavenly Bodies were at first put into Motion by their Creator) we are enabled to assign the Cause of the Continuation thereof, with all its Modifications and Irregularities; to determine the necessary Shape of those Bodies; and to account for the ebbing and flowing of the Sea, &c. Difficulties, too great seemingly for human Reason to surmount!

Some of the Phænomena which I have not been able to give a satisfactory Account of, from the abovementioned Principles, are the Reflection of Light, its Emission from luminous Bodies, and the Formation and Ascent of Vapour. This

a

may

may be only owing to the want of better Acquaintance with the Circumstances of those Phænomena; that is, more sufficient Data, or Facts to found their Solution upon: So that we are not to conclude immediately, that the Principles are insufficient; but rather to wait with Patience: The Diligence of others may render that easy, which our utmost Efforts at present are not able to surmount.

However, as a Reader unacquainted with Studies of this Kind, may wonder that so many of the Phænomena or Appearances of Nature, should be accounted for by so few Principles; and because it may be a Means of giving him some Insight into the Subject of these Sheets, I will here lay down the following Propositions, which are immediately deducible from the Principles, and also nearly connected with the Phænomena to be accounted for by them; by means of which,

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*which, he will more readily perceive the Connection or Relation between the one and the other.*

P R O P O S I T I O N I.

Matter being an *unactive Substance*, is utterly incapable of putting itself into Motion in any Direction whatever; and will therefore in all Cases move, or endeavour to move in that Direction only, in which it is urged with the greatest Force.

*Hence we have the true Idea of the Gravity or Weight of Bodies belonging to the Earth. Bodies are, here, by Virtue of the first Principle, attracted towards the Sun, the Moon, the rest of the Planets, and the Earth; but towards this last more strongly than towards any of the rest; and so they tend, gravitate, or are heavy towards that. The Reason that they are attracted more*

a 2 *forcibly*

*forcibly towards the Earth, than towards those other Bodies, is, that although it be one of the Laws of the first Principle, that it operates according to the Quantity of Matter in Bodies, and therefore the Attraction of the Sun should be the most prevalent, in as much as that Body contains the most Matter; yet it is another Law of that Principle or Disposition, that it acts more strongly according to the nearness of Bodies to each other: This latter Consideration in the present Case, overbalances the former; and so the Bodies about us tend towards the Earth.*

## P R O P O S I T I O N II.

If a large round Body be covered every where with smaller ones to an equal Height or Distance from its Surface; and if those smaller ones tend towards the large Body, by virtue of the first Principle, and are, at the same time,



time, disposed to fly from each other by virtue of the third; and supposing farther, that when they touch or are very near each other, their Disposition to avoid each other exceeds their Tendency to the large Body, and when they are at a certain greater Distance from each other, that Disposition is less than their Tendency to the large Body: Then will those smaller Bodies keep at certain Distances from each other, and constitute an *elastic, compressible* Substance surrounding that large Body, *gravitating* towards it on all Sides.

*Hence an Idea of the Nature and Condition of the Atmosphere surrounding the Earth, with all its Properties.*

N. B. *When I say a Body tends to another, I don't mean that it moves towards it, but only that it would move towards it, if nothing prevented. Thus, a Bird while mounting aloft into the*  
*Air,*



*Air, tends towards the Earth, as much as one that is falling down; for the one would fall as well as the other, if nothing prevented.*

## P R O P O S I T I O N III.

If, while the abovementioned smaller Bodies are in the State supposed in the foregoing Proposition, any one, or more of them, be made to move, (suppose for Instance half way) towards the next, it will by virtue of the third Disposition, drive or impel those it comes nearer to, closer together; which Bodies, when that other moves back again (as it will immediately do, being repelled by them) will recede from each other again: That is, a kind of tremulous Motion will be communicated to them by that other, and for the like Reason, by them to the next; and so on through the whole, or at least to a great Distance from where it began. *Hence*

*Hence we may form an Idea how Sound is excited by the Tremors of a Body during its Vibration, and propagated through the Air.*

## P R O P O S I T I O N IV.

One of the Laws by which the second Principle is observed to act, is, that Bodies act upon one another, not in Proportion to the Quantity of Matter they contain, as by the first Principle; but only according to the Breadth of their Surfaces, and the nearness of the Surface of one Body to that of another.

*From hence we may understand, that such Particles of Bodies as are flat or square, and so situated among each other as to touch, or be very near one another in many Points, will constitute what we call an hard Body,*  
*and*

*and those Particles which are more round, or so situated that less Portions of their Surfaces are near together, will attract one another with a less Force, and so form a softer Body; those which are round, or nearly so, will attract one another still less, and also slide more easily over one another, and so form what we call a fluid Body.*

## P R O P O S I T I O N V.

When two Bodies meet together, if the Particles which constitute the one, be disposed, by Virtue of the second Principle, to move towards those of the other with a greater Degree of Force, than the Particles of either Body are disposed to move towards themselves; those of the first will leave it, and run in among those of the second: And for the same Reason, those of the second will sever from that, leave it and enter in between those of the first. And if  
the

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the Motion with which this is done, be very violent, and the Bodies be of the inflammable Kind, their Particles by thus rubbing and clashing one against another, will be sufficiently heated to take Fire, and will burst out into Flame.

*Hence Diffolutions, Fermentations, Explofions, Eruptions of Vulcano's, Thunder, Lightning, Aurora Borealis. With all other Phænomena of that Tribe.*

## P R O P O S I T I O N VI.

If a Pipe, open at both Ends, and of a very small Bore, have one End dipped into Water, the Water will run up into the Pipe above the Surface of the Water on the outside (being drawn up by the Tendency it has by the second Principle to that Part of the inner Surface of the Pipe, which is just above it, as it rises) till that inner  
b
Surface

Surface which is still just above it, be loaded with as great a Weight of Water, as that Tendency can support.

*Hence a right Notion of the Ascent of Sap in Vegetables; the Suction of Fluids by Sponges: With all other Phænomena reducible to that Head.*

P R O P O S I T I O N VII.

If a Body moving right forwards, but obliquely with respect to the Surface of another Body, at length comes so near that Body, as to be disposed by the second Principle to tend towards it; instead of continuing to go right on, it will turn out of its Way towards that Body, before it comes at it; and consequently will strike or enter it in a nearer Place, and in a less oblique Direction, than it would have done, in case it had gone right on. If it enters the Body, it still keeps turning  
out



out of its Course the same Way as before, till it has got so far within it, that there shall be as many Particles of the Body behind it to attract it backwards, as there are before it near enough to attract it forwards: After which it goes right on in its last acquired Direction, till it comes near the other Side; for while it is surrounded with as many Particles to attract it one way as another, it is the same thing as if it were not attracted at all. When it has got so near the other Side, that there are fewer Particles before it to attract it forwards, than there are behind it, near enough to attract it backwards, it then begins to turn out of its Course towards the inside of the Body; that is, from that Side of the Body towards which it is going; and continues to bend its Course the same Way, till it has got so far out of the Body, that there are no Particles of the Body behind it, near enough to it to attract it any more.



After which it pursues an undisturbed Course in the Direction it acquired last of all.

*Hence we have a just Idea of the Refraction of Light with all the Phenomena arising therefrom; which are no other than so many Cases of this Proposition.*

P R O P O S I T I O N VIII.

If several Bodies be moving right forwards, and at length be attracted by another Body, as supposed in the foregoing Proposition, but some with greater Degrees of Force or Intensity, than others; those which are attracted with the greatest Force, will turn the farthest out of their Way towards that Body; and consequently if all of them, before this happened, were moving in one Direction, they will be made to part from each other, and move different Ways.

*Hence*



*Hence an Idea of the different Refrangibility of the Rays of Light.*

## P R O P O S I T I O N IX.

If a Body be made to move from another Body, towards which by the first Principle it tends, its Motion will be retarded continually; that is, it will move slower and slower: If it moves towards that Body, its Motion will be continually increased; and unless it be made to move directly to or from it, its Course will always keep bending towards it, so that it shall describe a Curve, concave, or hollow, on the Side next the Body.

*Hence all the Phænomena of falling Bodies, and of Projectiles.*

## P R O P O S I T I O N

## P R O P O S I T I O N X.

If a Body, that by the first Principle tends towards another Body, moves towards it on the Surface of an inclined Plane, its Motion (as in the foregoing Proposition) will be continually increased; and if it moves from it on the same Plane, its Motion will be retarded continually, but less in Proportion to the Obliquity of the Plane: (that is, less in Proportion as the Plane deviates from the Perpendicular) the Interposition of the Plane preventing in some Measure the Effect its Tendency to the other Body, would otherwise produce. And the Velocity it acquires by rolling down one Plane, will by virtue of its *Inactivity*, or that Disposition Bodies have to continue their State of Motion or Rest, enable it to roll up another fitly disposed.

*Hence*

*Hence the Solution of the Phenomena of Bodies descending on inclined Planes, and the Vibration of Pendulums.*

## P R O P O S I T I O N   X I.

If a Body acted upon by the first Principle, move to or from another Body with a competent Degree of Obliquity and Velocity, it will move quite round the other Body without touching it at all, returning to the Place from whence it set out: In which Case it will revolve round it over and over again in the same Path; for being indifferent either to Motion or Rest, and meeting nothing to take off from, or diminish its Motion, it will have the same Tendency to move on after any one Revolution, as it had at first.

*From*

*From hence we have the Solution of the Motion of the primary Planets round the Sun, and of the secondary ones round the Primary.*

P R O P O S I T I O N    X I I .

If a Body be revolving about another as in the last Proposition, and a third Body approaches them, towards which they both shall also tend, the Motion of the revolving Body will be disturbed: That is, its Path will be altered, and Irregularities in its Course will ensue; its Tendency to that third Body in some Parts of its Course conspiring with, and in others being opposite to its own Motion. And not only so, but the Tendency it has to the Body about which it revolves, will in some Situations be increased, and in others be diminished by the Action of the third; which Thing also conduces towards altering its Course.

*Hence*

*Hence the Lunar Irregularities, and all other Disturbances in the Motion of the Heavenly Bodies on their 100 near Approach towards each other.*

## P R O P O S I T I O N XIII.

Imagine a large Body cover'd all over with smaller ones tending to its Center : Suppose also a distant Body, towards which they all tend, but the little ones with less Degrees of Force than they do towards the Body they touch. Then will such of those smaller Bodies, as are nearest the distant one, lose Part of their Tendency to the Body they touch ; and so will those smaller ones which are farthest off, or placed on the opposite Side the large Body. But, as to those smaller Bodies, which are at the same Distance from the distant Body with the Center of the large Body itself, their  
c
Tendency

Tendency to the Body they lie upon, will be increased. The rest will have their Tendency increased or diminished more or less, according to their Nearness to those whose Tendency is increased or diminished. (d)

*Hence arises the Difference in the Weights Bodies have upon the Earth's Surface, at the Approach and Departure of the Heavenly Bodies, (but chiefly of the Moon,) to or from that Side of the Earth where the Bodies are; and consequently the ebbing and flowing of the Sea, the Water rising where its Weight or Tendency to the Earth is diminished, and sinking at the same Time in those Places where its Weight is augmented. That the Approach and Departure of the Moon should cause a greater Difference in the Weight of Bodies on the Earth, than the Approach and*

(d) What is affirmed in this and the foregoing Proposition, depends on a Train of Reasoning too long to be inserted here. To understand it thoroughly, read Chapters the 18th and 19th of Part the Fourth.

*Departure*



*Departure of the other Heavenly Bodies, is owing to the nearness of the Moon to the Earth; which Consideration in this Case overbalances the Consideration of her Smallness, the above-mentioned Effects depending in a great Measure, on the Proportion the Diameter of the Earth bears to the Distance of the Heavenly Bodies.*

## P R O P O S I T I O N XIV.

If a Body, whose Parts tend to the Center thereof, consists wholly of a Fluid, or be partly solid and partly fluid, provided some of the Fluid be at the Surface, and very distant Parts thereof communicate with each other; and the Body have no Motion about its Axis, it will settle into a spherical Form, the mutual Tendency of its Parts towards each other, contracting it into the least possible Shape. But if it revolves about its Axis, all its



Parts will endeavour to fly off from that Axis; but such as are farthest from the Axis, more than the rest: Consequently those Parts in its Surface, which are the farthest from the Extremities of that Axis, being also farthest from the Axis itself, will have a greater Endeavour to fly off, than such as are nearer those Extremities; besides, as is evident, the former will endeavour to fly off directly from the Center, but the latter not so. The abovementioned Endeavour therefore in the former will take off a much greater Degree of their Tendency to the Center than the Endeavour of the latter will; and since the same may be said of those which are at any other assignable equal Distances from the Center, all those which lie between the Center of the Body, and such, as are farthest from the Extremities of the Axis, will have their Tendency to the Center much more diminished,  
than

than those will, which lie between the Center and the said Extremities : These latter Parts therefore will press in towards the Center, overbalance the former, and raise them to a greater Distance from it than they were at before, restoring thereby an *Equilibrium* of the Parts of the Body one among another. On which Account the Body will assume a flattish or oblate Form. That is, supposing Lines drawn thro' the middle of the Axis at right Angles therewith, those Lines will be lengthen'd and the Axis will be shorten'd.

*Hence the Figures of the Heavenly Bodies.*

#### P R O P O S I T I O N XV.

The Impetus or Force wherewith a Body in Motion endeavours to proceed forwards, depends not only on the Quantity of Matter in that Body, but  
likewise

likewise on the Swiftneſs the Body moves with: Thus, the Stroke of an Hammer is not only according to the Bulk or Weight of its Head, but is alſo according to the Swiftneſs of the Motion it ſtrikes with. If therefore two Bodies of equal Quantities of Matter, be ſuſpended at the Ends of a Lever of equal Arms, each of them when the Lever turns on its Center, having equal Degrees of Swiftneſs or Velocity, will therefore have the ſame Impetus or Force whereby they endeavour to proceed (being in like Circumſtances with reſpect to both thoſe things, which alone can give the one a Force or Tendency to move on with, ſuperior to the other) and conſequently neither of them will pre-ponderate. If one of the Bodies be larger than the other, the larger Body having the ſame Velocity with the other, but more Matter, will have the Advantage, and preponderate. If the Arms of the Lever are unequal,  
and

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and the Bodies equal, that Body which is at the greatest Distance from the Center of Motion, moving the quickest, will have the Advantage over the other that way, and overpoise it. So that the least Body or Power, imaginable, may be made to equiponderate, overpoise, or keep in Motion the greatest, by being applied to such a Machine, and in such Manner, that when the Machine moves, what it wants in Weight or Force, may be made out by the Velocity it has, compared with the Velocity the Body has at the same Time, which is to be equiponderated, over-poised, or moved by it.

*This holds equally in all Machines, and is the Foundation of their Theory.*

## P R O P O S I T I O N    XVI.

Imagine the Surface of a large round Body to be covered every where, or in  
Part,

Part, with smaller ones to an equal Height, and that these smaller ones tend towards the large Body by the first Principle; imagine also the whole Mass of smaller ones divided into Columns reaching from top to bottom; those Columns, if their Bases be equal, will equiponderate, or be an equal Counterpoise to one another; and so they will, if their Bases be unequal: For in this Case the Columns being of unequal Size in Diameter, if a larger Column subsides, the lower Parts of that Column (to find Room for themselves) will raise a smaller Column farther than the larger one settled in the same time, and in such Proportion that, what the little Column wants in Weight, will be made out to it in Velocity; and consequently, according to what was shewn in the foregoing Proposition, the little Column will be a just Balance to the greater.

Farther

Farther, if in the abovementioned Supposition, there be a Body among those smaller ones, heavier than a Bulk of them equal to its own Bulk, a Column of which that Body is a Part, will be heavier than any other Column of an equal Base; it will therefore subside, permitting the Body to come to the Bottom: if the Body be lighter than a Bulk of the smaller ones equal to its own Bulk, a Column, of which it makes a Part, will be lighter than any other; the Body therefore will be buoy'd upwards, till it rises so far out above the Surface, that it, together with the Column below it, may be a Counterpoise to another Column of equal Base.

*Hence the Effects of the Pressure of Fluids upon one another, and upon Solids immersed in them.*



## P R O P O S I T I O N    X V I I .

Imagine the Surface of a large round Body cover'd every where, or in Part, with smaller ones to an equal Height; and that these smaller ones tend towards the large one by the first Principle, and that they are at the same time disposed to fly from one another by Virtue of the third, constituting thereby an elastic Substance surrounding that large Body, as in Proposition the second; and let them be divided into Columns, as in the last Proposition. And let it be farther supposed, that the Disposition in those smaller Bodies, whereby they endeavour to depart from each other, is capable of being increased by Heat; and that at the Bottom of some of these Columns, that Disposition is actually increased, but no where else, or at least not in so great a Degree: then

# P R E F A C E. xxxv

then will the Bodies, where that Disposition is increased, diffuse themselves into a larger Space, and so taking up more Room than an equal Number in the neighbouring Columns, a Column of which they are a Part, will become lighter than a neighbouring one of an equal Base. For, since the Bodies in the lower Parts of this Column, are more distant from each other, than such as are in other Columns, this Column cannot contain so many of them; that is, it cannot be so heavy as another of equal Base, unless it be longer; that is, unless the uppermost Parts thereof stand out above the Tops of the neighbouring Columns; but this they will not do; for by Virtue of the Tendency those Parts have to the large Body, they will immediately (like Water raised above the Banks, which before confined it) spread themselves every Way. This Column therefore, which, ac-

according to the foregoing Proposition, before this happened, was a Counterpoise to those, which are round about it, being now become lighter, is no longer so. The Consequence of which is, that the lowermost Parts of the neighbouring Columns, will press in under this from all Sides to restore the Equilibrium. Neither can the Equilibrium be restored, so long as the Place we have been considering remains hotter, than those which are round about it. For, since the Bodies, that come in, will spread themselves into a larger Space by Means of the Heat they receive there, and fill up more Room, than the like Number in another Column of equal Base, the Column to which they belong, will, for the Reasons abovementioned, always be lighter than another of equal Base. And consequently, according to the Tenour of the foregoing Proposition, the neighbouring Columns  
will

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will overpoise it, whatever Dimensions, as to their Bases, we suppose them to be of.

*Imagine the like to happen to a Column or Columns of the Earth's Atmosphere, and the lower Parts of the neighbouring Columns rushing in accordingly at the Bottom from all round, and you have an adequate Idea of the Cause and Nature of the Winds; every Stream of the Particles of the Atmosphere rushing in, as above, being a distinct Wind blowing from that Point of the Compass from which they come. And, if you conceive the Center of that warmer Space to shift its Place variously upon the Surface of the Earth, you then get the Idea of the several Sorts of them, as the Trade Winds, Monsoons, &c. For Instance, if it shifts regularly along the same Path, it causes Trade Winds; if now forwards,*

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*forwards, and then backwards, Monsoons, &c.*

*These are the Principal Phænomena in Natural Philosophy that are independent of each other; the rest are for the most Part, no other than so many particular Cases, Circumstances, or Consequences of these, or, in short one way or other related to them. For the Solution, of which I refer the Reader to the Book it-self.*

*From a due Consideration of the Propositions here laid down, the Reader will be able to form a true Judgment of the Nature and Business of Natural Philosophy; will see the Uniformity and Consistency of the several Parts thereof with each other, and therein the wonderful Wisdom and Contrivance of the supreme Being, in choosing so short and easy a Method of producing so great a Variety of Effects.*

*There*

*There is one Thing more I think proper to be taken Notice of, before I put an End to this Preface; viz. That it has been a standing Objection against all Natural Philosophy in general; that whereas it ascribes Effects to natural or mechanical Causes, acting by fixed and unalterable Laws, it therefore excludes a Providence and the immediate Care and Protection of the supreme Being, making him no other than an Idle Spectator of Things here below.*

*In Answer to this, it is to be considered in the first Place, that the Principles of the Philosophy which is now received, are so far from being mechanical Causes, at least those which are here made Use of, that, as above demonstrated, they are the very Reverse; and consequently can be no other than the continual acting of God upon Matter, either mediately or immediately. Since  
then*



*then Natural Philosophy, by accounting for the Phænomena of Nature by those Principles, tends to shew the Reality of them; it is evident that it is so far from excluding the Deity from being concerned in the Affairs of this World; that it tends to demonstrate that none are performed without his Order and Direction. Neither, secondly, does Natural Philosophy inculcate, that the Laws by which those Principles act, are fixed and unalterable: The Accusation is therefore foreign. But to consider this Matter a little more particularly.*

*When, in Natural Philosophy, a Principle is said to act according to a particular Law, the Meaning is not, that it acts necessarily and unalterably so; but only, that it does so ordinarily, and in common Cases. Doubtless the Author, both of Matter and of those very Principles by which it acts, can at any Time, in the Room of its ordinary Operation, substitute a different*  
one

one, and by that means produce Effects contrary to the common Course of Nature, whenever he shall think proper. That he has done so, when wise Ends required it, appears from History. That it may be done a Thousand Ways, unperceived by us, is evident; the Operation of natural Causes (as they are called) being to us almost always out of Sight. For Instance, though Lightning may be accounted for by these Principles; and in all Probability is ordinarily the Result thereof; yet who will affirm, in any particular Case, that those Principles formed that very Lightning, or that its Course was directed by them? Upon the whole therefore, to presume, that the ordinary and common Course of Nature is not sometimes altered, is hasty and unwarrantable.

e

T H E

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E R R A T A in the Preface.

P Age 10, Line 1; dele is; Page 24, Line 13; read, ensue; Page 26, last Line but 2; read, inserted.



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## COMPENDIOUS SYSTEM

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# Natural Philosophy.

PART I.

## The INTRODUCTION.

**S**O wild and extravagant have been the Notions of a great Part of Philosophers, both ancient and modern, that it is hard to determine, whether they have been more distant in their Sentiments from Truth, or from one another; or have not exceeded the Fancies of the most fabulous Writers, even Poets and Mythologists. This was owing to a precipitate Proceeding in their Enquiries, and a Neglect of Geometry and Experiment; without the Assistance of which, it is impossible the Powers of natural Agents should be discovered.

THE Manner of Philosophizing among the Ancients was to ascribe to Bodies certain arbitrary

## 2 The *INTRODUCTION*.

trary Properties, such as best served their Purpose in accounting for the Phænomena \* of Nature; from whence proceeded so many various Sects of Philosophers; every one assigning a different Cause to the same Appearance, as his particular Genius and Imagination led him.

THE chief Agreement observable among most of them, consists in this, *viz.* that they conceived all Bodies, as Compositions of Air, Earth, Fire, and Water, or some one or more of them, from whence these acquired the Name of Principles or Elements, which they still retain.

EPICURUS advanced a little farther, and asserted, that though Bodies consisted of some one or more of these, yet that they were not strictly Elements, but that these themselves consisted of Atoms; by an accidental Concourse of which (as they were moving through infinite Space in Lines nearly parallel) all Things received their Form and Manner of Existence †.

DES CARTES has contrived an Hypothesis very different from the rest: He sets out with a

\* By a Phenomenon of Nature, is meant any Motion or Situation of Bodies among one another, which offers itself to the Notice of our Senses, and is not the immediate Result of the Action of an intelligent Being.

† For the Opinions of the ancient Philosophers, consult *Diogenes Laertius*, and *Stanley's Lives*.

Suppo-



## The INTRODUCTION. 3

Supposition that the Universe at first was entirely full of Matter; that from this Matter, when first put in Motion, there would necessarily be rubbed off (by the grinding of the several Parts one against another) some Particles sufficiently fine to pass through the hardest and most solid Bodies without meeting with any Resistance: Of these consists his *Materia subtilis*, or *Materia primi Elementi*. He imagined, that from hence also would result other Particles, of a globular Form; to which he gave the Name of *Materia secundi Elementi*. Those which did not so far lose their first Figure, as to come under the Denomination of *Materia primi* or *secundi Elementi*, he called *Materia tertii Elementi*; and maintained, that all the Variety, which appears in natural Bodies, was owing to different Combinations of those Elements.

He likewise supposes, that God created a certain Quantity of Motion, and assigned it to this Mass of Matter; and that That Motion (being once created) could no more be annihilated, without an omnipotent Hand, than Body itself: In Consequence of which, he was obliged to teach, that the Quantity of Motion is always the same: So that, if all the Men and Animals in the World were moving, yet still there would be no more Motion than when they were at Rest, the Motion which they had not, when at Rest, being transferred

#### 4 The *INTRODUCTION*.

transferred to the *Æther*. So unaccountable are the Notions of this great Philosopher, that it is surprizing his Doctrine should have met with such universal Reception, and have got so strong a Party of Philosophers on its Side.

*DES CARTES* has been said, by a late Writer \*, to have joined to his great Genius an exquisite Skill in Mathematicks, and, by mixing Geometry and Physics together, to have given the World Hopes of great Improvements in the latter. But this Writer ought to have considered, that what he looked upon, in *DES CARTES*'s Book of Principles, as Demonstrations, are only Illustrations, there not being a Demonstration from Geometry in all his Philosophical Works †.

*THE* present Method of Philosophizing, established by Sir *ISAAC NEWTON*, is to find out the Laws of Nature by Experiments and Observations. To this, with a proper Application of Geometry, is owing the great Advantage the present System of Philosophy has over all the preceding ones, and the vast Improvement it has received within the last

\* *Mr. Wotton*, in his Reflexions on ancient and modern Learning.

† See this Subject discussed more at large in *Keil's* Introduction to his Examination of *Dr. Burnet's* Theory.

## The INTRODUCTION. 5

Age. It is indeed in vain to imagine, that a System of Natural Philosophy can be framed by any other Method; for without Observations it is impossible we should discover the Phenomena of Nature, without Experiments we must be ignorant of the mutual Actions of Bodies, and without Geometry we can never be certain whether the Causes we assign be proportionate to the Operations we would account for, as the various Systems of Philosophy built on other Foundations evidently shew.

THIS Way of searching into Nature was first proposed by my Lord BACON\*, prosecuted by the *Royal Society*, the *Royal Academy* at *Paris*, the Honourable Mr. BOYLE, Sir ISAAC NEWTON, &c.

WHAT wonderful Advancement in the Knowledge of Nature may be made by this Method of Enquiry, when conducted by a Genius equal to the Work, will be best understood by considering the Discoveries of that excellent Philosopher last mentioned. To Him it is principally owing, that we have now a rational System of Natural Philosophy; 'tis He, who, by pursuing the sure and unerring Method of reasoning from Experiment and Obser-

\* See his *Novum Organum*.



## 6 The *INTRODUCTION*.

vation, joined with the most profound Skill in Geometry, has carried his Enquiries to the most minute and invisible Parts of Matter, as well as to the largest and most remote Bodies in the Universe, and has established a System not subject to the Uncertainty of a mere Hypothesis, but which stands upon the secure Basis of Geometry itself,



CHAP.

( 7 )  
C H A P. I.

*The Properties of Body.*

**I**T being the Design of *Physics*, or Natural Philosophy, to account for the Phænomena of the Material World, it is necessary to begin with laying down the known Properties of Body.

THESE are, 1. Solidity. 2. Extension. 3. Divisibility. 4. A Capacity of being moved from Place to Place. 5. A Passiveness or Inactivity.

1. SOLIDITY, called also Impenetrability, is that Power which Body has of excluding all others out of its Place.

THAT Body, as such, must be endued with this Property follows from its Nature, for otherwise two Bodies might exist in the same Place, which is absurd. The softest are equally solid with the hardest; for we find, by Experiment, that the Sides of a Bladder, filled with Air or Water, can by no Means be made to come close together \*.

2. THAT

\* At Florence a hollow Globe of Gold was filled with Water, and then exactly closed; the Globe thus closed was put into a Press driven by the Force of Screws; the Water, finding no Room for a nearer Approach of its Particles toward each other, made its Way through the Pores of that close

## 8 *The Properties of Body. Part I.*

2. THAT Body is extended, is self-evident, it being impossible to conceive any Body which has not Length, Breadth, and Thickness, that is, Extension.

3. IT is no less evident, that Body is divisible; for, since no two Particles of Matter can exist in the same Place, it follows that they are really distinct from each other, which is all that is meant by being divisible.

IN this Sense, the least conceivable Particle must still be divisible, since it will consist of Parts which will be really distinct \*. To illustrate this by a familiar Instance: Let the least imaginable Piece of Matter be conceived lying on a smooth plain Surface; 'tis evident, the Surface will not touch it every where; those Parts, therefore, which it does not touch, may be supposed separable from the other, and so on as far as we please; and this is all that

Metal, standing in Drops like Dew on the Outside, before the Globe would yield to the violent Pressure of the Engine. *V. Acad. del Ciment.*

\* This Proposition is demonstrated Geometrically thus: Suppose the Line *AD* (*Fig. 1.*) perpendicular to *BF*, and another, as *GH*, at a small Distance from it, also perpendicular to the same Line; with the Centers *C, C, C,* &c. describe Circles cutting the Line *GH* in the Points *e, e, e,* &c. Now the greater the Radius *AC* is, the less will be the Part *eH*. But the Radius may be augmented in infinitum. So long, therefore, the Part *EH* may be divided into still less Portions; consequently it may be divided in infinitum. *Q. E. D.* *V. Keil's Introd. ad Phys. Præl. 3, 4, 5. Gravesande's Elem. Math. Phys. L. 1. C. 4. Schol.*

Chap. I. *The Properties of Body.* 9

is meant, when we say Matter is infinitely divisible.

How far Matter may actually be divided, may in some manner be conceiv'd from hence\*, that a Piece of Wire, gilt with so small a Quantity as eight Grains of Gold, may be drawn out to a Length of thirteen Thousand Feet, the whole Surface of it still remaining cover'd with Gold †.

A Quantity of Vitriol, being dissolved and mix'd with nine Thousand Times as much Water, will tinge the whole, consequently the Vitriol will be divided into as many Parts as there are visible Portions of Matter in that Quantity of Water §.

THERE are Perfumes, which, without a sensible Diminution of their Quantity, shall fill a very large Space with their odoriferous Particles, which must therefore be of an inconceivable Smallness, since there will be a suffi-

\* We have a surprizing Instance of the Minuteness of some Parts of Matter, from the Nature of Light and Vision. Let a Candle be lighted and placed in an open Plane, it will then be visible two Miles round, consequently was it placed two Miles above the Surface of the Earth, it would fill with luminous Particles a Sphere, whose Diameter was four Miles, and that before it had lost any sensible Part of its Weight. The Force of this Argument will appear better when the Reader is acquainted with the Cause of Vision.

† *Keil's* Introd. ad *Phys. Præl.* 5. Religious Philos. Contempl. 25.

§ *Mem. de l'Acad.* 1706.

cient

cient Number in every Part of that Space, sensibly to affect the Organ of Smelling.

4. THAT all Matter is moveable, follows from its being finite; and to suppose it positively infinite is absurd, because it consists of Parts\*.

5. By the Passiveness or Inactivity of Matter, (commonly call'd its *Vis Inertiæ*) is meant the Propensity it has to continue its State of Motion or Rest, till some external Force acts upon it. This will be farther explained under the first Law of Nature.

## CHAP. II.

### Of Vacuum.

I. **P**LACE void of Matter is called empty Space, or *Vacuum*.

II. IT has been the Opinion of some Philosophers, particularly the *Cartesians*, that Nature admits not a *Vacuum*, but that the Universe is entirely full of Matter: in consequence of which Opinion they were oblig'd to assert, that if every Thing contain'd in a Vessel could be taken out or annihilated, the Sides of that Vessel, however strong, would come together; but this is contrary to Expe-

\* See Mr. Law's Translation of Abp. King *de Origine Mali*.  
Not. 3

## Chap. II. *Vacuum.*

11

rience, for the greatest Part || of the Air may be drawn out of a Vessel by means of the Air-Pump, notwithstanding which it will remain whole, if its Sides are strong enough to support the Weight of the incumbent Atmosphere.

III. SHOULD it be objected here, that as it is impossible to extract all the Air out of a Vessel, and that there will not be a *Vacuum* on that Account; the Answer is, that since a very great Part of the Air, that was in the Vessel, may be drawn out, as appears by the more quick Descent of light Bodies in a Receiver \* when exhausted of its Air, there must be some Vacuities between the Parts of the remaining Air: which is sufficient to constitute a *Vacuum*. Indeed to this it may be objected by a *Cartesian*, that those Vacuities are fill'd with *Materia subtilis*, that passes freely through the Sides of the Vessel, and gives no Resistance to the falling Bodies: but as the Existence of this *Materia subtilis* can never be prov'd, we are not oblig'd to allow the Objection; especially since Sir ISAAC NEWTON has found, that all Matter affords a Resistance nearly in Proportion to its Density †.

THERE are many other Arguments to prove this, particularly the Motions of the Comets.

|| A Vessel cannot be entirely exhausted of its Air, because the Action of the Pump depends on the Spring of that which remains in the Vessel.

\* By this Term is meant any Vessel, out of which we extract the Air by the Air-Pump.

† *Newt.* Principia Lib. 2. Prop. 31. & 40. & Opt. Edit. 2. Book 3. Quer. 18, 19, 20, 21. *Desagul.* Lect. 1. Ann. 2.

through



## 12 *Attraction and Repulsion. Part. I.*

through the Heavenly Regions without any sensible Resistance \*; the different Weight of Bodies of the same Bulk, &c. but those, being not yet explain'd, are not so proper to be insist-  
ed on in this Place.

### *Of Attraction and Repulsion.*

#### C H A P. III.

I. **B**ESIDES the forementioned Properties of Matter, it has also certain Powers or active Principles, known by the Names of *Attraction* and *Repulsion*, probably not essential or necessary to its Existence, but impressed upon it by the Author of its Being, for the better Performance of the Offices for which it was design'd.

II. **ATTRACTION** is of two Kinds. 1. **Cohesion**, or that by which minute Bodies, (or the several Particles of the same Body) when placed asunder at very small Distances, mutually approach each other; and then adhere or stick together, as if they were but one. 2. **Gravitation**, or that by which distant Bodies act upon each other.

III. **THE Attraction of Cohesion** is prov'd from abundance of Experiments, of which some of the most obvious are as follows.

\* *Desagul. Lect. 1. Annot. 8.*

### Chap. III. *Attraction and Repulsion.* 13

1. LET a small glass Tube (commonly call'd a Capillary Tube) open at both Ends, be dipt into a Vessel of Water, the Water will immediately rise up in the Tube to a certain Height above the Level of the external Water. This Rise of the Water in the Glass Tube is manifestly owing to the Attraction of those Particles of the Glass, which lie in the inner Surface of the Tube immediately above the Water: Accordingly the Quantity of Water raised is always found to be proportionable to the Largeness of that Surface \*.

2. LET two Spheres of Quicksilver be placed near each other, and they will immediately run together, and form one Globule.

IV. THE Laws of this Attraction are, 1. That it acts only upon Contact, or at very small Distances; for the Spheres, mentioned in the last Experiment, will not approach each other,

\* The Heights the Water rises to in different Tubes, are observed to be reciprocally as the Diameters of the Tubes, from whence it follows that the Quantities raised are as the Surfaces which raise them.

*Dem.* Let there be two Tubes, the Diameter of the first double to that of the second, the Water will rise half as high in the first as in the second: now was it to rise equally high in both, the Quantity in the first would be four times as great as in the second, (Cylinders of equal Heights being as the Squares of their Diameters; 11. *El.* 14.) therefore since it is found to rise but half as high, the Quantity is but twice as much, and therefore as the Diameter; but the Surfaces of Cylinders are as their Diameters, therefore the Quantities of Water raised are also as the Surfaces. Q. E. D.

See the Dissertation on this Subject. Part II.

till

#### 14 *Attraction and Repulsion.* Part. I

till they are plac'd very near. 2. It acts according to the Breadth of the Surfaces of the attracting Bodies, and not according to their Quantities of Matter. For, let there be two polish'd Glass Plates laid one upon another, in such a Manner as to touch at one End, and there make a very small Angle: If two unequal Drops of Oil be put between these Plates, at equal Distances from the Line of Contact, so that the least may touch both Glasses, they will then both move towards the Ends that touch, because the Attraction of the Surfaces inclines that Way; but the largest, touching the Glasses in most Points, will move the fastest. 3. 'Tis observ'd to decrease much more than as the Squares of the Distances of the attracting Bodies from each other increase: That is, whatever the Force of Attraction is at a given Distance, at twice that Distance it shall be more than four Times less than before\*.

V. FROM hence it is easy to account for the different Degrees of Hardness in Bodies; those whose constituent Particles are flat or square and so situated as to touch in many Points will be hard; those Particles which are more round, and touch in fewer Points, will constitute a softer Body; those which are spherical or nearly of that Figure, will form a Fluid †

\* *Keilii Opera* Ed. 4to. p. 626.

† See *Robault* in the Notes, p. 105, 108. See Part II Chap. I. §. 2. in the Notes. *Newtoni. Optic.* p. 335.

Chap. III. *Attraction and Repulsion.* 15

VI. ATTRACTION of Gravitation is that, by which distant Bodies act upon each other. Of this we have daily Instances in the falling of heavy Bodies towards the Earth.

VII. THE LAWS of this Attraction are, 1. That it decreases, as the Squares of the Distances between the Centers of the attracting Bodies increase. Thus, a Body, which at the Surface of the Earth (*i. e.* about the Distance of four Thousand Miles from its Center) weighs ten Pounds, if it was plac'd four Thousand Miles above the Surface of the Earth, *i. e.* twice as far distant from the Center as before, would weigh four Times less; if thrice as far, nine Times less, &c. The Truth of this Proposition is not to be had from Experiments, (the utmost Distance we can convey Bodies to, from the Surface of the Earth, bearing no Proportion to their Distance from its Center,) but is sufficiently clear from the Motion observ'd by the heavenly Bodies. 2. Bodies attract one another with Forces proportionable to the Quantities of Matter they contain; for all Bodies are observ'd to fall equally fast in the exhausted Receiver, where they meet with no Resistance. From whence it follows that the Action of the Earth upon Bodies is exactly in Proportion to the Quantities of Matter they contain; for was it to act as strongly upon a less Body as upon a larger, the least Body, being most easily put into Motion, would

## 16 *Attraction and Repulsion.* Part. I.

would move the fastest. Accordingly, it is observable, that the Weight of a Body is the same whether it be whole, or ground to Powder\*.

VIII. FROM hence it follows, that, was a Body to descend from the Surface toward the Center of the Earth, it would continually become lighter and lighter, the Parts above attracting it, as well as those below; in which Case it is demonstrated by Mathematicians, that the Gravity would decrease with the Distance of the Body from the Center†.

*Scholium.*

\* *Gravesande* Lib. 4. Chap. 11. *Cotes's* Preface to *Newton's* Princip.

† *Dem.* Let there be a Body as *P*. (*Fig. 2.*) placed any where within a Concave Sphere, as *AB*, and let us suppose it divided into an infinite Number of thin concentric Surfaces; I say, the Body *P* will be attracted equally each way by any one of these, *v. g.* the interior *HIKLM*. Let there be Lines as *IL*, *HK*, &c. drawn through any Point of the Body *P*, in such a manner as to form the Surface of two similar Figures; suppose Cones, the Diameters of whose Bases may be *IH*, *KL*, which let us suppose infinitely small. These Bases (being as the Squares of the Lines *IH*, *KL*) (*2. Elem. 12.*) will be directly, as the Squares of their Distances from *P* (for the Triangles *IPH*, *KPL*, being infinitely small, are similar.) But those Bases include all the Particles of Matter in the interior Surface, that are opposite to each other; the opposite Attractions are therefore in the same Ratio with those Bases, that is as the Squares of the Distances *PK*, *PI*. But the Attraction is inversely, as the Squares of the Distances of the attracting Bodies, §. 7. *i. e.* inversely as the Squares of the same Distances *PK*, *PI*; these two Ratios therefore destroying each other, it is evident, that if the Concavity of the Sphere was filled with Matter, that alone, which lies nearer the Center than the Body, can effect it, the respective Actions of all the Parts, that are more distant, being equal.



### Chap. III. *Attraction and Repulsion.* 17

*Scholium.* It may be proper to observe here, that when Philosophers speak of Bodies gravitating to, or attracting each other, that Body is said to gravitate to another, which moves towards it, while the other actually is, or appears to be, at Rest, and this other is said to attract the former; though indeed the Force being mutual and equal on both Sides (as will be explain'd under the third Law of Nature) the same Term might be apply'd to either the gravitating or attracting Body.

It is farther to be observ'd, that when we use the Terms, Attraction or Gravitation, we do not thereby determine the Physical Cause of it, as if it proceeded from some supposed occult Quality in Bodies; but only use those Terms to signify an Effect, the Cause of which lies out of the reach of our Philosophy. Thus, we may say, that the Earth attracts heavy Bodies; or that such Bodies tend or gravitate to the Earth: although at the same time we

and in contrary Directions, since the same is demonstrable of any of the remaining concentric Surfaces. Let us see then what Effect that, which lies nearer the Center than the Body, will have upon it, which may be considered as a Sphere, on whose Surface the Body is plac'd. The Distances of each Particle of Matter from the Body, (taken collectively) will be as the Diameter of the Sphere, or as the Radius, *i. e.* as the Distance of the Body from the Center: their Action therefore upon the Body will be inversely as the Square of that Distance: but the Quantity of Matter will be as the Cube of that Distance, (18. *Elem.* 12.) the Attraction therefore will be also in that Proportion. Now, these two Ratios being compounded, the Attraction will be only as the Distance of the Body from the Center. *Q. E. D.*

B

are



18 *Attraction and Repulsion.* Part. I.

are wholly ignorant whether this is effected by some Power, actually existing in the Earth, or in the Bodies, or external to both; since it is impossible any Error in our Reasonings can follow from hence: it being evident, that all the Consequences of such Tendency must be the same, let the Cause be where, or what it will.

X. REPULSION is that Property in Bodies, whereby, if they are placed just beyond the Sphere of each other's Attraction of Cohesion, they mutually fly from each other.

If an oily Substance, lighter than Water, be placed on the Surface thereof, or if a Piece of Iron be laid on Mercury, the Surface of the Fluid will be depress'd about the Body laid on it: This Depression has been by some ascrib'd to a repelling Power in the Bodies, which hinders the Approach of the Fluid towards them.

BUT it is more generally imputed to this, that the Particles of the Fluid attract each other, in these and the like Instances, more forcibly than they are attracted by the Body laid on it; and so recede from the Body as far as the Gravity of the neighbouring Particles of the Fluid will permit them.

IT is possible in some Cases to press or force the Particles of different Substances, that seem to repel each other in this Manner, so near together, that they shall mutually attract each other: as

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### Chap. III. *Attraction and Repulsion.* 19

when we mix Oil and Water till they incorporate.

THIS Property is however exceedingly apparent in the Particles of the Air, which are endued with so strong a repulsive Force with regard to each other, that they never could, by an Experiment yet made upon them, be compressed or driven so close together, but they would still endeavour to fly from each other \*.

XI. BESIDES the general Powers forementioned, there are some Bodies that are endued with another, call'd *Electricity*. Thus Amber, Jet, Sealing-Wax, Agate, Glass, and most Kinds of precious Stones attract and repel light Bodies at considerable Distances.

THE chief Things observable in these Bodies are, 1. That they don't act, but when heated. 2. That they act more forcibly when heated by rubbing, than by Fire. 3. That, when they are well heated by rubbing, light Bodies will be alternately attracted and repell'd by them, but without any observable Regularity whatever. 4. If a Line of several Yards in length has a Ball, or other Body suspended at one End, and the other End be fixed to a Glass Tube; when the Tube is heated by rubbing, the Electrical Virtue of the Glass will be communicated from the Tube to the Ball, which will attract and repel light Bodies in the same Man-

\* See a further Proof of this Repulsive Force in Sir Isaac Newton's Opticks. B. 3. and Query 31.

ner, as the Glass itself does. 5. If the Glass Tube be emptied of Air, it loses its Electricity \*.

XII. LASTLY, the Loadstone is observ'd to have Properties peculiar to itself, as that by which it attracts and repels Iron, the Power it communicates to the Needle, and several others †.

### C H A P. IV.

#### *Of the Laws of Motion, commonly called Sir ISAAC NEWTON's Laws of Nature.*

I. **A**L L Bodies continue their State of Rest, or uniform Motion in a right Line, till they are made to change that State by some external Force impressed upon them.

THIS Law is no other, than that universal Property of Bodies, called Passiveness or Inactivity; whereby they endeavour to continue the State they are in, whatever it be. Thus a Top only ceases to run round on Account of the Resistance it meets with from the Air, and the Friction of the Plane whereon it moves. And a Pendulum, when left to vibrate in *vacuo*, where there is nothing to stop it, but the Friction arising from the Motion

\* See Hawksbee's Experiments. *Philosoph. Transact.* No. 326.

† Several Solutions of these Properties of Electricity and Magnetism have been attempted by different Philosophers, but all of them so unsatisfactory as not to deserve a particular Account in this Place. See Chambers's Dictionary in *Electricity*, and *Des Cartes Opera Philosophica*. P. IV. § 133, with several others refer'd to in *Quaestiones Philosophicae*, Desagul. Lect. I. § 33.

Chap. IV. *Laws of Motion.* 21

of the Pin on which it is suspended, continues to move much longer, than one in the open Air.

II. THE Change of Motion produc'd in any Body, is always proportionable to the Force, whereby it is effected; and in the same Direction, wherein that Force acts.

THIS is an immediate Consequence of this Axiom; The Effect is always proportionable to its Cause. For Instance, if a certain Force produces a certain Motion, a double Force will produce double the Motion; a triple Force triple the Motion, &c. If a Body is in Motion, and has a new Force impressed on it in the Direction wherein it moves, it will receive an Addition to its Motion, proportional to the Force impressed; but if the Force acts directly contrary to its Motion, the Body will then lose a proportional Part of its Motion: Again, if the Force is impressed obliquely, it will produce a new Direction in the Motion of the Body, more or less different from the former in Proportion to its Quantity and Direction \*.

III. RE-

\* This Case is expressed more accurately by Mathematicians thus. If the Proportion and Direction of two Forces, acting upon a Body at the same Time, be represented by the Sides of a Parallelogram, the Diagonal of that Parallelogram will represent the Proportion and Direction of their united Forces.

*Dem.* Let the Body *A* (*Fig. 3.*) be impell'd with a Force, which would carry it to *E*, in the same Time that another, acting upon it in the Direction *AD*, would carry it to *D*. Imagine that while the Body passes to *E*, the Line *AD* (in which

III. REACTION is always contrary, and equal to Action; or the Actions of two Bodies upon each other, are equal, and in contrary Directions.

THUS, suppose a Stone, or other Load to be drawn by an Horse; the Load reacts upon the Horse, as much as the Horse acts upon the Load; for the Harness, which is stretched equally between them both Ways, draws the Horse towards the Stone, as much as it does the Stone towards the Horse; and the progressive Motion of the Horse is as much retarded by the Load, as the Motion of the Load is

the Body moves by the other Force) moves to  $EB$ , in a Direction parallel to itself; when the Body has advanc'd to  $G$  in the Line  $AE$ , the Line  $AD$  will have got to  $GF$ , and the Body will have passed over  $GH$ , such a Part of it, as bears the same Proportion to the whole Line  $GF$ , as  $AG$  does to  $AE$ , that is  $GH$  (the shorter Side of the Parallelogram  $GM$ ), is, to  $GF$ , or, which is the same Thing, to  $EB$  (the shorter Side of the Parallelogram  $ED$ ) as  $AG$  (the longer Side of the former) is to  $AE$  the longer Side of the latter; from whence the Parallelograms are similar, *El. 6. Def. 1.* and consequently, by 24 *El. 6.* the Point  $H$  is in the Diagonal, that is, the Body will always be found in the Line  $AB$ . Q. E. D.

*Coroll.* From hence we have an easy Method of resolving a given Motion into any two, or more Directions, whatever; viz. by describing a Parallelogram about the given Direction as a Diagonal, the two Sides of which will represent the Directions sought. Thus, suppose a Body was impell'd in the Line  $AB$ , we may conceive it as acted upon by two Forces at the same Time, one towards  $E$ , the other towards  $D$ , or any other two whatever, provided the Lines be drawn of such length, that, when the Parallelogram is completed, the given Line  $AB$  shall be its Diagonal.

promoted



Chap. IV. *Laws of Motion.* 23

promoted by the Endeavour of the Horſe \*. This will be better explained from the following Inſtance; let a Perſon, ſitting in a Boat, draw another Boat equally heavy towards him, they will both move towards each other with equal Velocities: Let the Boat he ſits in be the lighteſt, and it will move the faſteſt; becauſe the Action being equal on both Sides, the ſame Quantity of Motion will be given to each Boat, that is, the leſſer will have the greater Velocity \*. *momentum*

WE have a farther Confirmation of this from Attraction. Suppose two Bodies attracting one another, but prevented from coming cloſe together by ſome other Body placed between them: If their reſpective Actions, by which they tend towards each other, were not equal on both Sides, then would the intermediate Body be preſſed more one Way than the other, and ſo all three would begin to move of themſelves the ſame Way; but that three Bodies ſhould be put into Motion after this Manner, when no foreign Force acts upon them, is

\* It may be thought perhaps, (two equal and contrary Forces deſtroying one another) the Horſe will in this Caſe not be able to move at all, becauſe the Load draws him back, as much as he draws the Load forwards. But it is to be obſerved that the Strength of the Horſe is not properly exerted upon the Load but upon the Ground; and conſequently the Ground, reacting and continuing at Reſt, pushes the Horſe forward with juſt ſo much Force as the Horſe exerts, above what is counteracted by the Load.

\* See the Diſtinction between Motion and Velocity. Chap. 9.  
contrary



contrary to Experience, consequently whatever different Degrees of Force, any two Bodies may be capable of exerting, their mutual Actions on each other, are always equal. This may be try'd with a Loadstone and Iron; which, being put into proper Vessels, contiguous to one another, and the Vessels made to float on the Surface of Water, will be an exact Counterbalance to each other, and remain at Rest, whatever be the attractive Power of the Loadstone, or the Proportion of their respective Magnitudes.

THESE Laws receive an abundant Additional Proof from hence, *viz.* that all the Conclusions that are drawn from them, in Relation to the Phænomena of Bodies, how complicated soever their Motions be, are always found to agree perfectly with Observation. The Truth of which sufficiently appears in all Parts of the *Newtonian Philosophy* \*.

## C H A P. V.

*The Phænomena of Falling Bodies.*

I. **T**HE Laws of Nature being thus explained, we proceed to account for some of those Phænomena, which are solvable by them.

\* See these Laws explain'd more at large by *Cheyne* in his Principles of Philosophy. *Keil's* Introd. ad Phys. Præl. 11. 12.

II. To

II. To begin with those of falling Bodies. Constant Experience shews, that Bodies have a Tendency towards the Earth, which is call'd Gravity, the Laws of which were enumerated in Chap 3. §. 7.

III. THE Height, Bodies can be let fall from, bears so small a Proportion to their Distance from the Center of the Earth, that it cannot sensibly alter their Gravity; which therefore may be conceived, as acting constantly and uniformly upon them, during the whole Time of their Fall: From whence they must necessarily acquire, at every Instant, an equal Degree of Velocity, which on that Account will constantly increase, in Proportion to the Time the Body takes up in falling.

IV. THE Spaces Bodies fall through in different Times, reckoning from the Beginning of their Fall, are as the Squares of those Times; thus, a Body will fall four Times as far in two Minutes, as it does in one, and nine Times as far in three, sixteen Times as far in four, &c.\*

V. FROM

\* In order to demonstrate this Proposition, it will be necessary to lay down the following Theorem, *viz.*

That the Space a Body passes over, with an uniform Motion, is in a Ratio compounded of the Time and Velocity. For the longer a Body continues to move uniformly, the more Space it moves over; and the faster it moves during any Interval of Time, the farther it goes; therefore the Space is in a Ratio compound of both, that is, is had by multiplying one into the other.

*Coroll.* Therefore the Area of a Rectangle, one of whose Sides represents the Celerity a Body moves with, and the other the Time of its Motion, will express the Space it moves through.

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V. FROM this Proposition it follows, that a Body falls three Times as far, in the second Portion of Time, as it does in the first; five Times as far in the third; seven Times in the fourth, and so on in the Series of the odd Numbers: For otherwise it could not fall four

This being premised, let the Line  $AB$  (Fig. 4.) represent the Time a Body takes up in falling, and let  $BC$  express the Celerity acquir'd by its Fall; farther let the Line  $AB$  be divided into an indefinite Number of small Portions,  $ei$ ,  $im$ ,  $mp$ , &c. and let  $ef$ ,  $ik$ ,  $mn$ ,  $pq$ , &c. be drawn parallel to the Base. Now it is evident from §. 3. (*viz.* that the Velocities are as the Times in which they are acquir'd) that the Lines  $ef$ ,  $ik$ ,  $mn$ ,  $pq$ , &c. being to each other (4 *El.* 6.) as the Lines  $Ae$ ,  $Ai$ ,  $Am$ ,  $Ap$ , &c. will represent the Celerities in the Times represented by these: that is,  $ef$  will be as the Velocity of the Body in the small Portion of Time  $ei$ , and  $ik$  will be as the Velocity in the Portion of Time  $im$ ; in like Manner  $pq$  will be as the Velocity in the Portion of Time  $po$ , which Portions of Time being taken infinitely small, the Velocity of the Body may be suppos'd the same, during any whole Portion: and consequently, by the Corollary of the foregoing Theorem, the Space run over in the Time  $ei$  with the Velocity  $ef$  may be represented by the Rectangle  $if$ : in like Manner the Space run over in the Time  $im$ , with the Celerity  $ik$ , may be express'd by the Rectangle  $mk$ ; and that run over with the Celerity  $mn$  in the Time  $mp$ , by the Rectangles  $pn$ ; and so of the rest. Therefore the Space run over in all those Times will be represented by the Sum of all the Rectangle, that is, by the Triangle  $ABC$ , for those little triangular Deficiencies, at the End of each Rectangle, would have vanished, had the Lines  $ei$ ,  $im$ ,  $mp$ , &c. been infinitely short, as the Times they were suppos'd to represent. Now as the Space, the Body describes in the Time  $AB$ , is represented by the Triangle  $ABC$ , for the same Reason the Space pass'd over in the Time  $Ao$  may be represented by the Triangle  $Aor$ , but these Triangles, being similar, are to each other, as the Squares of their homologous Sides  $AB$  and  $Ao$  (20 *El.* 6): that is, the Spaces represented by the Triangles are to each other, as the Squares of the Times represented by the Sides, Q. E. D.

Spaces

I. Chap. V. *Falling Bodies.* 27

Spaces in two Minutes, and nine in three, as the Proposition asserts \*.

VI. THE Spaces, describ'd by falling Bodies in different Times, are as the Squares of the last acquir'd Velocities. For by §. 4. the Spaces are as the Squares of the Times, and by §. 3. the Velocities are as the Times; therefore the Spaces are also as the Squares of the Velocities.

VII. THE Space a Body passes over, from the Beginning of its Fall in any determinate Time, is half what it would describe in the same Time moving uniformly with its last acquir'd Velocity †.

VIII. IN like Manner, when Bodies are thrown up perpendicularly, their Velocities decrease, as the Times they ascend in increase; their Gravity destroying an equal Portion of their Velocity every Instant of their Ascent.

\* This may also be shewn in the following Manner. Let the Triangle  $ABC$  (Fig. 4.) be divided into lesser ones, as in Fig. 5. each equal to  $Dbr$ , which represents the Space described by the falling Body in  $Db$  the first Portion of Time; 'tis evident that, in  $bc$  the second Portion of Time, there are three such Triangles described, viz. those that lie between the Lines  $br$  and  $cs$ ; in  $cd$  the third Portion of Time, five such, viz. all between  $cs$  and  $dt$ ; in  $df$  the next equal Portion of Time, seven such, &c.

† For let the Time be  $AB$ , (Fig. 4) and the last Velocity  $BC$ , the Space the Body runs over, while it is acquiring that Velocity, is as  $ABC$ , but the Space it would pass over in the Time  $AB$ , was it to move uniformly with the Celerity  $BC$ , is by the Theorem (Note p. 25.) as the Space  $ABCD$ , double the former.  $\mathcal{Q}$ . E. D.

IX. THE

IX. THE Heights Bodies rise to, when thrown perpendicularly upwards, are as the Squares of the Times spent from their first setting out, to the Moment they cease to rise. That is, if a Body is thrown with such a Degree of Velocity, as to continue rising twice as long as another, it shall ascend four Times as high; if thrice, nine Times as high, &c.

THESE two are the converse of the third and fourth Sections \*.

## CH A P. VI.

### *Of the Descent of Bodies on oblique Planes, and of Pendulums.*

WHEN a Body descends on an oblique Plane, its Motion is continually accelerated by the Action of Gravity, but in a less Degree, than when it descends perpendicularly; its free Descent in this Case being hinder'd by the Interposition of the Plane: From whence it follows, that what was said in the last Chapter, concerning the perpendicular Descent of Bodies, is true of such as fall on oblique Planes, Allowance being made for the Difference of Acceleration.

II. The Effect Gravity has upon a Body falling down an oblique Plane, is to that which

\* See *Keil's* Introd. ad. *Phys. Præl.* 11. *Gravesande* L. 1. Ch. 17.



it exerts upon another falling freely, as the perpendicular Height of the Plane is to its Length \*.

III. THE Space, through which a Body falls down the oblique Side of a Plane, is to that through which it would fall perpendicularly in the same Time, as the perpendicular Height of the Plane is to its Length †.

For the Space, a Body falls through in any determinate Time, whether down an inclined Plane, or not, is as the Effect of the Gravity with which it is acted upon during that Time; but the Gravity, with which a Body descends down the oblique Side of a Plane (by the last Proposition) is to that with which it falls perpendicularly, as the perpendicular Height of the Plane is to its Length: The Space therefore, which a Body falls through obliquely, is to

\* *Demo.* Let *AC* (Fig. 6.) be the inclin'd Plane, the Body at *A*, and the Action of Gravity, whereby it endeavours to fall perpendicularly represented by the Line *AB*; let *AD* be perpendicular to *AC*, *AD* will then represent the Direction by which the Plane acts upon the Body (for all Bodies act in Lines perpendicular to their Surfaces) let then those two Forces be resolved into one in the Direction *AC*, (as shewn in Note to §. 4. Chap. IV.) by completing the Parallelogram *BD*, whose Diagonal will be *AG*. In order to this *BG* must be let fall perpendicularly upon *AC* (that it may be parallel to the opposite Side of the Parallelogram *AD*) consequently (8 *Elem.* 6.) *AG* is to *AB* as *AB* to *AC*, that is, the Tendency of the Body down the Plane is to its perpendicular Tendency, as *AB* is to *AC*. Q. E. D.

† From this Proposition it follows, that supposing *BG* (Fig. 6.) perpendicular to *AC*, the Body would fall from *A* to *G*, in the same Time another would fall from thence to *B*, for, as was observed (Note the last) *AG* is to *AB*, as *AB* to *AC*.

that



that which it would pass through perpendicularly in the same Time, also in that Proportion.

IV. THE Velocity, a Body acquires by falling perpendicularly, is to that which it acquires by falling obliquely in the same Time, as the Space of its perpendicular Descent is to that of its oblique one \*.

V. THE Time, in which a Body descends through the oblique Side of a Plane, is to that in which it falls through the perpendicular Height of the same, as the Length of the oblique Side is to its Height †.

VI. A BODY acquires the same Velocity in falling down the oblique Side of a Plane, as

\* Since by the Note to Section the last, a Body falls to *G*, (Fig. 6.) in the same Time another falls to *B*, and (by Chap. V. §. 7.) the Space, a falling Body passes over in any Time, is half that which it would run over in the same Time moving uniformly with its last acquir'd Velocity, it follows that the Body falling down the oblique Plane would pass over double the Space *AG*, moving uniformly with its last acquir'd Velocity, in a Portion of Time equal to that in which it was acquir'd; likewise double the Space *AB* would be passed over by the other Body moving uniformly with its last acquir'd Velocity, in a Portion of Time equal to that in which it was acquir'd; but since the Velocities of Bodies moving uniformly are as the Spaces they run over in equal Times, the Velocities of the Bodies in *G* and *B* are to each other as double the Lines *AG* and *AB*, that is, as the Lines themselves, which by §. 3. are as the Spaces run through in the same Time, from whence the Proposition is clear.

† Dem. The Square of the Time in which *AC* (Fig. 6.) is run over, is to the Square of the Time in which *AG* is run over as *AC* to *AG*, (by Chap. V. §. 4.) that is, since *AC*, *AB*, *AG* are continually proportional (8 Elem. 6.) as the Square of *AC* to the Square of *AB* (by Def. 10. Elem. 5.) therefore the Times themselves are as the Lines *AC* and *AB*, that is, as the oblique Side of the Plane to the perpendicular Height, *Q. E. D.*

Chap. VI. *on oblique Planes.* 31

it would do, if it fell freely through the perpendicular Height of it\*.

VII. A Body takes up the same Time in falling through the Chord of a Circle, whether it be long or short, as it does in falling perpendicularly through the Diameter of the same Circle †.

VIII. UPON this is founded the Theory of Pendulums: For from hence it follows, that supposing a Pendulum could be made to vibrate in a Chord of a Circle, instead of an Arch, all its Vibrations would require the same Time, whether they were large or small ‡.

IX. FROM hence we see the Reason, why the shorter Arches a Pendulum describes, the

\* *Dem.* The Square of the Velocity which a Body acquires by falling to *G*, is to the Square of the Velocity it acquires by falling to *C*, as the Space *AG* to the Space *AC* (by Chap. V. §. 4.) that is (by 8. *Elem.* 6. and *Def.* 10. *Elem.* 5.) as *AGq* to *ABq*; consequently the Velocity itself at *G* is to the Velocity itself at *C*, as *AG* to *AB*: But since *AG* is run over in the same Time *AB* is (see Note to §. 3.) the Velocity in *G* is also to the Velocity in *B*, as *AG* to *AB*, (by §. 4.) and consequently since the Velocities both in *C* and *B* bear the same Proportion to that in *G*, they must be equal to each other. *Q. E. D.*

† *Dem.* It was demonstrated (§. 3.) that a Body will fall from *A* to *G*, (*Fig.* 7) on the inclin'd Plane *AC*, in the same Time another would fall freely to *B*, provided *AGB* is a right Angle, in which Case *AG* (by 31 *Elem.* 3.) is a Chord of that Circle of which *AB* is the Diameter; therefore a Body falls through the Chord, &c. *Q. E. D.*

‡ This may be illustrated by conceiving the last Figure inverted (as in *Fig.* 8) where supposing the Ball suspended in such a Manner, as to swing in the right Line *GA* instead of the Arch *GA*, it would always fall through it in the same Time, however long or short it was, for the Inclination of the Line *GA* to the horizontal Line *BC*, is not alter'd by inverting the Figure.

nearer

nearer its Vibrations come to an Equality, for small Arches differ less from their Chords than large ones. But if the Pendulum is made to vibrate in a Curve, which Mathematicians call a *Cycloid*; each Swing will then be perform'd in the same Time, whether the Pendulum moves through a larger or lesser Space. For the Nature of this Curve is such, that the Tendency of a Pendulum towards the lowest Point of it, is always in Proportion to its Distance from thence; and consequently let that Distance be more or less, it will always be run over by the Pendulum in the same Time\*.

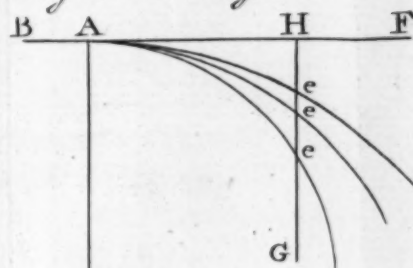
X. THE Time of the Descent and Ascent of a Pendulum, supposing it to vibrate in the Chord of a Circle, is equal to the Time in

\* The Description of a *Cycloid*:

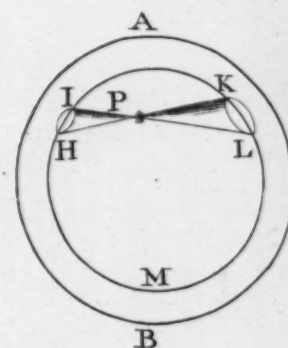
Upon the right Line *AB*, (*Fig. 9.*) let the Circle *CDE* be so plac'd, as to touch the Line in the Point *C*, then let this Circle roll along upon it from *C* to *H*, as a Wheel upon the Ground, then will the Point *C* in one Revolution of the Circle describe the Curve *CKH*, which is called a *Cycloid*. Now suppose two Plates of Metal bent into the form *HK* and *KC*, and placed in the Situation *LH* and *LC*, in such Manner, that the Points *H* and *C* may be apply'd to *L*, and the Points answering to *K* be apply'd to *H* and *C*. This done, if a Pendulum as *LP*, in Length equal to *LH*, be made to vibrate between the Plates or Cheeks of the Cycloid *LC* and *LH*, it will swing in the Line *CKH*; and the Time of each Vibration, whether the Pendulum swings through a small or a great Part of the Cycloid, will be to the Time a Body takes up in falling perpendicularly through a Space equal to *IK*, (half the Length of the Pendulum) as the Circumference of a Circle to its Diameter, and consequently it will always be the same. See these Things demonstrated in the Appendix.

which

Figure 1. Page 8.

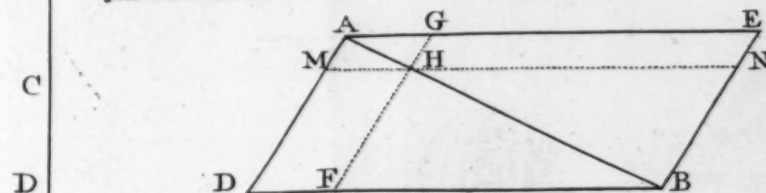


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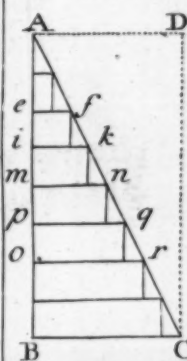


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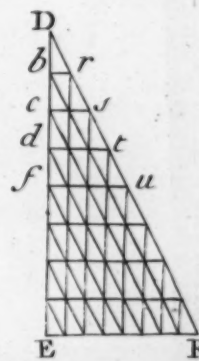
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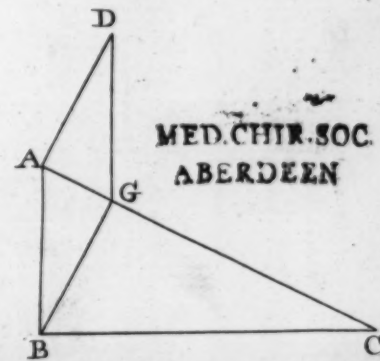
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F. 5. P. 26.



F. 6. P. 28.



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which a Body falling freely would descend through eight Times the Length of the Pendulum.

FOR the Time of the Descent alone upon the Chord is equal to that in which a Body would fall through the Diameter of the Circle (by §. 7.); that is, twice the Length of the Pendulum: But in twice that Time (*viz.* during a whole Vibration) the Body would fall four Times as far (Chap. V. §. 4.), that is, through eight Times the Length of the Pendulum.

XI. THE Times, that Pendulums of different Lengths perform their Vibrations in, are as the square Roots of their Lengths \*.

XII. THE Center of *Oscillation*, is a Point in which if the whole Gravity of a Pendulum was collected, the Time of its Vibration would not be alter'd thereby †; this is the Point from  
C whence

\* *Dem.* Let there be two Pendulums *A* and *B* (Fig. 10. and 11.) of different Lengths, the Time the first vibrates in (suppose through a Chord) is equal to the Time in which a Body would fall freely through *DA*, the Diameter of the Circle (as demonstrated §. 7.); in like Manner the Time *B* vibrates in is that in which a Body would fall through *FB*. Now the Times in which Bodies fall through different Spaces are as the square Roots of those Spaces, that is, of *DA* and *FB*, or of their Halves *CA* and *CB*, *i. e.* of the Lengths of the Pendulums.  
Q. E. D.

† The Rule for finding the Center of Oscillation.

If the Ball *AB* (Fig. 12.) be hung by the String *CD*, whose Weight is inconsiderable, the Center of Oscillation is found thus; suppose



whence the Length of a Pendulum is measur'd, which in our Latitude, in a Pendulum that swings Seconds, is thirty nine Inches and two Tenths.

XIII. THE Squares of the Times in which Pendulums, acted upon by different Degrees of Gravity, perform their Vibrations in, are to each other, inversly as the Gravities \*.

FROM whence it follows, that a Pendulum will vibrate slower when nearer the Equator, than the same when nearer the Poles ; for the

suppose *E* the Center of the Globe, take the Line *K* of such a Length, that it shall bear the same Proportion to *ED* as *ED* to *EC*, then *EH* being made equal to  $\frac{2}{3}$  of *K*, the Point *H* shall be the Center of Oscillation.

If the Weight of the Rod *CD* be too considerable to be neglected, divide *CD* (Fig. 13.) in *I*, so that *DI* may be equal to  $\frac{2}{3}$  of *CD*, and make a Line as *G*, in the same Proportion to *CD*, that the Weight of the Rod bears to that of the Globe, then having found *H* the Center of Oscillation of the Globe, as before, divide *IH* in *L*, so that *IL* may bear the same Proportion to *LH*, as the Line *CH* bears to the Line *G* ; then will *L* be the Center of Oscillation of the whole Pendulum. See the Method of finding a general Rule for determining the Center of Oscillation in all Cases whatever, in the Appendix.

\* *Dem.* The Spaces, falling Bodies descend through, are as the Squares of the Times, when the Gravity by which they are impell'd is given (Chap. V. §. 4.) ; and as the Gravity when the Time is given (for the Sum of the Velocities produced in any Time will always be as the generating Forces :) Consequently, when neither is given, they are in a Ratio compounded of both ; the Squares of the Times are therefore inversly as the Gravities. [For if in 3 Quantities *a*, *b*, *c* ; *a* is as  $\frac{a}{b}$  then *b* : - , i. e. if *a* is given, as - or as *c* inversly.] But

the Gravity of all Bodies is less, the nearer they are to the Equator; *viz.* on account of the spheroidal Figure of the Earth, and its Rotation about its Axis, as will be explain'd hereafter. To which we may add the Increase of the Length of the Pendulum occasion'd by the Heat in those Parts; (for we find by Experiment, that Bodies are enlarged in every Dimension in Proportion to the Degree of Heat that is given them;) for which Reason (§. 11.) the Vibrations of the Pendulum will also be slower.

## C H A P. VII.

## Of Projectiles.

**A** BODY, projected in a Direction parallel or oblique to the Horizon, would proceed on *in infinitum* in a right Line (by the first Law of Nature); but being continually accelerated toward the Earth by its Gravity, it will describe a Curve called a *Parabola* \*.

the Squares of the Times, in which Bodies fall through given Spaces, are inversly as the Gravities by which they are acted upon; then the Squares of the Times, in which Pendulums of equal Lengths perform their Vibrations, will be also in the same Ratio, on account of the constant Equality between the Time of the Vibration of a Pendulum, and of the Descent of a Body through eight Times its Length (§. 12.)

\* *Dem.* Let us suppose the Body thrown from *A* in the Direction *AB* horizontally (*Fig. 14.*) or obliquely (*Fig. 15.*) it would (if not attracted towards the Earth) move uniformly

from *A* towards *B*, that is, in equal Times it would describe equal Parts of the Line *AB*, as *AC*, *CD*, *DE*, &c. but, if in the first Portion of Time, while it would move from *A* to *C*, it would have descended from *A* to *G* by its Gravity, had it only been let drop from thence; it will by a Composition of these two Motions (Chap. IV. §. 2.) at the End of that Time be found in *H*, the opposite Angle of the Parallelogram *ACGH*. Then in twice that Time, *viz.* while it would have moved over two equal Portions, or from *A* to *D*, it would fall downwards to *M*, four Times as far as before (Chap. V. §. 4.) and will therefore be found in *I*, supposing *DI* equal and parallel to *AM*. Then again in three Portions of Time, or while it would have moved over three Divisions, that is from *A* to *E*, it would have fallen downwards nine Times as far as in the first Portion of Time, and therefore being carried by these two Motions will at the End of that Time be found in *K*, supposing *EK*, or its equal *AN*, nine Times as long as *AG* or *CH*, &c. Therefore the Lines *CH*, *DI*, *EK*, &c. which are to each other as the Numbers 1, 4, 9, &c. are as the Squares of the Lines *AC*, *AD*, *AE*; (these being only as the Numbers 1, 2, 3.) But this is the Property of the Parabolic Curve. (See De L'Hospital B. I. Prop. 1. Corol. 2. and Prop. 3. Corol. 1.) Consequently the Curve *AHIK*, &c. which the Body moves in, whether thrown horizontally or obliquely, is a Parabola. *Q. E. D.*

*Lemma 1.* The Quotient which arises from the Division of the Square of the Line *AC* by the Line *AG*, *viz.* the Quantity  $\frac{ACq}{AG}$  (in either of the Parabolic Curves, Fig. 14. or 15.) or

the Square of the Line *AD* divided by *AM*, *viz.*  $\frac{ADq}{AM}$  or the Square of *AE* divided by *AN*, *viz.*  $\frac{AEq}{AN}$  is equal to the

*Parameter* of the Point *A*, for *GHq* is equal to *AG* multiplied by the *Parameter* (De L'Hospital Con. Sect. B. I. Prop. 1 & 3.) therefore the *Parameter* is equal to *GHq* divided by *AG*, that is,  $\frac{ACq}{AG}$ . The same is demonstrable of *AD* divided by *AM*, &c.

and consequently any of these Quantities may be indifferently put to express the *Parameter* of the same Point.

*Lemma 2.* The Velocity a Body would acquire by falling from an Height equal to the fourth Part of the *Parameter* of the Point *A*, is to the Velocity it would acquire by falling from *A* to *N*, as *AE* is to twice *AN*. *Dem.*

*Dem.* Since we are comparing the Velocity, which a Body would acquire by falling through a fourth Part of the Parameter, with that which it would acquire by falling to  $N$ , let  $\frac{AEq}{AN}$

be made choice of to denote the Parameter. Then  $\frac{\frac{1}{2}AEq}{AN}$  will express a fourth Part of the Parameter. Now because the Velocities, acquir'd by falling Bodies, are as the square Roots of the Spaces they fall through (Chap. V. §. 6.) the Velocity, acquir'd by a Body in descending through  $\frac{\frac{1}{2}AEq}{AN}$ , is to that Velocity, which it would acquire by falling through  $AN$ , as the square Root of  $\frac{\frac{1}{2}AEq}{AN}$  to the square Root of  $AN$ ; that is, ex-

tracting the Roots of those Quantities, as  $\frac{\frac{1}{2}AE}{\sqrt{AN}}$  to  $\sqrt{AN}$ , and, multiplying each Term by  $\sqrt{AN}$ , as  $\frac{1}{2}AE$  to  $AN$ , or as  $AE$  to twice  $AN$ . Q. E. D.

*Prop.* The Velocity a Body ought to be projected with, to make it describe a given Parabola, is such as it would acquire by falling through a Space equal to the fourth Part of the Parameter belonging to that Point of the Parabola, from whence it is intended to be projected.

*Dem.* The Velocity with which a Body must be projected from  $A$  towards  $B$ , to make it describe the given Parabola  $AHIK$ , must be such, as would carry it to  $C$  by an uniform Motion, in the same Time that it would descend by its Gravity from  $A$  to  $G$ ; and to  $E$ , in the Time it would fall to  $N$ , &c. as was before observed. Now the Velocity, with which the Line  $AE$  is described with an uniform Motion, is to that which is acquired by the Body in falling to  $N$  in the same Time, as  $AE$  is to twice  $AN$ ; because (Chap. V. §. 7.) its Velocity in  $N$  would have carried it over twice  $AN$  in that Time, had it also been uniform. But by Lemma 2. the Velocity a Body would acquire, by falling through a Space equal to a fourth Part of the Parameter of the Point  $A$ , is to that which it would acquire by falling from  $A$  to  $N$ , also as  $AE$  to twice  $AN$ . Since therefore the Velocity, with which the Line  $AE$  is described (or, which is the same Thing, that whereby the Body is projected) and that which a Body would acquire by falling through a fourth Part of the Parameter of the Point

*A*, bear one and the same Proportion to that Velocity which a Body would acquire by falling from *A* to *N*, they must be equal. Q. E. D.

*Corol.* This affords us an easy Method of finding what Direction it is necessary to throw a Ball in with a given Velocity, in order to strike an Object in a given Situation, *v. g.* Let it be requir'd to strike an Object as *K*, with a Ball thrown from *A* with a given Velocity. Here it is only necessary to make the Triangle *ANK* (suppose a right Line drawn from *A* to *K*) such, that  $\frac{NK}{AN}$  or, which the same Thing,  $\frac{AE}{EK}$  in the Triangle *AEK*, may be equal to four Times the Space a Body must fall through, to acquire such a Degree of Velocity as that with which it is intended to be thrown, and then *AE* will be the Direction sought. In order to this we must lay down the following Lemma.

*Lemma.* Let there be a Circle as *ABC* (Fig. 16) *AK* a Tangent in the Point *A*, *AB* and *KI* parallel to each other, and let the other Lines be drawn, as in the Figure, I say  $\frac{AE}{EK} = AB$ .

For the Angle *ABE* is equal to the Angle *EAK* (32. Elem. 3.) and the Angle *BAE* is equal to the Angle *AEK* as alternate, therefore the Triangles *ABE* and *AEK* are similar; consequently *AB* is to *AE*, as *AE* to *EK*, and multiplying the extreme Terms together, and middle Terms together,  $AB \times EK = AE^2$ , and dividing both Sides of the Equation by *EK*,  $AB = \frac{AE^2}{EK}$ . Q. E. D. By the same Method of arguing

$\frac{AI}{IK}$  may be proved equal to *AB*.

#### THE PROBLEM.

Let it be requir'd to strike an Object in a given Situation as *K* (Fig. 17.) with a Ball projected from *A* with a given Velocity.

*Solution.* Erect *AB* perpendicular to the Horizon, and equal to four Times the Height a Body must fall from, to acquire the Velocity with which the Ball is to be thrown; bisect this in the Point *G*, through which draw *HC* perpendicular to *AB*, and meeting the Line *AC* (perpendicular to *AK*) in *C*. On *C* as a Center with the Radius *CA*, describe the Circle *ABD*; lastly, through *K* draw the Line *KEI* perpendicular

II. THE greatest horizontal Distance, to which a Body can be thrown with a given Velocity, is at the Elevation of 45 Degrees\*.

III. IF two Balls are thrown at different Elevations (but with equal Degrees of Velocity) the one as much above forty-five Degrees as the other below, the horizontal Distances (or Randoms) where they both fall, will be the same †.

## IV. THE

pendicular to the Horizon, cutting the Circle in the Points *E* and *I*; I say *AE* or *AI* will be the Direction sought.

For by the Lemma,  $AB = \frac{AEq}{EK}$  or  $\frac{AIq}{IK}$ , but (*ex constructione*) *AB* is equal to four Times the Height a Body must fall from, to acquire the Velocity with which it is to be thrown, therefore its Equal,  $\frac{AEq}{EK}$  or  $\frac{AIq}{IK}$ , is the same, which by the Corollary was the Thing requir'd to determine the Direction sought; consequently the Parabola, which the Body will describe, will pass through the Point *K*. Q. E. D.

Coroll. 1. From hence it is evident, that, if the Object to be struck be placed any where in the horizontal Line *AO* (*Fig. 18.*) beyond *Q*, the Problem is impossible; for then *QH* will not touch the Circle, and the Ball will not reach that Point with any Direction whatever.

\* And that when the Ball is directed towards *H*, it will fall on *Q* the greatest Distance it can possibly be thrown to; but the Angle *QAH* being equal to *ABH* in the opposite Segment (32. *Elem.* 3.) is equal to half *AGH* at the Center (20. *Elem.* 3.) which is a right one; consequently *QAH* is an Angle of 45 Degrees.

† Coroll. 2. If the Object is situated in the horizontal Line *AO* (*Fig. 19.*) but nearer to *A*, than the greatest horizontal Distance at which it may be struck, suppose in *K*; the two Directions *AE* and *AI*, with which it may be hit, are equally distant



IV. THE Height a Body will rise to, when thrown perpendicularly upwards, is equal to half the greatest horizontal Distance it can be thrown to, with the same Velocity \*.

FROM hence we may easily know how far a Mortar-Piece, or other such Machine, will carry a Ball. Let the Ball be shot perpendicularly upwards, note the Time of its Ascent and Descent, half that is the Time of Descent, from whence we learn the Height from which it falls (for Bodies are observ'd to fall in the first Second of Time sixteen Feet, consequently in two Seconds they fall four Times sixteen Feet (Chap V. §. 4.) in three, nine Times as much, &c. but the perpendicular Height from whence it falls is the same with that to which it ascended, consequently (§. 4.) the double of this is equal to the greatest horizontal Distance to which that Machine will carry the Ball with an equal Charge.

distant from the Direction  $AH$ ; for the Angles  $IAH$  and  $HAE$  are equal, as inscribing on equal Arches  $IH$  and  $HE$ , (28. Elem. 3.)

\* Coroll. 3. The Altitude of a perpendicular Projection is equal to a fourth Part of the Height  $AE$ ; for the Velocity, with which the Body is projected, is (*ex hypoth.*) such as it would acquire by falling through a fourth Part of the Line  $AB$ ; but a fourth Part of the Line  $AB$  is equal to half the Line  $GH$ , or  $AQ$  (Fig. 18.) that is, half the greatest horizontal Distance to which the Body can be thrown.

See Cotes's Harmonia Mensurarum, p. 87. Keil's Introduct. ad Phys. Præl. 16.

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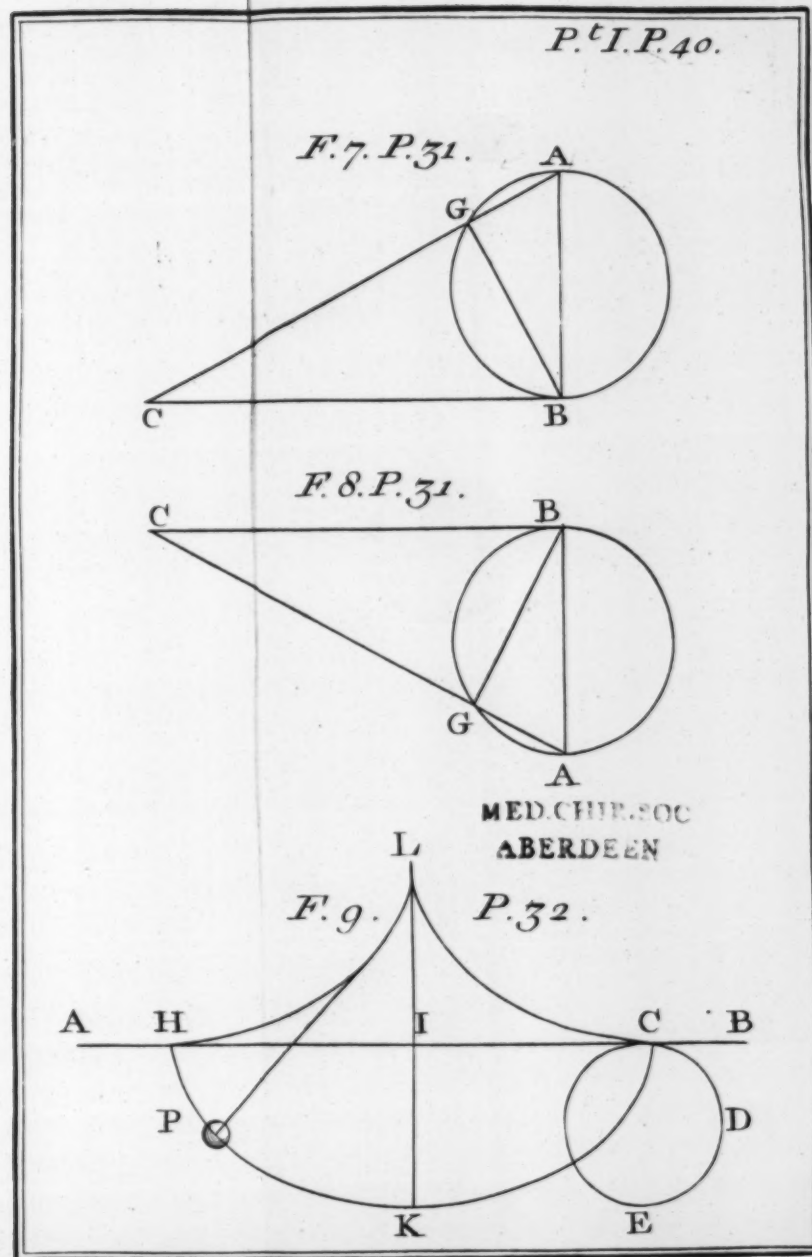
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V. THE Randoms of two Projectiles, having the same Degrees of Elevation, but thrown with different Velocities, are as the Squares of the Velocities: For by the last, the Randoms are equal to double the Heights to which the Bodies thrown perpendicularly upwards will ascend, but the Heights are (Chap. V. §. 6.) as the Squares of the Velocities, therefore the Randoms are so too.

VI. SUPPOSING the Motion of the Earth, all Bodies, when thrown perpendicularly upwards, describe *Parabola's*; notwithstanding they appear both to ascend and descend in the same right Line.

THIS may very easily be illustrated in the following Manner; let there be a Body carried uniformly along the Line AB (*Fig. 20.*) by the Motion of the Earth from A towards B; as it passes the Point C, let it be projected upwards, by some Force acting underneath it in the Direction CO perpendicular to the former: the Body will not thereby lose its Motion, which it had in common with the Earth, towards B (by the first Law of Nature) but will be carried by two Motions, one towards B, the other towards O; let us then suppose, that in the Time it would have advanced forwards to P in the Line AB, it rises upwards to M in the Line CO; it will then be found in D (Chap. IV. §. 2.) In like Manner, supposing it would have advanced forward to Q  
while

while it rises to N, it would then be found in E, afterwards in F, then in G, &c. describing the Curve CGL, which (from what was demonstrated under §. 1.) is a Parabola \*.

THE Reason, why it appears to a Spectator to rise and fall perpendicularly, is because he is carried uniformly along with it by the Motion of the Earth in the Direction AB; *v. g.* Suppose the Spectator at C at the Instant the Body is thrown from thence, when it arrives at D, he will be moved to P, when the Body is at E he will be at Q, &c. as is evident from what was observed about the Motion of the Body in the Curve; and they will both meet in L. Therefore since the Spectator imagines himself standing still, and sees the Body always perpendicularly over his Head, he must of Course think that it rises right up, and falls right down.

It may be proper to observe here, that Experiments, relating to the Motion of projected Bodies, do not exactly answer the Theory, the Resistance of the Air destroying Part of their Motion; for which a small Allowance is to be made.

\* *Dem.* Suppose the Motion the Body had in common with the Earth towards B (*Fig.* 21) and that with which it is projected towards O, such, as being compounded (*Chap.* IV. §. 2) would have produced a Motion in the Direction CX; it will follow from thence, that the Path described by it will be the same, as if it had been thrown in that Direction from a Point as C at Rest; but in that Case it would have described a Parabola as CGL (§. 1.) therefore also in this. *Q. E. D.*

## C H A P. VIII.

*Of Centripetal and Centrifugal Forces.*

**W**HEN a Body is projected in an horizontal Direction, and by its Gravity made to describe a Parabola, as demonstrated Chapter the last ; the Curvature of that Parabola will vary in Proportion to the Velocity with which the Body is thrown, and the Gravity which impels it towards the Earth. For the less its Gravity is in Proportion to the Quantity of Matter it contains, or the greater the Velocity is with which it is projected; the less it will deviate from a straight Line, and the further it will go, before it falls to the Earth. For Instance, if a Bullet be shot out of a Cannon from the Top of a Mountain with a given Velocity in an horizontal Direction, and goes in a Curve Line, suppose to the Distance of two Miles from the Foot of the Mountain before it falls to the Ground; the same Bullet, shot with a much greater Velocity, would fly to a much greater Distance before its Fall. And by encreasing the Velocity, the Distance to which it is projected may be encreased as much as you please; so that it will not fall to the Ground, till it is arrived at the Distance of ten, or thirty, or ninety Degrees; or till it has even surrounded the whole



whole Earth, and arrives at the very Top of the Mountain from whence it was projected: In this Case it will perform a second Revolution, and so on *in infinitum*, without a new Projection, provided the Resistance of the Air is taken away. And were the Sphere of the Earth's Attraction a limited one, it might be projected with such Violence, that it would continually recede from the Earth in a Curve, till at length it would go out of that Sphere; after which it would go on in a straight Line without ever returning. Which may thus be illustrated.

LET ABC (*Fig. 22.*) represent the Earth, M the Top of the Mountain from whence the Body is projected in the Direction MQ: It may be thrown with such Force as to carry it to B before it falls, or to C, or even to go round to M, describing the Circle MDM; or lastly, it may be made to describe the Curve MO, by which Means, if it gets out of the Sphere of the Earth's Attraction, suppose at O, it will go on afterwards in the infinite straight Line OX, there being nothing to stop or alter its Course. Farther it may be projected with such a Force from M (*Fig. 23.*) as will cause it continually to recede from the Earth, till it arrives at the opposite Point G, describing the Curve MKG; and if the Point G is within the Sphere of the Earth's Attraction, the Body will return to M, describing the Curve GLM exactly

actly similar to MKG; and in moving nearer and nearer to the Earth, till it comes to M, will regain what Velocity it lost in going from M to G, its Gravity conspiring with its Motion from G to M in the same Degree in which it opposed it from M to G; consequently the Body when at M, having recovered the Velocity with which it set out, will be enabled to perform a second Revolution in the same Curve as before; and so on.

AGAIN, suppose it had been projected from the Point M, with a less Degree of Force than would have carried it round in the Circle MDM (*Fig. 22.*) but greater than would have suffered it to have fallen to the Earth at the opposite Point F (*Fig. 23.*) it would also in this Case have arrived at the Point of M, from whence it set out; for the Excess of Velocity it would have gained in F, by its Tendency towards the Earth in its Way thither, over and above that, with which it was projected from M, would be sufficient to carry it off again from the Earth, till it arrived at M; and to make it describe the Path FPM exactly similar and equal to the former, losing in its Way from F to M just so much Velocity, as it gained by passing from M to F; and thereby it would be enabled to perform an infinite Number of Revolutions in the same Curve, without requiring a second Projection.

FROM

FROM hence it follows, that supposing a Body projected from a Point at any Distance within the Sphere of the Earth's Attraction, with a Force sufficient to carry it half round without falling to the Surface, it is impossible it should fall upon any Part of the other half; but will return to the Point from whence it set out, making continual successive Revolutions in the same Curve; provided it meets with no Resistance from the Medium through which it passes, nor any other Obstacle to obstruct its Motion \*.

FROM hence also it is clear, that, the nearer the revolving Body approaches to the Earth, the faster it moves; its Velocity being continually increased during the Time of its Access towards the Earth, and as much retarded during its Recess from it. And this Acceleration and Retardation will always be such, that the Body will describe equal Areas in equal Times: The Meaning of which is, that if we imagine a Line constantly extended from the Center of the Earth to the Center of the Body, that Line will always describe or pass through equal Surfaces or Spaces in equal

\* Gravity is here supposed to be inversely as the Squares of the Distance from the Earth, for 'tis possible that the Force, by which a Body tends towards another, may vary in such a Manner at different Distances, that the projected Body shall describe a spiral Line, continually approaching to, or receding from that about which it revolves.

# Chap. VIII. Central Forces.

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Times, for it constantly becomes shorter the faster it moves, and *vice versâ* \*.

AND for the same Reason that a Body, projected with a sufficient Velocity, may by the Force of Gravity be made to describe a Curve round the Earth, and perform continual successive Revolutions therein; it follows, that the Moon may by the same Force of Gravi-

\* *Dem.* Let the Time in which the Body performs one Revolution be divided into equal Parts, in the first of which let the Body describe the right Line  $AB$  (Fig. 24.) in the second Part of Time, if not prevented, it would go straight on to  $c$ , describing the Line  $Bc$  equal to  $AB$  by the first Law of Nature; the Lines  $SA$ ,  $SB$ ,  $Sc$  being drawn, the Triangles  $SBA$ ,  $ScB$ , will be equal to each other, their Bases  $AB$  and  $Bc$  being equal, and their Heights  $S$  the same (38. *Elem.* 1.) When the Body arrives at  $B$ , let the centripetal Force by one single Impulse turn it out of the straight Line  $Bc$  into the Line  $BC$ ; in which let it move on uniformly without receiving a second Impulse till it comes to  $C$ . Let  $Cc$  be drawn parallel to  $SB$  meeting  $BC$  in  $C$ ; then at the End of the second Part of Time, the Body will be found in  $C$ , having described the Diagonal of the Parallelogram  $Nc$  (Chap. IV. §. 2.) Draw  $SC$ , and the Triangle  $SCB$  will be equal to the Triangle  $ScB$ , (each having the same Base  $SB$  and being between the same Parallels  $Cc$  and  $SB$ ) and therefore also equal to the Triangle  $SBA$ . For the same Reason, if the centripetal Force acts in the Points  $C$ ,  $D$ ,  $E$  successively, so as to make the Body describe the straight Lines  $CD$ ,  $DE$ ,  $EF$ , &c. in so many equal Parts of Time, the Triangles  $SCD$ ,  $SDE$ ,  $SEF$ , &c. will be all equal to one another and to the Triangle  $SAB$ . Consequently equal Areas are described in equal Times. Let us then suppose the Bases of those Triangles, *viz.*  $AB$ ,  $BC$ ,  $CD$ ,  $DE$ , &c. diminished in *infinitum*, and likewise the Times in which they are described; then will the Perimeter  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $F$ , &c. become a Curve, and any Number of those Triangles taken together (or their Areas) will be proportionable to the Times in which they are described. Q. E. D.

ty

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ty be made to revolve about the Earth, or any other Planet, by the like Force, about the Sun; if the Velocities with which they move are duly adjusted to the Forces by which they are acted upon.

WHEN a Body revolves about another in this Manner, that Force or Power by which it is prevented from flying off (as it otherwise would do in a Tangent to the Curve which it describes) is call'd the *Centripetal*; the Counter-action of this, by which it endeavours to fly off, the *Centrifugal*; these, by the third Law of Nature, being equal to each other, are called by one common Name, *Central Forces*; that with which the Body is at first projected, or continues its Motion from any Point, is the *Projectile Force*; and the Time in which it performs one Revolution, the *Periodical Time*.

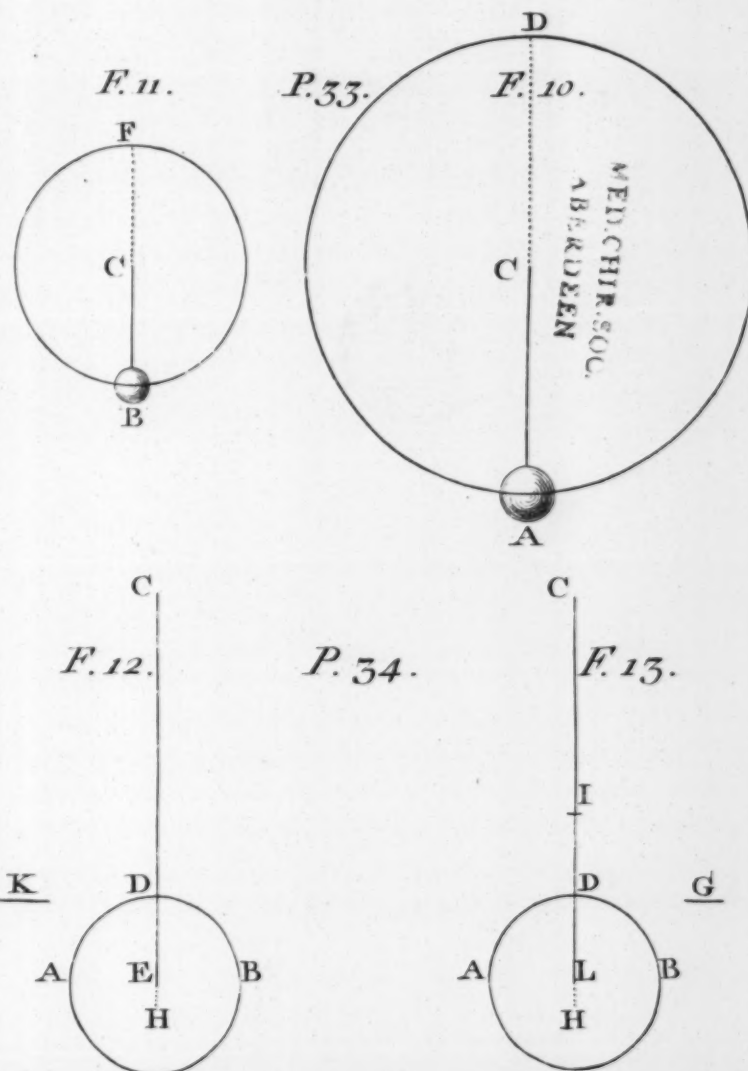
THESE Forces, properly relating to the Motions of the Heavenly Bodies, will be more largely treated of in the fourth Part of this Treatise.

## C H A P. IX.

### *Of the Communication of Motion.*

I. **B**EFORE we proceed to explain the Laws, by which Bodies communicate their Motion from one to another, it is very necessary

*P.<sup>t</sup> I. P. 48.*







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necessary to make a Distinction between Motion and Velocity; which ought to be well observ'd, and is as follows.

By the Motion of a Body (sometimes called its Quantity of Motion, sometimes its *Momentum*) is not to be understood the Velocity only, with which the Body moves; but the Sum of the Motion of all its Parts taken together: Consequently the more Matter any Body contains, the greater will be its Motion, though its Velocity remains the same. Thus, supposing two Bodies, one containing ten Times the Quantity of Matter the other does, moving with equal Velocity; the greater Body is said to have ten Times the Motion, or Momentum, that the other has: For 'tis evident, that a tenth Part of the larger has as much, as the other whole Body. In short, that Quality in moving Bodies, which Philosophers understand by the Term Momentum or Motion, is no other than what is vulgarly called their *Force*, which every one knows to depend on their Quantity of Matter, as well as their Velocity. This is that Power, a moving Body has to affect another in all Actions that arise from its Motion, and is therefore a fundamental Principle in Mechanics.

II. Now, since this Momentum, or Force, depends equally on the Quantity of Matter a Body contains, and on the Velocity with which it moves; the Method, to determine how  
D great

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great it is, is to multiply one by the other. Thus, suppose two Bodies, the first having twice the Quantity of Matter, and thrice the Velocity, which the other has; any two Numbers, that are to each other as two to one, will express their Quantities of Matter (it being only their relative Velocities and Quantities of Matter which we need consider) and any two Numbers that are as three to one their Velocities; now multiplying the Quantity of Matter in the first, *viz.* two by its Velocity three, the Product is six; and multiplying the Quantity of Matter in the second by its Velocity, *viz.* one by one, the Product is one; their relative Forces therefore or Powers will be as six to one, or the Moment of one is six Times greater than that of the other. Again if their Quantities of Matter had been as three to eight, and their Velocities as two to three, then would their Moments have been six to twenty four, that is, as one to four.

THIS being rightly apprehended, what follows, concerning the Laws of the Communication of Motion by Impulse, and the mechanical Powers, will be easily understood.

The Communication of Motion.

I. In Bodies not Elastic.

III. THOSE Bodies are said to be not Elastic, which, when they strike against one another

Chap. IX. *Communication of Motion.* 51

ther, do not rebound, but accompany one another after Impact, as if they were joined. This proceeds from their retaining the Impression, made upon their Surfaces, after the impressing Force ceases to act. For all rebounding is occasioned by a certain Spring in the Surfaces of Bodies, whereby those Parts, which receive the Impression made by the Stroke, immediately spring back, and throw off the impinging Body; now, this being wanting in Bodies void of Elasticity, there follows no Separation after Impact.

IV. WHEN one Body impinges on another which is at Rest, or moving with less Velocity the same Way, the Quantity of the Motion or Momentum in both Bodies taken together remains the same after Impact, as before; for by the third Law of Nature, the Reaction of one being equal to the Action of the other, what one gains, the other must lose.

THUS, suppose two equal Bodies, one impinging with twelve Degrees of Velocity on the other at Rest: The Quantities of Matter in the Bodies being equal, their Moments and Velocities are the same; the Sum in both twelve; this remains the same after Impact (§. 4.) and is equally divided between them (§. 3.) they have therefore six a-piece, that is, the impinging Body communicates half its Velocity, and keeps half.

## 52 *Communication of Motion.* Part I.

V. WHEN two Bodies impinge on each other by moving contrary Ways, the Quantity of Motion, they retain after Impact, is equal to the Difference of the Motion they had before ; for by the third Law of Nature, that, which had the least Motion, will destroy an equal Quantity in the other, after which they will move together with the Remainder, that is the Difference.

THUS for Instance, let there be two equal Bodies moving towards each other, the one with three Degrees of Velocity, the other with five, the Difference of their Moments or Velocities will be two ; this remains the same after Impact (§. 5.) and is equally divided between them (§. 3.) they have therefore one a-piece : That is, the Body, which had five Degrees of Velocity, loses three, or as much as the other had, communicates half the Remainder, and keeps the other half\*.

\* From these Positions it is easy to reduce a Theorem, that shall shew the Velocity of Bodies after Impact in all Cases whatever, Let there be two Bodies *A* and *B*, the Velocity of the first *a*, of the other *b* ; then (§. 2.) the Moment of *A* will be expressed by *Aa*, and of *B* by *Bb* ; therefore the Sum of both will be  $Aa+Bb$  ; and  $Aa-Bb$  will be the Difference when they meet. Now these Quantities (by §. 4. and 5.) remain the same after Impact ; but knowing the Quantities of Motion and Quantities of Matter, we have the Velocity (which §. 3. is the same in both) by dividing the former by the latter (as follows from §. 2.) therefore  $\frac{Aa+Bb}{A+B}$  or  $\frac{Aa-Bb}{A+B}$  will in all Cases express the Velocity of the Bodies after Impact.

II. In

Chap. IX. *Communication of Motion.* 53

II. In Elastic Bodies.

VI. BODIES perfectly *Elastic* are such as rebound after Impact with a Force equal to that with which they impinge upon one another: Those Parts of their Surfaces, that receive the Impression, immediately springing back, and throwing off the impinging Bodies with a Force equal to that of Impact.

VII. FROM hence it follows, that the Action of Elastic Bodies on each other (that of the Spring being equal to that of the Stroke) is twice as much as the same in Bodies void of Elasticity. Therefore, when Elastic Bodies impinge on each other, the one loses, and the other gains twice as much Motion as if they had not been Elastic; we have therefore an easy Way of determining the Change of Motion in Elastic Bodies, knowing first what it would have been in the same Circumstances, had the Bodies been void of Elasticity.

THUS, if there be two equal and Elastic Bodies, the one in Motion with twelve Degrees of Velocity impinging on the other at Rest, the impinging Body will communicate twice as much Velocity as if it had not been Elastic, that is (by §. 4.) twelve Degrees, or all it had; consequently it will be at Rest, and the other will move on with the whole Velocity of the former.



## 54 *Communication of Motion.* Part I.

VIII. It sometimes happens, that in Bodies not Elastic, the one loses more than half its Velocity, in which Case, supposing them Elastic, it loses more than all; that is, the Excess of what it loses, above what it has, is negative, or in a contrary Direction; thus, suppose the Circumstances of Impact such, that a Body, which has but twelve Degrees of Velocity, loses sixteen; the overplus four is to be taken the contrary Way, that is, the Body will rebound with four Degrees of Velocity. *v. g.* Let it be required to determine the Velocity of a Body after Impact against an immoveable Object. Let us first suppose the Object and Body both void of Elasticity: 'Tis evident the impinging Body would be stopp'd or lose all its Motion, and communicate none; if they are Elastic, it must lose twice as much (by §. 7.) and consequently will rebound with a Force equal to that of the Stroke.

IX. It is sufficient if only one of the Bodies is Elastic, provided the other be infinitely hard; for then the Impression in the Elastic Body will be double of what it would have been, had they both been equally Elastic: And consequently the Force with which they rebound, will be the same as if the Impression had been equally divided between the two Bodies.

X. THERE are no Bodies, that we know of, either perfectly Elastic, or infinitely hard; the

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Chap. IX. *Communication of Motion.* 55

the nearer therefore any Bodies approach to Perfection of Elasticity, so much the nearer do the Laws, which they observe in the mutual Communication of their Motion, approach to those we have laid down.

XI. Sir ISAAC NEWTON made Trials with several Bodies, and found that the same Degree of Elasticity always appeared in the same Bodies, with whatever Force they were struck, so that the Elastic Power, in all the Bodies he made Trial upon, exerted itself in one constant Proportion to the compressing Force. He found the Celerity with which Balls of Wool, bound up very compact, receded from each other, to bear nearly the Proportion of five to nine to the Celerity wherewith they met; and in Steel, he found nearly the same Porportion; in Cork the Elasticity was something less, but in Glass much greater; for the Celerity, with which Balls of that Material separated after Percussion, he found to bear the Proportion of fifteen to sixteen to the Celerity wherewith they met\*.

XII. WE have hitherto supposed the Direction, in which Bodies impinge upon one another, to be perpendicular to their Surfaces: When it is not so, the Force of Impact will be less, by how much the more that Direction varies from the Perpendicular; for it is manifest that a direct Impulse is the greatest of all

\* Newt. Princip. Phil. pag 21.

56 *Communication of Motion.* Part I.  
 others that can be given with the same Degree  
 of Velocity \*.

XIII. THIS is the Case, when Bodies impel  
 one another by acting upon their Surfaces;  
 but in Forces, where the Surfaces of Bodies  
 are not concerned, as in Attraction, &c. we  
 must not consider the Relation which the Di-  
 rection of the Force has to the Surface of the  
 Body to be moved, but to the Direction in  
 which it is to be moved by that Force. Here  
 the Force of Action will be less, by how  
 much the more these two Directions vary  
 from each other †. My Meaning in both  
 Cases

\* The Force of oblique Percussion is to that of direct, as the  
 Sine of the Angle of Incidence to the Radius.

*Dem.* Let there be a Plane as  $AD$  (*Fig. 25.*) against which let  
 a Body impinge in the Point  $D$  in the Direction  $BD$ : which Line  
 may be supposed to express the Force of direct Impulse, and may  
 be resolved into two others (*Chap. IV. §. 2.*)  $BC$  and  $BA$ ; the  
 one parallel, the other perpendicular to the Plane; but that Force  
 which is exerted in a Direction parallel to the Plane can no Way  
 affect it; the Stroke therefore arises wholly from the other Force  
 expressed by the Line  $BA$ ; but this is to the Line  $BD$ , as the Sine  
 of the Angle of Incidence  $ADB$  to the Radius; from whence  
 the Proposition is clear.

If the Surface of the Body to be struck is a Curve, then let  $AD$   
 be made a Tangent to  $D$  the Point of Incidence, and the De-  
 monstration will be the same.

† The Force of oblique Action is to that of direct, as the Co-  
 Sine of the Angle comprehended between the Direction of the  
 Force, and that wherein a Body is to be moved thereby, to the  
 Radius.

*Dem.* Let  $FD$  (*Fig. 26.*) represent a Force acting upon a  
 Body as  $D$ , and impelling it towards  $E$ ; but let  $DM$  be the  
 only Way in which it is possible for the Body to move; the  
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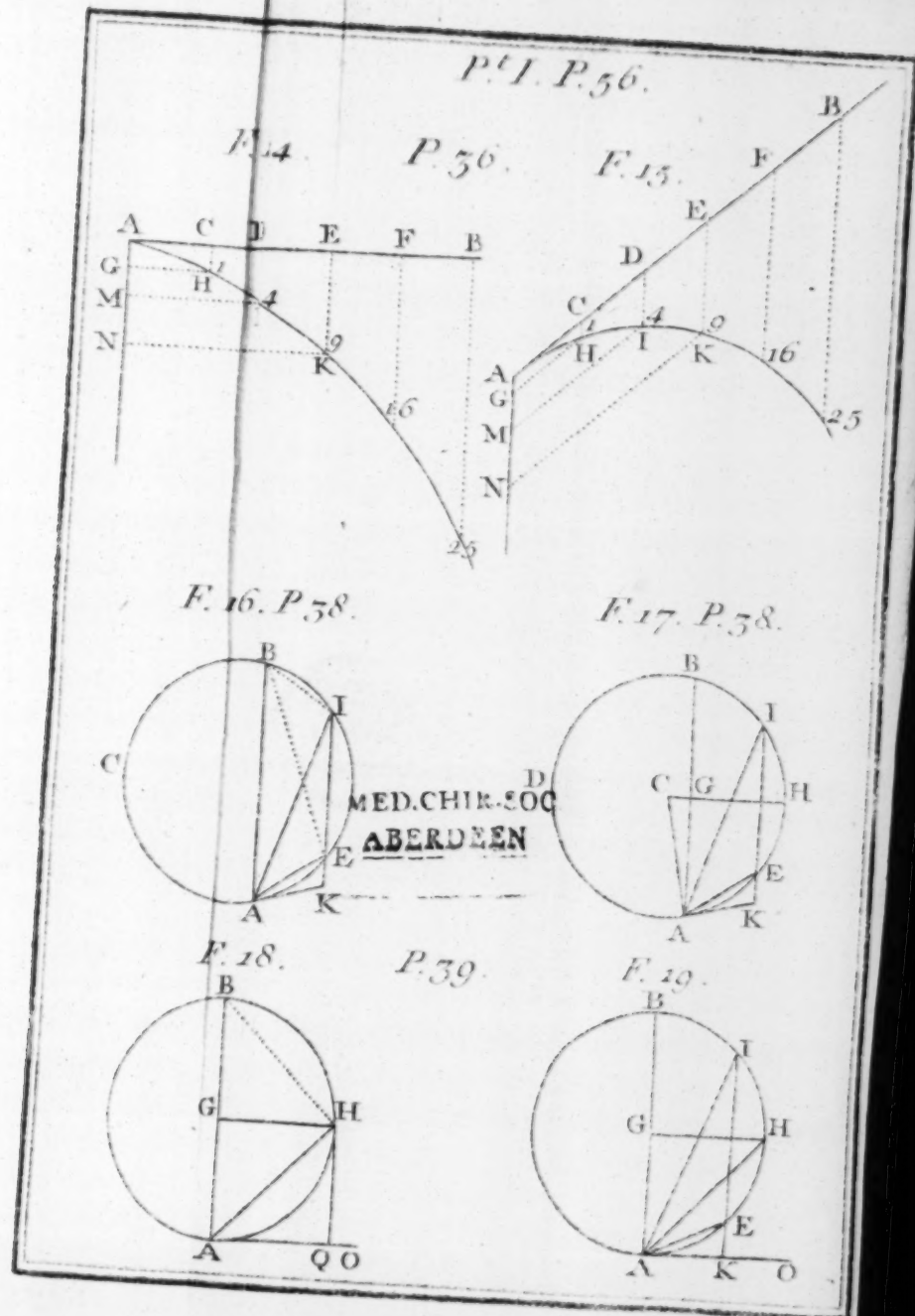
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## Chap. IX. *Communication of Motion.* 57

Cases will be understood from the Instance of a Ship under Sail. The Force, by which the Wind acts upon the Sail, will be less, by how much the more the Direction of the Wind varies from one that is perpendicular to the Surface of the Sail: But the Force of the Sail, to move the Ship forward, will be less, by how much the more the Direction of the Ship's Course varies from that, in which she is impell'd by the Sail.

XIV. To this we may add the following Proposition, relating to oblique Forces, *viz.* that, if a Body is drawn or impelled three different Ways at the same Time by as many Forces acting in different Directions; and if the Quantity of those Forces is such, that the Body is kept in its Place by them: Then will the Forces be to each other, as the several Sides of a Triangle, drawn respectively parallel to the Directions in which they act\*.

### CHAP.

Force  $FD$  may be resolved (Chap. 4. §. 2.) into two others  $FG$  and  $FH$ , or which is equal to  $GD$ ; but 'tis evident that only the Force  $GD$  impels it towards  $M$ . Now,  $FD$  being the Radius,  $GD$  is the Co-Sine of the Angle  $FDG$  comprehended between the two Directions  $FE$  and  $GM$ ; from whence the Proposition is clear.

\* *Dem.* Let the Lines  $AB$ ,  $AD$ ,  $AE$ , (Fig. 27.) represent the three Forces acting upon the Body  $A$  in those Directions, and by that Means keeping it at Rest in the Point  $A$ . Then the Forces  $EA$  and  $DA$  will be equivalent to  $BA$ , otherwise the Body would be put into Motion by them (*contra Hypoth.*) But these Forces are also equivalent to  $AC$  (Chap. IV. §. 2.) consequently  $AC$  may be made use of to express the Force  $AB$ ; and  $EC$ , which is parallel and equal to  $AD$ , may express the Force





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## Chap. IX. *Communication of Motion.* 57

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## C H A P. X.

*Of the Mechanical Powers.*

I. **H**AVING, in the foregoing Chapter, accounted for the Communication of Motion by Impulse; we proceed next to consider Motion as communicated by Means of certain Instruments, commonly known by the Names of *Mechanical Powers*. The Use of these Powers consists chiefly in managing great Weights, or performing other Works with a determinate Force.

II. THEY are usually reckoned five, viz. The Lever, the Wheel and Axis, the Pulley, the Screw, and the Wedge; to which some add the inclined Plane. To these all Machines how complicated soever are reducible.

III. THESE Instruments have been of very ancient Use; for we find that *Archimedes* was well acquainted with the Extent of their Power; as may be inferred from that celebrated Saying of his,  $\Delta\omicron\varsigma\ \pi\epsilon\varsigma\ \epsilon\tilde{\alpha}\nu$ , καὶ ἡ γυνὴ κινῆσαι. By which he meant, that the greatest imaginable Weight might be moved with the smallest Power.

Force *AD*, while *AE* expresses its own: But *ACE* is a Triangle whose Sides are all parallel to the given Directions; therefore the Sides of this Triangle will express the Relation of the Forces by which the Body is kept at Rest. Q. E. D.

IV. THA

Chap. X. *Mechanical Powers.* 59

IV. THAT Body, which communicates Motion to another, is called the *Power*; that which receives it, the *Weight*.

V. THAT Point in a Body, which remains at Rest, while the Body is turning round, is called the *Center of Motion*. Besides this, there are two other Centers in Bodies, 1. That of *Magnitude*, which is a Point, as near as possible, equally distant from all the external Parts of the Body; 2. That of *Gravity*, or that, about which all the Parts of the Body, in whatever Situation it is placed, exactly balance each other.

VI. WHEN a Body communicates Motion to another, it loses just so much of its own, as it communicates to that other; the Action of one being equal to the Reaction of the other. See Chapter the last, §. 4. and 5.

VII. WHEN two Bodies have such Relation to each other (suppose them fixed to different Parts of the same Machine) that if one be put into Motion, the other will thereby necessarily have such a Degree of Velocity given it, that their Moments will be equal; it will then be impossible, that one should begin to move without communicating to the other a Quantity of Motion equal to its own; 'tis evident therefore from the last Proposition, that if we suppose it to begin to move, in that very Instant it must lose all its own Motion by communicating the whole to the other Body,

Body, and therefore, being left to itself, will remain at rest, and communicate none at all.

Now the Moments of two Bodies are equal (Chap. IX. §. 2.) when the Velocity of the first is to that of the second, as the Quantity of Matter of the second to that of the first; or if we suppose their Quantities of Matter as one to three, then by the Supposition their Velocities are as three to one; and if we multiply the Quantity of Matter in the first, *viz.* one, by its Velocity three, and that of the other, *viz.* three, by its Velocity one; their Products are equal: their Moments are therefore by the Definition (Chap. XI. §. 1. and 2.) equal. They will also be equal, when the Spaces the Bodies pass over are in that Proportion; for the Times they both move in being the same, the Spaces will always be as the Velocities.

VIII. FROM hence it follows, that in any Machine, whether simple or compound, the Power however small may have a Moment equal to that of the Weight; provided the Machine be such, that when it is in Motion, the Velocity of the Power shall bear such Proportion to that of the Weight, as the Weight does to the Power; for then, what the Power wants in Quantity of Matter or Weight, will be made up in Velocity; consequently their Moments will be equal by §. the last, and therefore by §. 7. they will exactly balance each other; or be in *Æquilibrio*.

IX. BUT

IX. BUT if the Power bears a greater Proportion to the Weight, than the Velocity of the Weight to that of the Power ; it will then have a greater Momentum than the other ; so that though the other receives all its own Moment from it when the Machine moves, yet there will some remain, which, if it be sufficient to overcome the Friction of the Machine, will keep it moving.

WE proceed now to treat of each Mechanical Power in its Order, and

# I. Of the LEVER.

X. THE Lever is a right Line (or Bar whose Weight in Theory is not consider'd) moveable on a Center, which is called its *Fulcrum*, or *fixed Point*.

XI. THE *Æquilibrium* in this Machine is, when the Distance of the Power from the fixed Point is to that of the Weight from the same, as the Quantity of Matter in the Weight is to that in the Power.

FOR, supposing the Lever placed on its Fulcrum with the Weight to be raised at one End, and the Power applied to the other ; 'tis evident, the farther the Power is placed from the Fulcrum or Center of Motion, the larger will be its Sweep when the Machine is put in Motion ; that is, it will move over proportionably more Space in the same Time than the Weight to be raised: now, if it is placed just

so



so many Times farther from the Fulcrum, as it is less than the Weight, it will move just so many Times faster; their Moments therefore will be equal (§. 7.) and consequently the Power and Weight will exactly balance each other or be in *Æquilibrium* \*. And, if the Power is sufficiently augmented to overcome the Friction of the Machine, it will put it in Motion.

THE Lever is of three Kinds. 1. When the fixed Point is between the Weight and the Power, as in the last Case. 2. When the Weight is between the fixed Point and the Power. 3. When the Power is between the fixed Point and the Weight.

IN all which Cases the *Æquilibrium* will be, when their Distances from the fixed Point are such, that their Velocities shall be inversely as their Quantities of Matter; for then (by §. 7.) being at Rest, neither of them will communicate any Motion to the other.

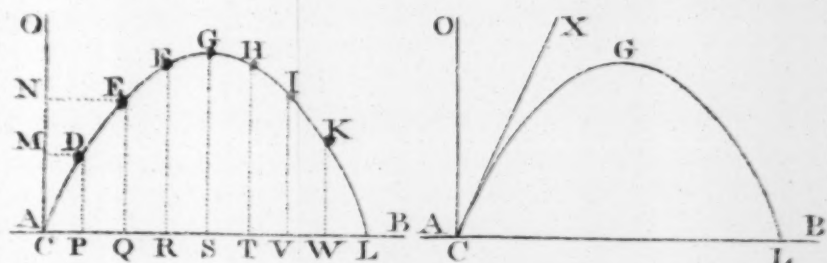
THE common Scales may be considered as

\* Geometrically thus. Let *AB* (Fig. 28.) represent the Lever, *F* the Fulcrum, *W* the Weight, *P* the Power, the one suspended at the Extremity of the Lever *A*, the other at *B*, and let *BF* be to *FA* as *W* to *P*; then while the Lever moves from the Situation *AB* into that of *CD*, the Point *B* which sustains the Power will move as many Times farther than *A* which sustains the Weight (and consequently as many Times faster, since they perform their Motions in the same Time) as the Arch *BD* is longer than *AC*; that is, the Triangles *BFD* and *AFC* being similar as the Arm *BF* is longer than *AF*, which (*ex Hypoth.*) is as many Times as the Weight exceeds the Power, they will therefore (§. 7.) be in *Æquilibrium*. Q. E. D.

*P. I. P. 62.*

*F. 20. P. 41.*

*F. 21. P. 42.*

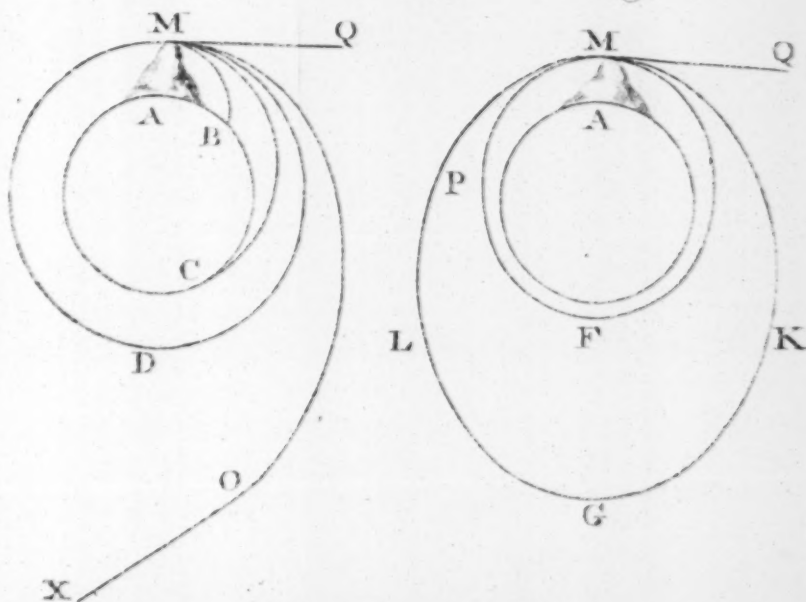


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Chap. X. *The Wheel and Axis.* 63

Lever of the first Kind, where the Weight and Power are applied at equal Distances from the fixed Point.

The Steelyard is also a Lever of the first Kind, but whose Arms are unequal.

THE Difference between the Use of the Scales and the Steelyard consists in this ; that as in one you make use of a larger Power (or more Weights) to estimate the Weight of an heavier Body ; in the other you use the same Power, but give it a greater Velocity with respect to that of the Weight by applying it farther from the fixed Point, which by §. 7. will have the same Effect.

## II. The WHEEL and AXIS.

XII. THIS Machine is a Wheel, that turns round together with its Axis ; the Power in this is applied to the Circumference of the Wheel, and the Weight drawn up by Means of a Rope wound about the Axis.

XIII. In this there will be an *Æquilibrium*, when the Weight is to the Power, as the Diameter of the Wheel to the Diameter of the Axis.

'Tis evident, the Velocity of the Power will exceed the Velocity of the Weight, as many Times as the Circumference of the Wheel exceeds that of its Axis ; because the Spaces they pass over in one Revolution will be as those Circumferences ; that is, as many Times as the Diameter

meter of one exceeds that of the other (the Circumferences of Circles being as their Diameters;) what therefore in this Case the Power wants in Weight will be made up in Velocity, from whence (§. 7.) there will be an *Æ*-quilibrium\*.

THE Use of this Machine is to raise Weights to greater Heights than the Lever can do, because the Wheel is capable of being turned several Times round, which the Lever is not; and also to communicate Motion from one Part of a Machine to another; accordingly there are few compound Machines without it.

### III. The PULLEY.

XIV. A Pulley is an Instrument composed of one or more Wheels moveable on their Axes.

XV. A simple Pulley, if its Axis is fixed, is of no other Use, than to alter the Direction of the Power; for the Power and Weight will both move through an equal

\* Geometrically thus. Let *AB* (Fig. 29.) be the Diameter of the Wheel, *DE* that of the Axis, *W* the Weight, and *P* the Power; when the Wheel begins to move, the Point *B* and *D* will describe similar Arches about the Center *C*, in the same Manner the Point *A* and *B* in the Lever were shewn to do about the fixed Point *F* (Fig. 28.) that is, the Point *B* will move as many Times faster than *D*, as *CB* is longer than *CD* or *AB* than *DE*, the Motion therefore of *P* (§. 7.) will be equal to that of *W*. From whence the Proposition is clear.

Space in the same Time. But in a Pulley not fixed, as in *Fig. 30.* where the Rope runs under it, or in a Combination of Pullies as in *Fig. 31.* the *Æquilibrium* will be, when the Power is to the Weight, as one to the Number of Ropes, that pass between the upper and lower Pullies.

FOR, suppose one End of the Rope fixed in B (*Fig. 30.*) and the other supported by the Power P, it is evident, that in order to raise the Weight W one Foot, the Power must rise two, for both Ropes, *viz.* BC and CP, will be shortened a Foot a-piece, when the Space run over by the Power will be double to that of the Weight; if therefore the Power is to the Weight as one to two, their Moments will be equal: for the same Reason, if there be four Ropes passing from the upper to the lower Pullies as in *Fig. 31.* the Velocity of the Power will be quadruple to that of the Weight, or as four to one, &c. In all Cases therefore when the Power is to the Weight, as one to the Number of Ropes passing from the upper to the lower Pullies (§. 7.) there will be an *Æquilibrium*.

XVI. If the Pullies be disposed as in Figure the 32d, each having its own particular Rope, the Action of the Power will be very much increased; for here every Pulley doubles it, wherefore the Power is four Times greater with two Pullies, eight Times with three,

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sixteen



sixteen Times with four, &c. For, it is evident from the Consideration of the Figure, the first will move half as fast as the Power, the second half as fast as that, and so on; wherefore (§. 7.) the Power is doubled by each Pulley.

THE Use of the Pulley is nearly the same with that of the Wheel and Axis, but it is more portable, and easier to be fixed up.

#### IV. The SCREW.

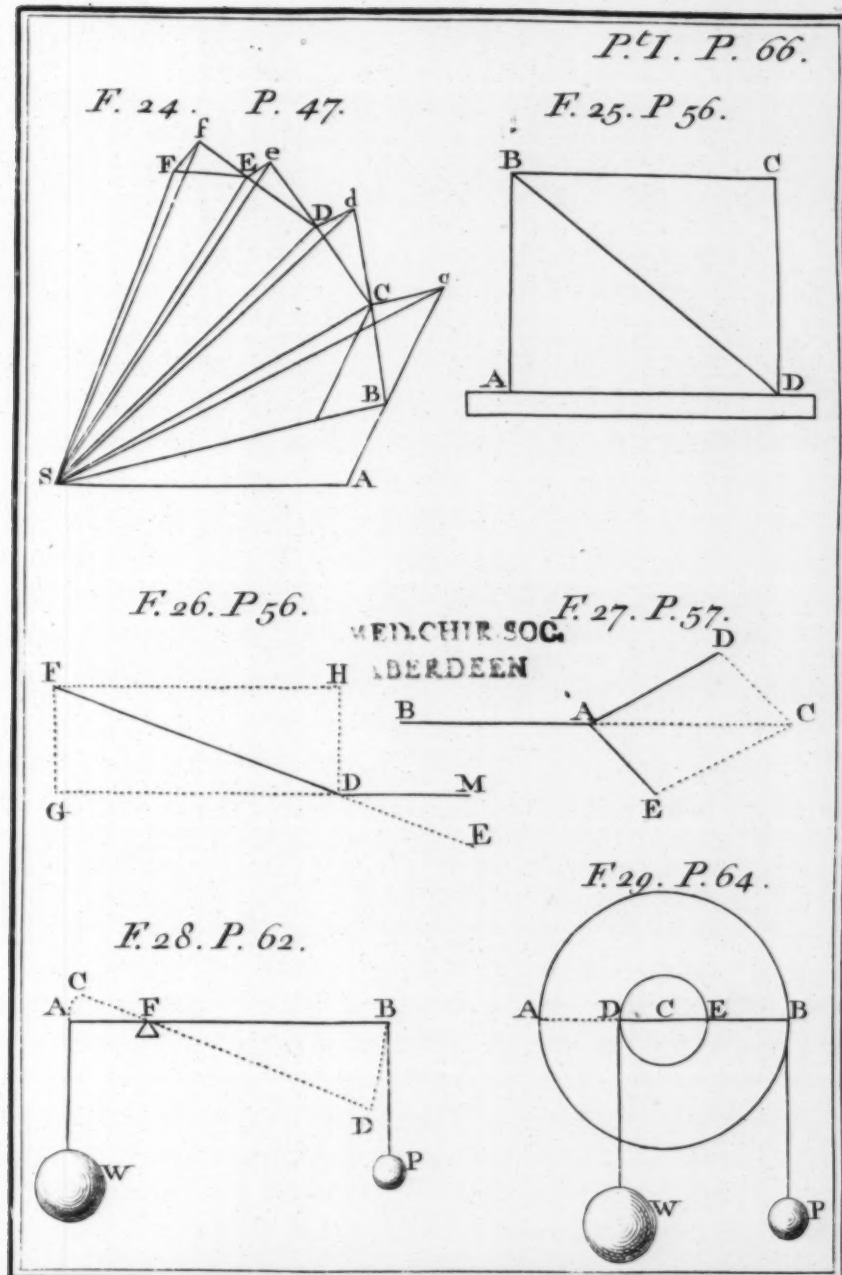
XVII. IN this Machine the *Æquilibrium* will be, when the Power is to the Weight, as the Distance between any two contiguous Threads or Spirals in the Screw, to the Way described by the Power in one whole Revolution. It is manifest from the Form of the Machine (*Fig. 33.*) that in one Revolution of the Screw, the Weight will be moved through a Space equal to the Distance of two contiguous Threads, and that the Power will run through a Space equal to the Compass it takes in one Revolution, therefore (§. 7.) if the Weight is to the Power in that Proportion, there will be an *Æquilibrium*.

THIS Machine is of great Force, and very useful in retaining Bodies in a compressed State, because it will not run back, as the three foregoing will, when the Power is removed. This arises from the great Friction

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of those Parts in the Screw, which during its Motion slide upon those that are at Rest.

## V. The WEDGE.

XVIII. THIS Instrument is formed by two equal Rectangles, joined at their lower Bases, and separated at their upper ones, by a third; which is called the *Back* of the *Wedge*; the other two, its *Sides*.

XIX. In the foregoing Mechanical Powers we have all along considered the Weight, as moved in the same Direction with that, in which it is acted upon by the Machine, as is commonly the Case; but in this, the Weight is generally applied in such a Manner, as to be made to move in a Direction different from that, in which it is protruded by the Wedge; hence it is, that Mathematicians have widely differed in their Determination of the Power of this Machine, some considering the Weight as moved by it in one Direction, and some in another. Nay, there are some, even among the late Writers, that have been led into manifest Errors by it. We will therefore lay down the several Proportions, they have given us, for the determining the Power of this Machine, and examine them one by one. I. It is demonstrated by some, that the Power will be equivalent to the Resistance of the Weight, when it bears such Proportion to it, as the Breadth of the Back of the Wedge

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of those Parts in the Screw, which during its Motion slide upon those that are at Rest.

## V. The WEDGE.

XVIII. THIS Instrument is formed by two equal Rectangles, joined at their lower Bases, and separated at their upper ones, by a third; which is called the *Back* of the *Wedge*; the other two, its *Sides*.

XIX. In the foregoing Mechanical Powers we have all along considered the Weight, as moved in the same Direction with that, in which it is acted upon by the Machine, as is commonly the Case; but in this, the Weight is generally applied in such a Manner, as to be made to move in a Direction different from that, in which it is protruded by the Wedge; hence it is, that Mathematicians have widely differed in their Determination of the Power of this Machine, some considering the Weight as moved by it in one Direction, and some in another. Nay, there are some, even among the late Writers, that have been led into manifest Errors by it. We will therefore lay down the several Proportions, they have given us, for the determining the Power of this Machine, and examine them one by one. 1. It is demonstrated by some, that the Power will be equivalent to the Resistance of the Weight, when it bears such Proportion to it, as the Breadth of the Back of the Wedge



does to the Sum of its Sides ; or, which is the same Thing, as half that Breadth to one of its Sides. 2. Others make it somewhat larger, and demonstrate, that it ought to be, as half the Breadth of the Back to the perpendicular Height of the Wedge. 3. Some are of Opinion, that there will not be an *Æquilibrium* in this Machine, unless the Power is to the Weight, as the whole Breadth of the Back to the perpendicular Height ; *viz.* WALLIS, KEIL, GRAVESANDE, &c.

THOSE, who lay down the first Proportion for determining the Power of this Machine, suppose the Parts, which are separated from each other thereby, to recede from their first Situation in Directions perpendicular to the Sides of the Wedge. Thus let BAC (*Fig. 34.*) represent a Wedge ; P, P, two Bodies to be separated by it, the one to be moved towards I, the other towards F, in the Directions CI, and CF, perpendicular to AC and CB ; then 'tis evident, that when the Wedge is driven into the Situation MNO, the two Bodies will be moved to Q and Q ; that is, one will have passed through the Space CK, the other through CL, but these Spaces being equal, their Velocities are the same as if they had both passed over one of them, *v. g.* CL, or, which is equal to it, DG (drawn perpendicular to CB) ; therefore the Power, which we suppose applied at D, moves through DC, while the

the Obstacle moves through DG, consequently (§. 7.) when the Power is to the Weight as DG to DC, that is, as DB to CB\*, or half the Back of the Wedge to one of its Sides, they will be in *Æquilibrium*. This Proportion therefore, when the Parts of the Weight are supposed to be moved by the Wedge in the Directions CI and CF, is true.

2. THE second Proportion is also true, supposing the Bodies P, P, to recede from each other in the Directions CN and CM, parallel to AB the Back of the Wedge; for, when the Wedge is driven in between them, to the Situation MNO, the Bodies will have moved through a Space as CN, or, which is equal to it, DB, half the Back of the Wedge, and the Power through a Space equal to its Height as before; consequently (§. 7.) in this Case, the *Æquilibrium* will be, when the Power is to the Weight, as half the Back of the Wedge to its Height †.

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3. THOSE

\* For (8 *Elem.* 6.) the Triangles DCG and DCB are similar, and consequently  $DG : DC :: DB : CB$ .

† The same may be otherwise demonstrated from Section 14. Chapter 9. thus. Let there be a Body as L (*Fig.* 35.) drawn against the Wedge ABC by the Weight W, in the Direction LF, parallel to the Back of the Wedge AB; but prevented from sliding down towards C, by a Plane (whose upper Surface we may suppose represented by EF) lying under it. I say, the Power will be to the Weight, when they are in *Æquilibrium*, as DA to DC.

*Dem.*

3. THOSE, who imagine there will not be an *Æquilibrium*, unless the Power be to the Weight, as the whole Breadth of the Back of the Wedge to its Height, suppose, as in the last Case, that the Bodies to be separated recede from each other in Directions parallel to the Back of the Wedge; and endeavour to support their Opinion by the following Argument: *viz.* that, when the Wedge is driven into the Situation *MNO* (*Fig. 34.*) as before, each Part of the Weight having moved through a Space equal to half the Back of the Wedge, the whole Weight has therefore moved through

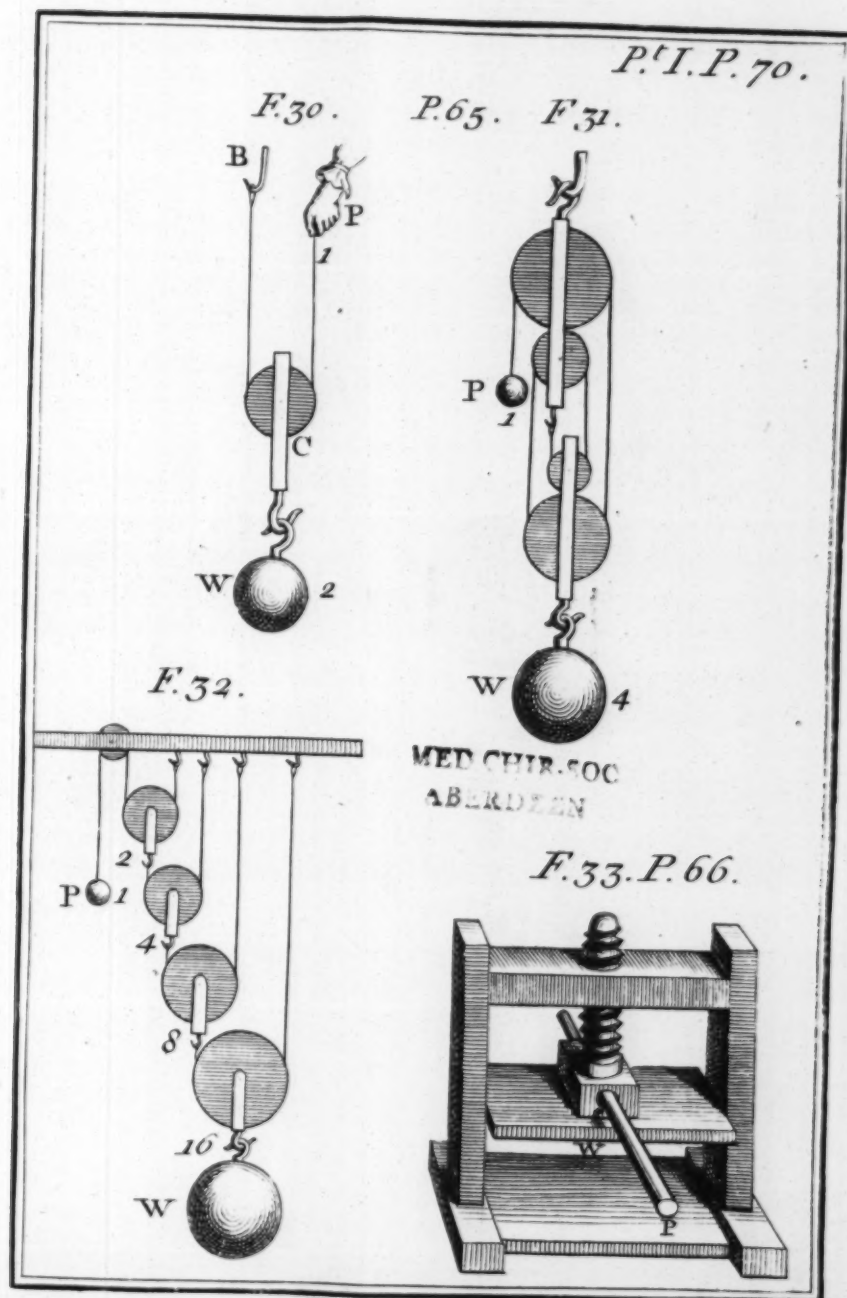
*Dem.* The Body *L* is here acted upon in three Directions, *viz.* by the Force of the Weight *W* in the Direction *LF*, by the two Planes *CA* and *EF*, in the Directions *LG* and *LI*, perpendicular to their Surfaces; let *GE* be drawn parallel to *LI*, then will the Triangle *LGE* have all its Sides respectively parallel to those Directions; consequently (*Chap. 9. §. 12.*) if we suppose *LE* to express the Force of the Weight, *GE* will express that of the supposed Plane, or which is the equal to it, because they act in contrary Directions to that of the Power: But *GE* is to *EL*, as *DA* to *DC* (for the Triangles *EGL* and *DAC* are similar, the Sides of one being *ex Construct.* respectively perpendicular to those in the other; *v. g.* *LG* to *CA*, *EL* to *DC*, and *GE* to *DA*); consequently the Power is to the Weight, when they balance each other, as half the Breadth of the Back of the Wedge to its Height. *Q. E. D.*

*Corol.* Suppose the Body *L* had been drawn against the Wedge in the Direction *GL* perpendicular to its Surface, and were to be moved by it in the contrary Direction towards *G*, as in the first Case; then if *GL* expresses the Force with which it is drawn towards the Wedge, *GE* will be that with which it resists the Power; but *GE* is to *GL* as *DA* to *AC*, the Triangles *EGL* and *DAC* being similar; consequently in this Case, the Power will be to the Weight, as half the Breadth of the Back of the Wedge to one of its Sides; as was before demonstrated.

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twice so much, or a Space equal to the whole Back : as much as to say, the Whole has moved farther than its Parts ; which is absurd.

XX. To determine the Power of this Machine in general, let  $ABC$ , in Fig. 34. represent an isosceles Wedge, whose Sides are  $AC$  and  $BC$ , and its Back  $AB$  ; draw the Line  $CD$  perpendicular to  $AB$  ; and let  $R, R$ , represent two equal Bodies to be separated by the Wedge, and suppose them confined by Planes put under them, or otherwise, to move in the Directions  $mT$  and  $sZ$  only, that are equally inclined to the Sides  $AC$  and  $BC$  respectively : and let the Wedge be driven in the Direction  $DC$ . Let  $m$  be the Point where the Body  $R$  touches the Wedge, and through  $n$  any Point in the Line  $mT$  draw the Line  $rn$  parallel to  $DC$ , and parallel to  $mT$  draw  $DX$  meeting  $CB$  (produced if necessary) in  $X$  ; then the Triangles  $mnr$  and  $CDX$  will be similar, the Sides of the one being respectively parallel to those of the other. And because the Bodies to be separated are equal, and the Wedge isosceles, the Lines  $mT$  and  $sZ$  equally inclined to the Sides  $CB$  and  $CA$ , and the Wedge driven in the Direction  $DC$ , the Bodies will be both moved with equal Degrees of Velocity ; and while the Wedge moves downwards a Space equal to  $rn$ , each of the Bodies will be driven in the Directions  $mT$  and  $sZ$  over a Space equal to  $mn$  : if  $rn$  therefore be taken to measure the Velocity of the



Wedge,  $mn$  will measure that of both the Bodies; for as to their moving different ways, that makes no Difference, because they are as free to move one way as another. Since then the Triangles  $mnr$  and  $DCX$  are similar, and therefore the Proportion of  $rn$  to  $mn$  the same as that of  $DC$  to  $DX$ , the Velocity of the Wedge will be to the Velocity with which the Bodies  $R, R$ , are driven by it, as  $DC$  to  $DX$ ; and consequently, whatever Forces, acting in the Directions  $Tm$   $Zs$ , oppose the Motion of the Wedge, a Force of the same Kind, acting upon the Back of the Wedge at  $D$  in the Direction  $DC$ , and pushing it forwards, will be in  $\text{\AA}quilibrium$  therewith, when it bears such Proportion to them, as  $DX$  does to  $DC$ : And therefore, if  $DC$  be taken to measure the Resistance of the Bodies to be moved by the Wedge,  $DX$  will always be the Measure of the Power requisite to make an  $\text{\AA}quilibrium$  therewith.

From this general Theorem the particular Cases that fall under it are readily deduced. Thus, if the Bodies  $R, R$ , are confined to move upwards in Directions parallel to the Sides of the Wedge,  $mT$  will lie parallel to  $CB$ ; and therefore  $DX$  the Measure of the Power necessary to make an  $\text{\AA}quilibrium$ , being to be drawn parallel to  $mT$ , must be drawn parallel also to  $CB$ ; it will therefore meet  $CB$  produced at no less than an infinite Distance, and will consequently itself be infinite: that is to say,

no

no finite Power will be able to drive the Wedge any farther between the Bodies thus confined. If the Bodies are confined to an horizontal Motion,  $mT$  becomes parallel to  $AB$  the Back of the Wedge, and  $DX$  (parallel to  $mT$ ) falls upon  $DB$ , and  $DB$  half the Back of the Wedge is the Measure of the Power requisite to constitute the *Æquilibrium*. If the Bodies are confined to move off at right Angles to the Sides of the Wedge,  $DX$  falls upon  $DG$ , and  $DG$  measures the Power; consequently,  $DG$  being shorter than any other Line that can be drawn from  $D$  to the Line  $CB$ , the Power in this Case necessary to make an *Æquilibrium* with the Resistance of the Bodies, is what the Mathematicians call a *Minimum*; that is, it is less in this than in any other Supposition whatever. And if the Bodies are to be moved perpendicularly downwards,  $X$  falls upon  $C$ , and  $DC$  becomes the Measure of the Power: but  $DC$  is all along supposed to measure the Resistance of the Bodies to be moved by the Wedge; the Power therefore and the Resistance are now equal to each other; which implies that the Bodies in these Circumstances are capable of being moved either with or without the Wedge with equal Ease. If the Bodies are confined to move downwards in Directions parallel to the Sides of the Wedge,  $DX$  becomes parallel to  $BC$ , the Point  $X$  falls upon it infinitely below  $C$ , and therefore  $DX$ , and consequently

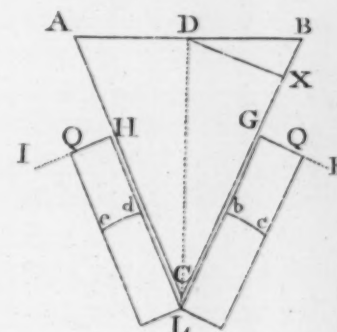
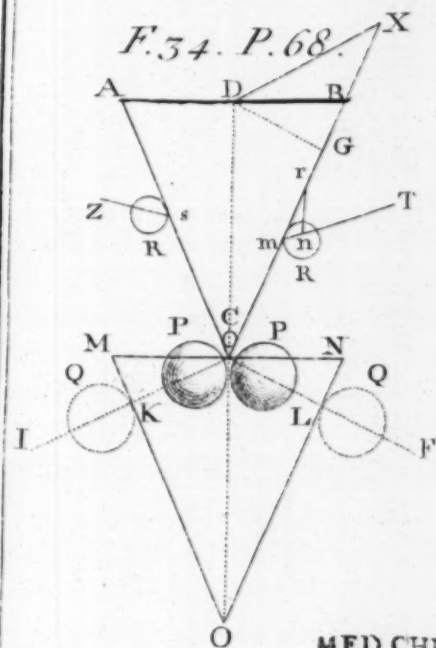
quently the Power to be measured by it, becomes infinite again; that is, the Bodies cannot be driven by the Wedge in those Directions by any Power whatever. When the Bodies R, R, are not confined, but are at Liberty to move in any Direction whatever, they will naturally take that in which they may be driven by the Wedge with the greatest Ease; that is, they will take that which is perpendicular to its Sides; and therefore the Perpendicular DG, which was the Measure of the Power necessary to make the *Æquilibrium*, when the Bodies were confined to move in that Direction, and was the *Minimum*, will be the Measure of it also in this Supposition.

XXI. The considering the Bodies to be separated by the Wedge as confined to move from each other in particular Directions, leads us to an easy Solution of a Case relating to one of the Uses of this mechanical Power, I mean the Dividing of Bodies therewith. To determine the Power of it in this, let us conceive the two Pieces LQ and LQ as turning upon a Joint at L, which Joint is to be opened by the Wedge ACB, driven in between the Pieces, and pressing against them only at their Extremities G and H, as represented in Fig. 36. And let us suppose that there is a certain Degree of Stiffness in the Joint, by which Stiffness may aptly be denoted the Tenacity or Cohesion of the Parts of the Body to be divided. Let the Lines GF  
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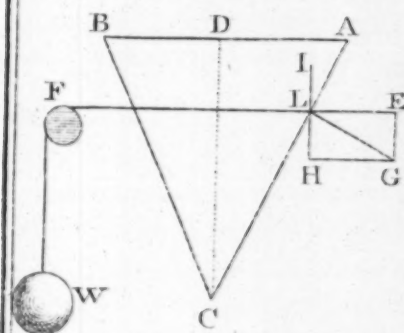
*P.<sup>t</sup> I. P. 74.*

*F. 36. P. 71.*

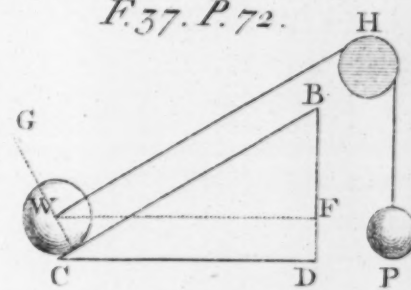


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*F. 35. P. 70.*



*F. 37. P. 72.*



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and HI be drawn perpendicular to the Lines LG and LH, which will therefore be the Directions, which those Extremities will be confined to, when they begin to move: parallel to one of these, as GF, draw DX; then since, as we have just been shewing, if two Bodies, confined to move in the Directions GF and HI only, were to be separated by the Wedge, the Power necessary to make an *Æquilibrium* with the Resistance of those Bodies would be to that Resistance as DX to DC; or that, if DC be taken to measure the Resistance they make, DX shall measure the Power that shall cause an *Æquilibrium* between them, it follows, that, in this Case, if we measure the Force, which, if applied at G and H in the Directions GF and HI, would overcome the Tenacity at L, and rend the Pieces asunder without a Wedge, DX will be the Measure of the Force, which would, with equal ease, perform the same thing by the Help of the Wedge. And when the Point comes infinitely near to L, and DX, parallel to GF, becomes therefore perpendicular to CB, and consequently the Triangles DCX and DCB become similar (8. *Elem.* 6.) the Power necessary to rend the Pieces with the Wedge will be to that which would do the same without it, as DX to DC; that is, because of the Similarity of the said Triangles, as DB to BC, or as half the Back of the Wedge to one of its Sides.

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But,



But, if we have no Regard to the Stiffness of the Joint L, and consider only the Quantity of Matter in the Power that drives the Wedge, and in the Bodies LQ and LQ to be turned upon that Joint, necessary to make an *Æquilibrium* between them, as is the usual Method in Theory, and observed in all the other mechanical Powers, and the Thickness of the Pieces be the same from End to End, and their Substance uniform, so that their Centers of Gravity may be in the Mid-way between L and G, or L and H, or any where in the Arches *bc* and *de*, whose Centers are at L, and which cut the Lines LG and LH in the Middle, the *Æquilibrium* will be, when the Quantity of Matter in the Power is to the Quantity in the Bodies LQ and LQ, as half DX to DC, and when the Point of the Wedge comes infinitely near to L, and DX becomes perpendicular to BC, and the Triangles DCX and DCB similar (as above) as half DB to BC, or as a Quarter of the Back of the Wedge to one of its Sides: Because, as is evident, in this Supposition the Centers of Gravity of the Pieces will have but half the Velocity they would have had, had all Parts of them been to be moved as fast as the Extremities G and H; half the Power therefore will be sufficient to move them that would have been required in that Case; but in that the Quantity of Matter in the Power must have been to the Quantity in the resisting Bodies, as DX to DC

Chap. X. *The Inclined Plane.* 77

and, when C comes infinitely near to L, as DB to BC; it is therefore, in the present Supposition, in the former of these Cases, as half DX to DC, and in the latter, as half DB to BC, or as a fourth Part of the Back of the Wedge to one of its Sides.

Or this Proposition may be considered in another Light, thus: The Centers of Gravity of the two Pieces LQ and LQ being in the Arches *bc* and *de*, that cut LG and LH in the Middle, they are carried upon the Levers LG and LH in the Mid-way between their common Fulcrum L and their Extremities G and H; the Wedge therefore has not only a mechanical Advantage over them, in Consequence of its own Form, in the Proportion of DX to DC, but in Consequence also of their being carried upon the said Levers, in the Proportion of two to one; the Wedge therefore has twice the Power over them it would otherwise have; and consequently half the Quantity of Matter in the Power that drives the Wedge is sufficient to constitute an *Æquilibrium* that is requisite where no such Levers are concerned.

XXII. THE Form of the *Inclined Plane* being no other than that of half a Wedge, as is manifest from the Representation of it (*Fig. 37.*) it follows, that what has been demonstrated of the one, may be applied to the other, and the Properties of both will be found the same. For Instance, if the Weight W is to be drawn up

up the Plane CB, by the Power P, in a Direction parallel to the Plane, instead of that, we may suppose the Weight prevented from running off the Plane by the String WH, and the inclined Plane driven under it like a Wedge in the Direction DC: then will the Weight rise towards G in a Direction perpendicular to CB, for we must always suppose the String WB parallel to the Plane (as it would have been, if the Weight had been drawn up by it;) then will the Action of the Plane upon the Weight be similar to that of the Wedge in the first Case, and consequently the Power will bear such Proportion to the Weight, as DB to BC; that is, as the Height of the Plane to its Length. Again, suppose the Weight was to have been drawn up the Plane by a String in the Direction WF parallel to CD the Base of the inclined Plane; then if the Plane be driven under the Weight as before, it must rise in a Direction perpendicular to CD, that is, parallel to DB: then the Case will be analagous to the second of the Wedge; consequently, the Power will be to the Resistance of the Weight, when there is an *Æ*quilibrium, in the Proportion of DB to DC, as there demonstrated.

XXIII. THESE are the Powers or Machines, which, under different Forms, constitute all other how complicated soever; and as the *Æ*quilibrium, in any one of these, is, when the Power

Chap. X. *The Conclusion.*

79

Power and Weight are inverfely as their Velocities; fo in a Machine however compounded, the Power and the Weight will exactly balance each other, when they are in this Proportion; for by §. 7. their Moments will then be equal, and the Machine, if at Reft, will continue in that State; and, if put into Motion by an external Force will gradually lofe it, when that Force ceafes to act; on account of the unavoidable Friction of the Machine, and the Refiftance of the Air, which it muft neceffarily meet with, unlefs its Motion could be performed in a perfect Vacuum. From hence we fee the Impossibility of contriving an Engine, whose Motion fhould be *perpetual*, that is, fuch as does not owe its Continuance to the Application of fome external Force; a Problem that has given Birth to an almoft infinite Number of Schemes and Contrivances. For unlefs fome Method could be found out of gaining a Force, by the artful Difpofition and Combination of the Mechanical Powers, equivalent to that which is continually destroyed by Friction, and the Refiftance of the Air, the Motion, which was at firft given to the Machine, muft at length be neceffarily loft. But we fee, that thofe Inftuments are only different Means, whereby one Body communicates its Motion to another, and not defigned 'to produce a Force which had no Exiftence before. 'Tis for Want of a due

due Consideration of this, that so many Mechanical Designs have proved abortive, so many Engines unequal to the Performance for which they were designed, and so many Impossibilities attempted.

“ If it were possible, says Bp. WILKINS,  
 “ to contrive such an Invention, whereby  
 “ any conceivable Weight may be moved by  
 “ any conceivable Power, both with equal  
 “ Velocity (as it is in those Things which are  
 “ immediately stirred by the Hand, without  
 “ the Help of any other Instrument) the  
 “ Works of Nature would be then too much  
 “ subjected to the Power of Art ; and Men  
 “ might be thereby encouraged (with the  
 “ Builders of *Babel*, or the Rebel Giants) to  
 “ such bold Designs, as would not become  
 “ a created Being. And therefore the Wisdom  
 “ of Providence has so confined these Human  
 “ Arts, that what an Invention hath, in the  
 “ *Strength* of its Motion, is abated in the *Slowness*  
 “ of it ; and what it has, in the extraordinary  
 “ *Quickness* of its Motion, must be abated  
 “ for in the great *Strength* requisite in  
 “ the Power which is to move it \*.”

\* *Wilkins's Mathem. Magick.* p. 104.

*The End of the First Part.*

## APPENDIX to Part I.

## C H A P. I.

*Of the Vibration of a Pendulum in a Cycloid.*

## P R O P O S I T I O N I.

**I**F a Pendulum be made to vibrate in a Cycloid, all its Vibrations however unequal, will be *isocronous*; that is, they will be performed in equal Times (*a*).

(*a*) In order to demonstrate this Proposition, it will be proper to lay down the following Lemma's.

## L E M M A I.

If a Body descends from A along the Line AX, (Appendix Plate, Fig. 1.) by virtue of a Force which decreases in Proportion as the Distance of the Body from X, decreases; that is, if when the Body comes to M, N, O, &c. the Action of that Force upon the Body, be as the Distances XM, XN, XO, &c. respectively: And if the last acquired Velocity of the Body; that is, its Velocity when it comes to X, be expressed, or set off, by the Perpendicular XB equal in Length to the Line AX, and its Velocities at M, N, O, &c. be set off there by the Lines MD, NP, OQ, &c. in Length proportionable to each other and to the Line XB, as the Velocities of the falling Body at M, N, O, &c. are to each other and to its last Velocity at X: And if through the Extremities of these Lines, the Curve ADB be drawn; I say, that Curve will be a Portion of a Circle: And the Time in which the Body will descend through the whole Space or Line AX [or any Part of it, as MO] will be such Time, as would be requisite for it to describe the whole Arch AB [or

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## 2      A P P E N D I X to Part I.

any Part as DQ, corresponding to MO] in, with its last acquired Velocity at X.

*Demonstration of the Lemma.* Parallel and contiguous to the Line MD, draw NP, in which Case the Line MN, becomes a Point, and the Arch DP a Tangent to the Curve: Produce PD till it meets XA produced, in T; draw the Line XD; and let fall the Perpendicular DL. Then the Lines DL and TM being parallel, the Angles PDL and DTM are equal, as being alternate (by 27 Elem. 1.); and the Angles at L and M, as being right ones; the Triangles therefore PDL and DTM are similar, which for the Sake of referring to it afterwards, let us make the first Step of the following Process

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| <p>From the first Step we have this<br/>Proportion (5 Elem. 6.)<br/>By the Figure<br/>But MD being the Velocity when<br/>the descending Body comes to<br/>M, the Point MN is described<br/>with that Velocity; for there<br/>is no Acceleration during the<br/>Passage of a Body over a Point;<br/>consequently MN is proportion-<br/>able to MD: that is,</p> <p>Comparing the second, third and<br/>fourth Steps</p> <p>But MD and NP being the Velo-<br/>cities of the descending Body at<br/>M and N, LP the Difference of<br/>those Lines, expressing the In-<br/>crease of Velocity in the Body,<br/>will be proportionable to the<br/>moving Force at the Point MN;<br/>that is, by the Supposition, to<br/>the Distance XM; therefore</p> <p>Comparing the fifth and sixth<br/>Consequently (5. Elem. 6.)</p> | <p>1<br/>2<br/>3<br/><br/><br/><br/><br/><br/>4<br/>5<br/><br/><br/><br/><br/>6<br/>7<br/>8</p> | <p>The Triangles PDL and<br/>DTM are similar</p> <p><math>PL : LD :: DM : MT.</math></p> <p><math>LD = MN</math></p> <p>MN is as MD.</p> <p><math>PL : MD :: DM : MT</math></p> <p>PL is as XM</p> <p><math>XM : MD :: DM : MT</math></p> <p>The Triangles XMD and<br/>DMT are similar.</p> |
|---|---|---|

And therefore, since their Angles at M are right ones, the Triangle TDX is (by the Converse of Prop. 8. Elem. 6.) right angled at D. Consequently since the same is true of any other Point of the Curve, as well as D, the Arch ADB is a Portion of a Circle (16 Elem. 3.). Which is the first Part.

Secondly

Secondly, comparing the first and eighth Steps, the Triangles PDL and XMD are similar; therefore ADB being a Portion of a Circle, as already proved  
Comparing the 3d, 9th and 10th Steps

$$9 \quad LD : DM :: DP : DX$$

$$10 \quad DX = XB$$

$$11 \quad MN : DM :: DP : XB.$$

Since then the Point MN bears the same Proportion to MD, or the Velocity it is described with by the falling Body, that the Point DP does to the last acquired Velocity XB, it follows that the former, MN, is described in the same Time with the Velocity the Body has when there, that the latter, DP, might be with the last acquired Velocity XB. And since the same is true of every other Part of the Arch ADB, it is obvious that the Time in which the Body will descend through any other Part of the Space AX, [or the whole of it,] will be such as would be required for it to describe any corresponding Part of the Arch ADB, [or the whole of it,] with the last acquired Velocity XB. Which was the other Part.

*Coroll.* Hence it follows, that if a Body descends along the Line AX, by Virtue of Forces acting upon it at A, M, N, O, &c. proportionably to the Length of the Lines XA, XM, XN, XO, &c. and if on X as a Center, and with the Radius XA a Portion of a Circle, as ADB, be described; and if the Radius or whole Sine XB, be put to represent the Velocity of the Body when it comes to X, the other Sines MD, NP, OQ, &c. will represent the respective Velocities of the Body at the several Points M, N, O, &c. And conversely, if one of the Sines, as MD, be put to express its Velocity at M, the other Sines NP, OQ, and the Radius or whole Sine XB, will express the Velocity of the Body at those other Points N, O and X.

LEMMA II.

If a Body moves along the Line AX, (Fig. 2.) and be urged all the Way by Forces proportionable to its Distance from the Point X; whatever Point of that Line it sets out from, it will come to the Point X in the same Time. Which Time will bear such Proportion to the Time it would move over the whole Line AX in, with the Velocity it shall acquire by falling through the whole Line AX, as the Semicircumference of a Circle does to its Diameter.

A 2

Dem.

# 4 APPENDIX to Part I.

*Dem.* Let two Bodies A and P set out from the Points A and P at the same Time; and let them be urged by Forces proportionable to their Distances from the Point X: I say, those Bodies will come to X at the same Instance of Time; that is, they will overtake one another at that Point. On X as a Center, and with the Radius's XA and XP describe the two Quadrants AB and PQ; and draw the Line SX, and the Sines RS and MN; and let the whole Sine or Radius XB express the Velocity the Body A will acquire by falling to X: Then by Corollary of Lemma 1. will the Sine RS, if taken as near as possible to A, express the first Velocity of the Body A. But the Force, which urges the Body A is supposed to be to that which urges the Body P, as XA to XP (or because the Archs AS and PN are similar) as RS to MN: As therefore RS expresses the first Velocity of A, MN will express the first Velocity of the other Body P: And therefore by the same Corollary, its Velocity when it comes to X, will be expressible by XQ. Farther, the Time the Body A falls to X in, is by Lemma 1. equal to the Time the Arch AB would be described in with the Velocity XB; and the Time the other Body falls from P to X in, is equal to the Time the Arch PQ would be described in, with the Velocity XQ. But a Body will be as long in moving over the Arch PQ with the Velocity XQ, as over the Arch AB with the Velocity XB, the Lines XQ and XB having the same Proportion to each other, that the Archs have. Therefore the Time the Body A falls to X in, is equal to the Time the other Body P would fall to that Place in. Which was the first Part.

Again by Lemma 1.

Axiom, or self evident Proposition

Comparing the first and second

- 1 The Time a Body would fall from A to X in, is equal to the Time it would move over the Arch AB in, with its last acquired Velocity at X.
- 2 The Time a Body would move over the Arch AB in with the last acquired Velocity at X, is to the Time it would move over AX in with the same Velocity, as AB is to AX.
- 3 The Time a Body would fall from A to X in, is to the Time it would move over AX in with the last acquired Velocity, as AB is to AX.

Axiom

# APPENDIX to Part I. 5

Axiom	4	AB is to AX as twice AB is to twice AX.
By the Figure	5	Twice AB is to twice AX as the Semicircumference of a Circle is to its Diameter.
Comparing the 3d, 4th and fifth Steps.	6	The Time a Body would fall from A to X in, is to the Time it would move over AX in with its last acquired Velocity, as the Semicircumference of a Circle is to its Diameter. Which was the second Part.

## LEMMA III.

If from the lowermost Point of a Circle, as X (Fig. 3.) be drawn the Chords XQ and XO, the Power of Gravity whereby it shall cause a Body to descend along the former, will be to the Power whereby it shall cause it to descend along the latter, as the Length of the former is to the Length of the latter.

*Dem.* Draw the Diameter XD, the Perpendiculars QR and OS; and join the Points QD and OD. Then (by 31 Elem. 3.) the Triangle XQD is right-angled at Q; and therefore (by 8.

Elem. 6.)	1	XR : XQ :: XQ : XD.
And for like Reasons	2	XS : XO :: XO : XD
But by Part I. Chap 6. § 2.	3	The Effect or Power of Gravity upon a Body descending along the Chord QX, is to that which it exerts upon another falling freely; that is, to its whole Power, as XR to XQ.
And also	4	The Power of Gravity upon a Body descending along the Chord OX, is to its whole Power, as XS to XO.
Comparing the 1st and 3d	5	The Power of Gravity upon a Body descending along the Chord QX is to its whole Power, as XQ to XD.
Comparing the 2d and 4th	6	The Power of Gravity upon a Body descending along the Chord OX is to its whole Power, as XO to XD.

Comparing

## 6 APPENDIX to Part I.

Comparing the 5th and  
6th Steps

7 The Power of Gravity upon a Body descending along the Chord QX, is to the Power of Gravity upon a Body descending along the Chord OX, as XQ to XO.  
Q. E. D.

The Description of a Cycloid, with the Definitions relating thereto. If a Circle as FCH (Fig. 4.) be rolled along the Line AB, till it has turned once round; the Point C in its Circumference, which at first touched the Line at A, will describe the Curve Line ACXB, which Curve is called a *Cycloid*. The right Line AB is its *Base*: The middle Point X is its *Vertex*: And a Perpendicular, as XD, let fall from thence to the Base, is its *Axis*: And the Circle FCH, or any other as XGD, equal thereto, is called the *Generating Circle*.

### LEMMA IV.

If on XD, the Axis of the Cycloid, as a Diameter, the generating Circle XGD be described; and if from a Point in the Cycloid, as C, the Line CIK be drawn Parallel to the Base, the Portion of it CG, will be equal to the Arch GX.

*Dem.* Draw the Diameter HF, then the Circles FCH and DGX being equal

Adding GI to each of them

By the Figure

Comparing the two last

By the Description of the Cycloid

By the Figure

Comparing the 5th and 6th

By the Description of the Cycloid

Comparing the 7th and 8th with the Figure

Comparing the 4th and the 9th

$$1 \quad KG = CI$$

$$2 \quad KI = CG$$

$$3 \quad KI = DF$$

$$4 \quad CG = DF$$

$$5 \quad \text{The Arch } CF = AF$$

$$6 \quad \text{The Arch } CF = DG$$

$$7 \quad AF = DG$$

$$8 \quad AFD = DGX$$

$$9 \quad FD = GX$$

$$10 \quad CG = GX. \quad Q. E. D.$$

### LEMMA V.

The same Things being supposed as in the foregoing Lemma, a Tangent to the Cycloid at the Point C, is parallel to GX a Chord of the Circle DGX.

*Dem.* It appears from the Description of the Cycloid, that since the Angle FCH is a right one, (as it is by 31 Elem. 3.) the Chord CH is a Tangent to the Curve at the Point C, but

CH

# APPENDIX to Part I. 7

CH is parallel to GX; a Tangent therefore at the Point C, is parallel to GX, the Chord of the Circle DGX. Q. E. D.

## LEMMA VI.

Things remaining as before, if from a Point of the Cycloid, as L, the Line LMK be drawn parallel to the Base AB, the Arch XL of the Cycloid, will be double of XM the Chord of the Circle corresponding thereto.

*Dem.* Draw the Line SQ parallel and contiguous to LK, crossing the Circle in R, and the Chord XM produced, in P, then will LS, MR and MP become Points, the first having the Property of a Tangent to the Cycloid at LS, the second that of a Tangent to the Circle at MR, and the third, the Properties of a Production of the Chord XM. Join the Points X and R, and on MP let fall the Perpendicular RO: Produce also the Point RM, till it meets XN, a Tangent to the Circle at X. Then will the Lines XN and QS, being each perpendicular to the Diameter DX, be parallel; and the Triangles MNX and MPR will be similar; as having their Angles at M vertical, and at P and X alternate. But the Tangents NX and NM are equal (by 36. Elem. 3.) the corresponding Lines therefore PR and RM in the other Triangle, are so too: This last Triangle is therefore an Isosceles one; and therefore RO being perpendicular to its Base MP, MP is equal to twice MO. The Tangent LS is parallel to MP, (as being by Lemma 5. parallel to MX) and therefore equal to it, the Lines LK and SQ being parallel: It is therefore equal also to twice MO. But LS is the Difference between the cycloidal Archs XL and XS; and MO is the Difference between the Chords XM and XR, for since XO and XR are close together, RO which is perpendicular to one of them, may be considered as perpendicular to both: The Difference therefore between any two Archs of the Cycloid is twice that which is between two corresponding Chords of the Circle; and consequently any Arch, as XL, is double of the corresponding Chord XM. Q. E. D.

*Coroll.* Since when the Arch XL becomes XB, the corresponding Chord XM becomes XD the Diameter of the Circle DMX; its obvious, that the Semicycloid BX, or AX, is equal to twice DX the Diameter of the generating Circle DMX.

## LEMMA VII.

If a Body descends in a Cycloid, the Force of Gravity (so far as it acts upon it in causing it to descend along the Cycloid) will be



# 8 APPENDIX to Part I.

## PROPOSITION II.

The Time in which a Pendulum vibrating in a Cycloid, performs a Vibration, is to the

be proportionable to the Distance of the Body from the lowest Point of the Cycloid.

*Dem.* Let the Cycloid be AXB (Fig. 5.) whose Base is AB and its Axis DX, on which last as a Diameter, describe the generating Circle DQX: Draw the Chords OX and QX; through the Points O and Q, and parallel to the Axis AB, draw the Lines LS and MR; draw also the Tangents LV and MY. The because by Lemma 5. the Tangent LV is parallel to OX, and the Tangent MY parallel to QX, its obvious that Gravity exerts the same Power or Force upon a Body descending in the Cycloid at L (because it then descends in the Tangent LV) as it would do upon the same Body descending along the Chord OX: And for the like Reason, it exerts the same Force upon it when it comes to M, that it would do if it were descending along QX. But (by Lemma 3.) the Power or Force of Gravity upon Bodies descending along the Chords OX and QX, are as the Lengths of those Chords; that is, by Lemma 6. (halves being proportionable to their wholes) as the Length of the Cycloidal Arches LX and MX. The Force therefore of Gravity upon a Body descending in the Cycloid at the Point L [or any other] is to the Force upon the same when at M [or any other Point] as the Space or Distance it has to move over in the former Case, before it gets to the lowest Point X, to that it has to run over in the latter before it arrives at the same Point. Q. E. D.

*Demonstration of the Proposition in the Text to which this Note refers.*

By Lemma 7. The Force of Gravity so far as it causes a Body to descend in a Cycloid is proportionable to the Distance of the Body from the lowest Point; imagine then that Body to be a Pendulum vibrating in the Cycloid, then whatever Point it comes out from, it will by Lemma 2. come to the lowest Point in the same Time: And consequently since the like is true as to it descending from that Point, all its Vibrations be they large or small, will be perform'd in the same Time. Q. E. D.

# APPENDIX to Part I. 9

Time in which a Body would fall freely thro' half the Length of the Pendulum, as the Circumference of a Circle is to its Diameter (b).

PROB.

(b) To demonstrate this, the following Lemma's will be of Use.

## LEMMA VIII.

If in a right-angled Triangle, as BFG (Fig. 6.) the Perpendicular FI be let fall from the right Angle to the Hypothenuſe BG, the Line BI multiplied by BG will be equal to BF<sup>2</sup>.

Dem. By 8. Elem. 3. the Triangle BFI and BFG are ſimilar, conſequently BI is to BF, as BF is to BG, and therefore  $BI \times BG = BF^2$ . Q. E. D.

## LEMMA IX.

If a Body deſcends along a curve Line, as AX (Fig. 7.) it will acquire the ſame Velocity that another, or the ſame Body, would do, by falling from an equal perpendicular Height in the Line DX.

Dem. Parallel to the horizontal Line AD, draw the Lines BM and FN contiguous to each other; in conſequence of which, the Lines MN and BG are capable of being conſidered as Points; and therefore the Velocity the deſcending Bodies paſs over them with, as uniform; and the curve Line BG, as a ſtraight Line alſo, and as a Tangent to the Curve AX at the Point BG. Things being thus, let it be ſuppoſed that the Bodies begin their Fall at B and M, or, which comes to the ſame Thing, that they have equal Velocities at thoſe Points: Then the Velocities of the Bodies being uniform and equal to each other, (for there is no Acceleration in a Point) the Lines BG and MN may repreſent the Relation the Times they are paſſed over in bear to each other. Parallel to DX draw BF, and let the equal Lines BF and MN repreſent the Force of Gravity acting perpendicularly at thoſe Points; and let the Force BF be reſolved into two others, viz. BI and IF, the one parallel, the other perpendicular to the Curve of the Body at B: It is only the former of theſe, viz. BI, that accelerates the Body along the Curve BG; the other, viz. IF, neither accelerates it nor retards it, but is wholly ſpent in preſſing the Body cloſe to the Surface BG, if it be a Surface; or in ſtretching the String which keeps the Body in the Courſe AX, if it be a String. Now the Velocity a Body acquires by paſſing over any Space, is proportionable to the Force that acts

B

upon

upon it, multiplied by the Time that Force acts. Since then BI represents the Force in one Case, and MN the Time in the other, it follows that the Velocity generated in one Case, is as  $BI \times BG$ ; and in the other, as  $MN \times MN$ ; or since BF and MN are equal, as the Quantities  $BI \times BG$  and  $BF \times BF$ , (or  $BF^2$ ) which Quantities by Lemma 8. are equal to each other. The Velocity therefore the one Body acquires by descending along BG, is equal to that which the other acquires by falling through MN: But the Lines BM and GN being parallel, it is obvious there is the same Number of BG's in the Curve AX, as of MN's in the perpendicular DX; the Velocity therefore which a Body would acquire by falling through one, is equal to that which it would acquire in falling through the other. Q. E. D.

*Demonstration of the Proposition.* Let AXB (Fig. 5.) be the Cycloid the Pendulum vibrates in. Then by Lemma 2. compared with Lemma 7, we have

By the Corol. of Lemma 6.

By Lemma 9.

From the three last compared

By Part I. Chap. 5. § 7.

Comparing the 4th and 5th

- 1 The Time a Body would descend from A to X in, is to the Time it would move over the same Space in with its last acquired Velocity, as the Semicircumference of a Circle is to its Diameter.
- 2 AX is equal to twice DX.
- 3 The Velocity a Body acquires by falling from A to X, is equal to the Velocity it would acquire by falling from D to X.
- 4 The Time a Body would descend from A to X in, is to the Time it would move over twice DX in with the Velocity acquired by Fall from D to X, as the Semicircumference of a Circle is to its Diameter.
- 5 The Time a Body would move over twice DX in, with the Velocity acquired by falling from D to X, is equal to the Time it would fall from D to X in.
- 6 The Time a Body would descend from A to X in, is to the Time it would fall from D to X in, as the Semicircumference of a Circle is to its Diameter.

From

PROBLEM.

To make a Pendulum *vibrate* in a given Cycloid.

*Solut.* Let AXB (Fig. 5.) be the given Cycloid; its Base AB, its Axis DX, and its generating Circle DQX, as before: Produce XD to C, till DC be equal to DX: Through C draw the Line EF parallel to AB, and take CE and CF, each equal to AD or DB; and on the Line CE as a Base, and with the generating Circle AGE equal to DQX, describe the Semicycloid CTA, whose Vertex will therefore touch the Base of the given Cycloid in A.

From the Figure

From the Solution of the following Problem it will appear, that

Comparing the three last Steps

Doubling the Antecedents of the last Step

7 The Time of Descent from A to X is half a Vibration.

8 DX is half the Length of a Pendulum, which in vibrating shall describe the Cycloid AXB.

9 The Time of half a Vibration is to the Time in which a Body would fall freely through half the Length of the Pendulum, as the Semicircumference of a Circle is to its Diameter.

10 The Time of an whole Vibration is to the Time in which a Body would fall freely through half the Length of the Pendulum, as the Circumference of a Circle is to its Diameter. Q. E. D.

B 2

And

And on the Line CF also as a Base, describe an equal Semicycloid CB. On the Point C, hang the Pendulum CTP equal in Length to the Line CX: And let the upper Part of the String of it, (as CT, in its present Situation in the Figure) as it vibrates this way and that, apply itself to the cycloidal *Cheeks* CA and CB: Then will the Ball of it P oscillate in the given Cycloid AXB. Q. E. F. (c).

## CHAP.

(c) Draw TG and PH, each parallel to the Base AB; and join the Points AG and DH. Then by the Corollary of Lemma 6.

By the Figure (DC being equal to DX)

Comparing the 1st and 2d Steps

By Construction

Comparing the 3d and 4th

From the 5th Step compared with the Figure

(The String touching the Cycloid at T) by Lemma 5.

By Construction

From the two last Steps compared, GATK is a Parallelogram, consequently

By Lemma 6.

Comparing the two last Steps

Comparing the 6th and 11th

From the 12th Step compared with the Figure

Comparing the last Step with the Figure

From the last compared with the Figure

Comparing the last with the Figure

$$1 \quad AC = 2 \quad AE$$

$$2 \quad 2 \quad AE = CX$$

$$3 \quad AC = CX$$

$$4 \quad CTP = CX$$

$$5 \quad AC = CTP$$

$$6 \quad AT = TP$$

$$7 \quad GA \text{ is parallel to } TK$$

$$8 \quad GT \text{ is parallel to } AK$$

$$9 \quad GA = TK, \text{ and } GT = AK$$

$$10 \quad GA = \frac{1}{2} TA$$

$$11 \quad TK = \frac{1}{2} TA$$

$$12 \quad TK = \frac{1}{2} TP$$

$$13 \quad TK = KP$$

$$14 \quad \text{The parallel Lines } GT \text{ and } PH \text{ are equally distant from } AD$$

$$15 \quad \text{The Arch } GA = \text{the Arch } DH$$

$$16 \quad \text{The Chords } GA \text{ and } DH \text{ are parallel, and } GE = HX.$$

From

CHAP. II.

*Of the Center of Oscillation and Percussion.*

**T**HE Center of Oscillation is that Point in a Pendulum, in which, if the Weight of the several Parts thereof were collected, each Vibration would be performed in the same Time, as when those Weights are separate.

*The Point or Center, of Suspension* is the Point on which the Pendulum hangs.

*A general Rule for finding the Center of Oscillation.*

If several Bodies be fixed to an inflexible Rod suspended upon a Point, and each Body

From the 7th and 16th Steps compared with the Figure

And therefore (KD being by Construction parallel also to PH) KDHP is a Parallelogram, consequently

By Lemma 4.

Comparing the 9th and 19th

By the Description of the Semi-cycloid CTA

From the two last compared with the Figure

Comparing the 18th and 22d

Comparing the 16th and 23d

But by Lemma 4. if PH be equal to HX, P is a Point in the Cycloid AXB; the Ball of the Pendulum CTP therefore being at that Point, is in the given Cycloid. The Problem therefore was rightly solved. Q. E. D.

17 KP is parallel to DH.

18  $KD = PH$

19  $GT = \text{the Arch } AG$

20  $AK = \text{the Arch } AG$

21  $AKD = AGE$

22  $KD = GE$

23  $PH = GE$

24  $PH = HX.$

be



# 145 APPENDIX to Part I.

be multiplied by the Square of its Distance from the Point of Suspension, and then each Body be multiplied by its Distance from the same Point; and all the former Products when added together, be divided by all the latter Products added together, the Quotient which shall arise from thence, will be the Distance of the Center of Oscillation of those Bodies from the said Point.

Thus, if CF Fig. 8. be a Rod on which are fixed the Bodies A, B, D, &c. at the several Points A, B, D, &c. and if the Body A be multiplied by the Square of the Distance CA, and B be multiplied by the Square of the Distance CB, and so on for the rest: And then if the Body A be multiplied by the Distance CA, and B be multiplied by the Distance CB, and so on for the rest; and if the Sum of the Products arising in the former Case, be divided by the Sum of those which arise in the latter, the Quotient will give CQ, the Distance of the Center of Oscillation of the Bodies A, B, D, &c. from the Point C (*d*).

(*d*) *Dem.* That the Process may be less complicated, let us suppose but two Bodies, as A and F, fixed to the Rod CF; and let AI and FL be the Arcs which the Bodies A and F describe when the Pendulum vibrates, and let the Pendulum be removed into the Situation CL. Contiguous to the Line CL draw CR; then may the Arcs IP and LR be considered as Tangents at the Points I and L, and those Tangents as inclined Planes, down which the Bodies I and L are to roll: These Tangents being each perpendicular to CL, are equally inclined to the Horizon, the Bodies therefore will endeavour to roll down with equal Velocities; but this they cannot do, because being fixed to the inflexible

## APPENDIX to Part I. 15

flexible Rod, they will describe the unequal Arch IP and LR in the same Time. That is, the Body L will oblige the Body I to describe a less Arch than it otherwise would have done; and the Body I will occasion the Body L to describe a larger Arch than it would have done. And the Effects of the Forces by which they act thus upon each other, like those of Action and Reaction, will be equal. It remains to determine these Effects.

In order to which, parallel to LI draw MN, and let the equal Spaces LM and IN be those the Bodies would move over in the least Time possible, had they been independent of each other. And let the Archs LR and IP be those which the Bodies join'd to the Rod describe in the same Time. For the Reason just mention'd, the former of these *viz.* LR, will be larger, and the latter, *viz.* IP, will be less than LM or IN; and the Arch which the Center of Oscillation describes will be equal to LM or IN, because the Center of Oscillation describes that Arch, which the Bodies would describe in the same Time, if they were both together, and neither of them an hindrance or furtherance to the other. Consequently the Center of Oscillation is at Y, where the Lines MN and PR cross.

Now the Motion which the Body I loses by being retarded, is its Motion over the Arch PN; and the Motion the other Body gains by being accelerated, is its Motion over MR: The Force or Moment of the first of these Motions, is the Product of the Body I multiplied by the Space PN; and the Force or Moment of the last is the Product of the Body L multiplied by the Space MR. These are the Forces, Moments or Actions, which retard the one Body, and promote the Motion of the other. But observe, that these Forces or Moments, in as much as they act at different Distances from the Center C, about which the Bodies I and L, when the Pendulum swings, do revolve; have each their *Mechanical Advantage*; but the one a greater than the other: For instance, L has an Advantage which is as LC, its Distance from the *Fulcrum* C; and I only the Advantage IC. As then in determining the Effect of a Power applied to a Lever, we multiply it by its Distance from the *Fulcrum*; so the above-mentioned Forces or Moments (*viz.* I multiplied by PN and L multiplied by MR) must be multiplied by their respective Distances from C; and then we have I multiplied by PN multiplied by IC, and L multiplied by MR multiplied by LC for the Effects, which, as things are circumstantiated, these Forces or Moments have upon the Bodies I and L. But, as observed above, these Effects are equal, consequently we have for the first Step

$$I \times PN = L \times MR$$

# 16 APPENDIX to Part I.

*The Center of Percussion* is that Point in a Pendulum, or in an inflexible Rod moving

But the Triangles PNY  
and MRY are similar,  
consequently

Comparing the two last  
Or taking the Pendulum  
in the Situation CPR,  
in which I coincides  
with P, and L with R,  
we have

Or, which is the same  
thing

$$1 \quad I \times PN \times IC = L \times MR \times LC$$

$$2 \quad PN : MR :: PY : RY$$

$$3 \quad I \times PY \times IC = L \times RY \times LC$$

$$4 \quad P \times PY \times PC = R \times RY \times RC$$

$$5 \quad A \times AQ \times AC = F \times FQ \times FC.$$

That is, in Words, if one of the Bodies were multiplied by its Distance from the Center of Oscillation, and the Product arising from thence were multiplied by the Distance of the same Body from the Center of Suspension, this last Product would be equal to the Product of the other Body multiplied by its Distance from the Center of Oscillation, multiplied by its Distance from the Center of Suspension. And, since the same would be true if there were more Bodies, if each Body be multiplied by its Distance from the Center of Oscillation, and that Product by the Distance of the same Body from the Center of Suspension, all the Products relating to the Bodies on one Side the Center of Oscillation taken together, will be equal to all those which relate the Bodies on the other Side thereof taken together. Let then the Distances of any Number of Bodies, as A, B, D, F, from the Center of Suspension be called *a, b, d, f*, respectively, and the Distance of the Center of Oscillation Q from the Center of Suspension C, be called *x*: And suppose the Distances of the Bodies A, B, D, less than the Distance CQ, or *x*; and that of the Body F greater, as in the Figure: Then will the Distances of A, B and D from the Center of Oscillation be expressible by  $x-a$ ,  $x-b$ , and  $x-d$ ; and the Distance of F, by  $f-x$ ; multiplying then each Body by its Distance from one Center, and the Product arising therefrom by the Distance of the same Body from the other Center, we shall have  $Aax - Aaa + Bbx - Bbb + Ddx - Ddd = Fff - Ffx$ , which reduced gives  $x = \frac{Aaa + Bbb + Ddd + Fff}{Aa + Bb + Dd + Ff}$ . Which latter Equation is the Sense of the Rule above laid down.

round

round a Point, with which, if the Pendulum or Rod strikes against an Obstacle, no Jar or Shock at the Point of Suspension shall be occasioned thereby.

Thus, let CF (Fig. 8.) be an inflexible Rod, having the Bodies A, B, D, &c. fixed in it at the Points A, B, D, &c. and let O be an Obstacle against which, as it vibrates or swings round the Point of Suspension C, it may strike against: then, if there be no Jar or Shock occasioned thereby at the Point C, the Point that strikes against O, (as the Point Q suppose) is called the Center of Percussion.

# PROPOSITION.

The Center of Percussion is the same with the Center of Oscillation; and consequently may be determined by the same Rule (e).

P R O B.

(e) *Dem.* From the Definition of the Center of Percussion above laid down, it appears, that the Forces, with which the Bodies A, B and D, which would pass above O, move; must be a Counterbalance to the Force of the Body F, which would pass below it: and that the Force of F must be a Counterbalance to them. But the Forces wherewith those Bodies move, are as their Masses multiplied by their Distances from C, their Velocities being as these Distances. Farther, when the Point Q comes to O, and is stopt there, the Bodies A, B and D, endeavouring to go on, sway or bear against F, and F against them; just as if they were fixed to a Lever, as AF, having its Fulchrum at Q. Consequently the Forces of the former Bodies, so far as they act against the latter, are as their Distances from the Point Q; and the Force of the latter, so far as it acts against

C

the

## P R O B L E M.

Let it be required to find the Center of Oscillation, or Percussion of an inflexible Rod AB (Fig. 9. as a Bar of Iron, or the like) every where of equal Size, and vibrating in, or revolving round the Point A, as a Center of Suspension. (*f*)

the former, is as its Distance also from Q: the abovementioned Forces must therefore be multiplied by the Distances of the Bodies from Q: but the former of them, as observed above, balances the latter; and the latter them. So many therefore of the last Products as relate to the Bodies above Q taken together, must be equal to that which relates to the Body (or Bodies) below it. But the like Products were equal to each other, when the Point Q was looked upon as the Center of Oscillation (as in the 5th Step of the foregoing Process) consequently the Center of Percussion is the same with that of Oscillation. Q. E. D.

(*f*) *Solut.* Imagine the Rod to be divided into the least possible Parts B, C, D, &c. each of which call *One*. These Parts we may consider as so many Bodies contiguous to one another; so that the Center of Oscillation or Percussion of these Bodies will be the Center of Oscillation or Percussion of the whole Rod. To find this, we are by the Rule above laid down in the Text, to multiply each of these Bodies by the Square of its Distance from A. The first of these Products then will be B (or One) multiplied by AB squared; but one multiplied by AB squared, is the same with AB squared; now AB squared is a square Area or Surface, one of whose Sides is AB. In like manner the Body C, when multiplied by the Square of its Distance from A, is a Square Area, one of whose Sides is AC, somewhat less than the former. Imagine this Area laid upon the former; and the next, which will be less still, laid upon that; and so on till you come to the least of all. These will make a Pyramid, whose Base is the first Area, and its perpendicular Height will be equal to the Thickness of them all together; which Thickness will be as the Length of the Line BA. The Value or solid Content

# APPENDIX to Part I. 19

Content of this Pyramid will be  $AB$ , (*viz.* its Base) multiplied by a third Part of  $AB$  (its perpendicular Height). In the next Place we are to multiply each of those Bodies by its Distance from  $A$ : Now the Body  $B$  (or One) multiplied by  $AB$ , give a Line, as  $AB$ ; so the Product of  $C$ , multiplied by its Distance  $AC$ , give a Line, as  $AC$ ; these Lines heaped one upon another (as the Areas were before) will make a Triangle, whose Base will be  $AB$ , and its perpendicular Height also  $AB$ ; the Value, or Area of which, will be  $AB$  multiplied by  $\frac{2}{3} AB$ . In the last Place, by the Rule, we are to divide the Sum of the Products in the first Case, by the Sum of the Products in the latter; that is, the Content of the Pyramid by the Area of the Triangle; that is,  $AB^2 \times \frac{1}{3} AB$ , by  $AB \times \frac{2}{3} AB$ , which gives  $\frac{\frac{1}{3} AB^3}{\frac{2}{3} AB^2}$ ; that is,  $\frac{2}{3} AB$ , or two Thirds of  $AB$ : so that the Distance of the Center of Oscillation or Percussion, (as  $E$  suppose) from  $A$  the Center of Suspension, must be equal to two Thirds of  $AB$ , the whole Length of the Rod. Q. E. I.

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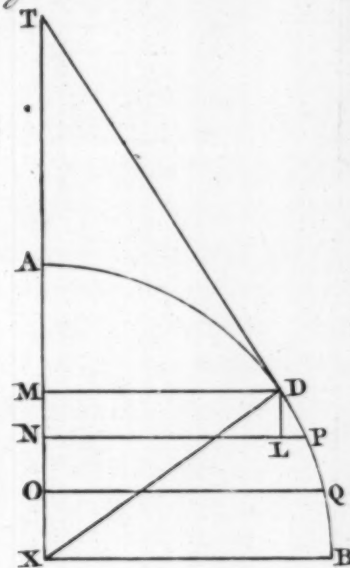
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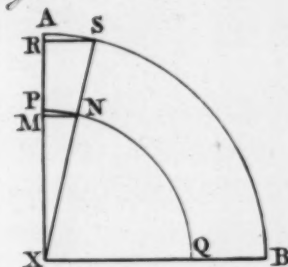


Appendix to part I. *Plate I.*

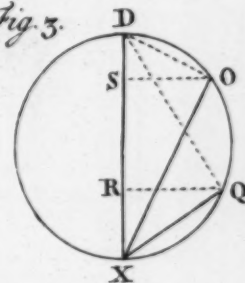
*Fig. 1.*



*Fig. 2.*



*Fig. 3.*



*Fig. 4.*

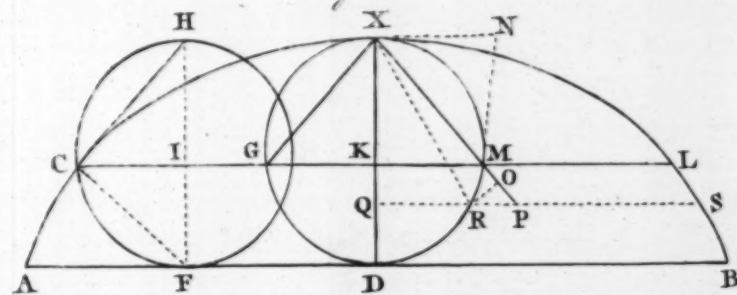
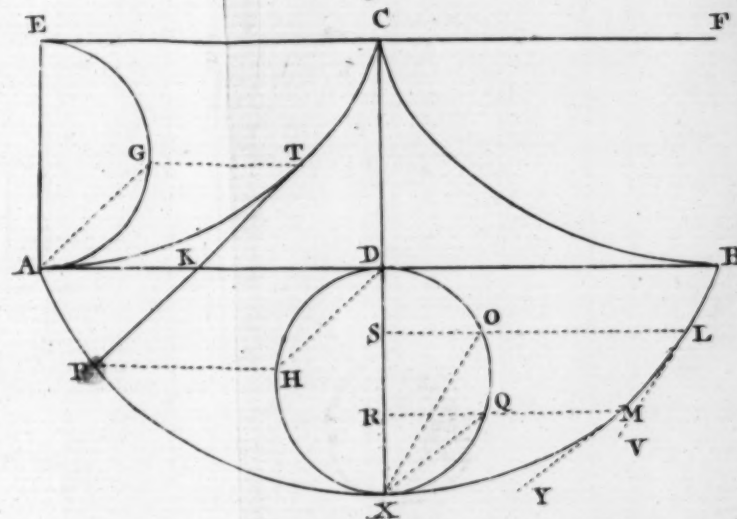




Fig. 5.



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Fig. 6.

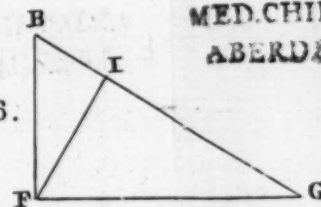


Fig. 8.

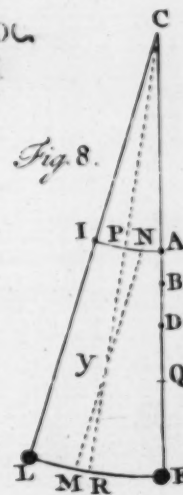
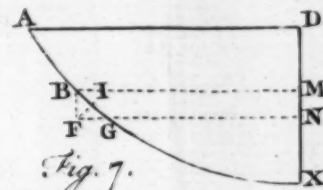


Fig. 9.



Fig. 7.





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PART II.  
HYDROSTATICS.

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CHAP. I.

*Of the Phænomena which arise from  
the mutual Action of the Particles  
of Fluids upon one another.*

I **I**N the former Part of this Essay, I have  
laid down and explained the general  
*Laws of Nature*, and from thence de-  
duced those Phænomena, which are in a strict  
and proper Sense \* denominated *Mechanical*:  
I pro-

\* In a larger Sense, all the Effects and Operations of natural  
Bodies upon one another may be called Mechanical; as being  
all subject to the general Laws of Motion. In Hydrostatics  
Fluids are governed by the Laws of Mechanism, as much as  
the Mechanical Powers themselves are; the same holds of the  
A Rays



I proceed now to an Explanation of such, as Philosophers have comprehended under the Name of *Hydrostatics*; the Intention of which is to explain the Nature of *Fluids*, and the Manner wherein they act upon one another, and upon Solids.

II. The Nature of a *Fluid*\*, as distinguished from that of a *Solid*, or hard Body, consists in this, *viz.* that its Particles are so loosely connected together, that they readily move out of their Places, when pressed with the least Force one Way more than another †. From whence Philosophers conclude, that they

Rays of Light, as will be seen when we come to Optics; and in the larger Bodies of the Planetary System, Mechanism equally prevails, as has been demonstrated by Sir *Isaac Newton*; which we shall endeavour to make out when we treat of the Physical Cause of the Motion of the Heavenly Bodies.

\* Some Philosophers make the following Distinction in Fluids; those which flow or spread themselves till their Surfaces become level or horizontal, they call *Liquid*; in Contradistinction to Flame, Smoak, Vapour, &c. which are also Fluids, but do not acquire such a Surface. Those, which are capable of exciting in us the Idea of Moistness, as Water, &c. they call *Humid*, distinguishing them thereby from Air, Quicksilver, and melted Metals. But those Distinctions are quite unnecessary in a Philosophical Sense; the Surfaces of all Fluids being level, when not prevented by the Bodies about them; and Humidity is only a *relative* Quality; for tho' Quicksilver will not moisten, or stick to a Man's Hand, it will to Silver or Gold.

† The common Definition, *Fluidum est cujus partes impressis cuicunque cedunt, & cedendo facillime moventur inter se*, though it expresses very well the Nature of a *compressible* Fluid, as Air, yet does not correspond to that of Water, whose Parts have been found to yield to no Force, with which they have been compressed,

## Chap. I. *among themselves.*

5

they are exceedingly minute, smooth, and round \*; it being otherwise impossible they should

compressed, unless it was greater on one Side than on the other. The Definition therefore seems imperfect, as not expressing that *Inequality* of Pressure, which is requisite to move the Parts of some Fluids one among another.

\* It is commonly observed that the Roundness of the Particles conduces very much to Fluidity, not only as it disposes them to move one among another with greater Facility, but because round Bodies touching one another in few Points only, the Force with which they mutually attract each other, is the weaker. But, upon this Supposition, the Particles of a Fluid ought to move with less Freedom one among another, by how much the greater the Weight is, with which they are compressed (for it is the same thing in this respect, whether they press against each other by Virtue of their own Attraction, or by some external Force) but of this we have no Experience. A Diver, upon plunging out of his Bell at the Bottom of the Sea, never finds the Water less fluid, notwithstanding the great Pressure from above. Mr. Boyle having caused a Tadpole to be put into a Vessel of Water, and to be pressed with a very great Force, tells us, that in Appearance it found no Inconvenience from thence, but swam about with the same Freedom and Briskness as ever.

Quere, Whether the Particles of which Fluids consist, are in Contact with each other, or not? Perhaps, they are prevented from approaching, nearer than to a certain Distance, by a repelling Power, diffused around each single Particle. The Observation, that Water is not render'd less fluid by Pressure, seems to favour this Opinion; and the Property, which the Air has of expanding or contracting itself, according to the Weight which it sustains (as shall be shewn, Chap. 3.) proves beyond Contradiction, that its Particles are endued with such a Power. But then if the Particles of all Fluids have this Power, it will follow that they ought to be in some Measure capable of being reduced into less Space by Pressure, as Air is; which they have not as yet been shewn to be. Further, since it has been proved (Part I. Chap. 3.) that if the Parts of Fluids are placed just beyond their natural Distances from each other, they will approach and run together, and if placed farther asunder still, will repel each other; it follows, upon the foregoing Supposition, that each

## 6 *The Action of Fluids* Part II.

should move with such Freedom one among another, upon the least Inequality of Pressure.

III. Those Particles, considered separately, are endued with all the common Properties of Matter, and subject to the same Laws of Motion

Particle of a Fluid must be surrounded with three Spheres of Attraction and Repulsion one within another: the innermost of which is a Sphere of Repulsion, which keeps them from approaching into Contact; the next, a Sphere of Attraction diffused around this of Repulsion, and beginning where this ends, by which the Particles are disposed to run together into Drops; the outermost of all, a Sphere of Repulsion, whereby they repel each other, when removed out of that Attraction.

Now, if this Hypothesis should be found to be true; and we might, agreeably to the above-mentioned Consequence of it, suppose, that the Particles of all Bodies attract and repel each other *alternately* at different Distances, perhaps we might be able to solve a great many Phenomena relating to small Bodies, which now lie beyond the reach of our Philosophy. However, upon the Supposition of the three Spheres of Attraction and Repulsion just mentioned, nothing is more easy, than to see how Solids may be converted into Fluids, and Fluids into Solids (as is done in Liquefaction and Freezing;) for allowing, that the first or innermost Sphere of Repulsion is capable, like that of the Particles of Air, of being augmented by Heat, and diminished or totally suspended by Cold, it follows that Bodies must be more or less fluid, in Proportion to the Degree in which they are affected by Heat or Cold: for when the Action of the first Sphere of Repulsion is diminished or destroyed by Cold, the Particles of the Fluid must necessarily be brought into closer Contact with each other by the Force of the circumambient Attraction, and by that means constitute an harder Body than before. But, we must not dwell too much upon an Hypothesis which wants Proof; I shall only add, that altho' some Fluids, as Water, have not been as yet contracted in their Dimensions, or made to take up less Space than they naturally do, by any Force with which they have been compressed by *Art*; yet there are none but are *naturally* contracted by Cold, from whence it seems reasonable to infer, that their Particles are at least capable of being brought into closer Contact, which is some Confirmation of this Doctrine.

water has now been compressed 5 per cent. ---

Motion and Gravitation with larger Bodies. To enquire therefore into the Nature of Fluids, is to consider, what Appearances a Collection of very small round Bodies, subject to those Laws, will exhibit under different Circumstances. In order to which, it is usual with Hydrostatical Writers to consider a Fluid, as divided into several perpendicular Columns contiguous to each other. Sometimes it is convenient to conceive it divided into thin Plates or *Strata* lying upon one another. In some Cases, the same Fluid is considered, as distinguished both these Ways, *viz.* into perpendicular Columns, and also into thin *Strata* or Plates. Figure 1. represents a Vessel filled with a Fluid to the Height EF, and divided into the Columns GH, IK, LM, &c. and also into the *Strata* RS, TV, XY, &c.

IV. From this Observation, concerning the Properties of the Particles considered separately, immediately results the following Proposition, *viz.* that in a Vessel, whose Form is such as represented by ABCD, (*Fig. 1.*) the Quantity of Pressure, which each Stratum sustains from the Weight of the incumbent Fluid, is in Proportion to the Number of those

It is an obvious Objection to this, that Water by freezing is augmented in its Bulk; but this may be owing to those Bubbles or Vacuities, observable in the Water after it is frozen, which were not in it before; and not to any general and uniform Removal of the Particles of the Fluid from each other, which the Objection, if it is of any Force against what has been advanced, must suppose.

*Strata,*

Strata, which rest upon it, that is, as the Height of the Surface of the Fluid above it; for if we suppose the Strata of equal Thickness, the Quantity sustained is proportionable to the Number of Strata of which it consists.

V. When the Surface of a Fluid is horizontal or level, each Particle thereof is disposed to continue in its Place, being sustained therein by the contiguous ones.

Let the Fluid be supposed to be divided into Strata, each of the Thickness of a Particle of the Fluid; and if the Truth of this Proposition be denied, let the Particle *mn* be one of those which is not sustained in its Place by the contiguous ones, but is moving from thence towards some other Part of the Vessel, *v. g.* towards D. Now, since all the other Particles of that Stratum are at an equal Depth below the Surface of the Fluid with this, they also sustain an equal Degree of Pressure (by the last Proposition,) consequently for the same Reason that one of them is moving towards D, the rest may all be said to be moving in the same Direction: but this cannot be true of the whole Stratum, while the Vessel is entire, and therefore of none of its Parts. Now, the like Reasoning will hold against the Motion of the Particle *mn* towards any other Part of the Vessel; from whence it follows, that each Particle of the Fluid is sustained in its Place by the contiguous ones

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and therefore disposed to continue at Rest.\*

VI. From hence is derived a fundamental Proposition in Hydrostatics, *viz.* That when the Surface of a Fluid is level, whatever Pressure any single Particles or small Portion of it sustains from the contiguous ones on one Part, it sustains the same on all the rest; that is, it is pressed by them with an equal Degree of Force on all Sides †.

For, by the Definition of the Fluid (§. 2.) each Particle is disposed to give Way, and move out of its Place, when the Pressure is not equal on all Sides; and (§. 5.) each Particle is pressed by the contiguous ones in such a Manner, that it is sustained in its Place thereby; it is therefore pressed with an equal Degree of Force on all Sides.

*Corol.* From hence it follows, that each Particle or small Portion of a Fluid presses with the same Degree of Force in all Directions on those which are contiguous to it. For, by the third Law of Nature, every Par-

\* This shews us the Absurdity of some Philosophers, who make the Nature of a Fluid, as contradistinguished from that of a Solid, to consist solely in the intestine Motion of its Particles.

† This Proposition, with its Corollary, is not strictly speaking true, unless the Particle or Portion of Fluid we speak of is supposed void of Gravity, for it presses downwards with a Force equal to the Weight of those Particles which rest upon it, added to its own; whereas the Force with which it presses upwards is only equal to the Weight it sustains, *viz.* that of the incumbent Fluid. But the Particles of Fluids are so exceedingly minute, and the Gravity of each so very small, that the Error arising from hence is insensible.



icle presses upon the contiguous ones with the same Degree of Force, with which it is pressed upon by them.

VII. The Surface of a Fluid becomes level by its own Gravity, when no external Force prevents it being so.

For, the Particles of Fluids press in all Directions with Forces proportionable to the Height of their Surfaces (Cor. §. 6. and §. 4.) If then the Surface be not level, the different Parts of the same inferior Stratum will be pressed not only downwards, but sideways against each other with unequal Forces; the greater Pressure therefore overcoming the weaker, the Particles which sustain the least Pressure, will be driven out of their Places and raised up, till the Surface becomes level; the Surface being level, each Particle will be equally pressed in every Direction, (§. 8.) all therefore will remain at Rest, and the Surface continue in that State \*.

VIII.

\* This Demonstration, and also both the foregoing, are founded upon a Supposition, that Bodies tend downwards by their Gravity in Lines parallel to each other, which though *physically* true, is not strictly so, their Tendency being towards the Center of the Earth, and consequently in Lines which meet in a Point: and therefore, if we would be accurate, the Fluid contained in a Vessel should be considered, as divided into Columns and Strata, as represented Figure the second, where ABD is the Earth, C its Center, EFGH a Fluid contained in a Vessel, and divided into Columns, which if continued down to the Center of the Earth, would there terminate in a Point C, and into the concentric Strata, *ab*, *cd*, &c. having the Center of the Earth for the Center of their Convexity. And then w

should

Chap. I. *among themselves.*

II

VIII. Fluids gravitate in Fluids of the same Kind.

This Proposition is a necessary Consequence of what has been observed about the Nature of the Particles of which Fluids consist, *viz.* that they are solid, and endued with the same Properties with other Bodies. The Reason why their Gravity is not sensible in the Fluid, is because the lower Parts sustain the upper, and hinder them from descending. But, it does not follow from thence, that the Gravity of the uppermost is entirely taken away, as some \* Philosophers have imagined; for, as

should find that the Particles of the Fluid will not be in *Æquilibrium* with each other, till all the Parts of its Surface are at equal Distances from the Center of the Earth, forming thereby the Surface EF, concentric to that of the Earth. For, supposing the Fluid EFGH continued down to C, so as to fill the Space ECF; it is evident the Columns, into which the Fluid is divided, cannot be of equal Lengths, and therefore cannot be of an exact Counterpoise to each other, unless the Surface EF is a Portion of a Sphere, whose Center is C: but the Action of the Parts of the Fluid upon each other at the Surface, is the same, whether the lower Part GCH be a Fluid, or not. Consequently the Surfaces of Fluids are not level or plain, but convex, having the Center of the Earth for the Center of their Convexity.

This Convexity, by Reason of the great Distance of the Earth's Center, approaches so near to a Plane, that in small Portions of it, the Difference is not sensible, and therefore may be neglected: but at Sea, 'tis evident to Sense; for when the Mariners put to Sea, the Shore first disappears, then the lower Buildings, afterwards the Towers, Mountains, &c. in like Manner, when they approach a distant Ship, the Top of its Mast and Sails appear first, while the Ship itself is intercepted from their View, by the Convexity of the Water between them.

\* This was the Notion of the *Cartesians*, who held, that when a Fluid is mixed with another of the same Kind, it loses its own Weight thereby.

B

much

much as the lower Parts press against the upper in sustaining them; just so much do the upper by their Re-action press upon the lower. Thus, the Weight of the whole Fluid is communicated to the Vessel, which therefore weighs according to the Quantity it contains, notwithstanding the Pressure of each Portion of the Fluid, taken separately, seems not to affect it.

IX. The Pressure of a Fluid is in Proportion to its perpendicular Height, and the Quantity of Surface against which it presses.

This Proposition admits of four Cases.

1. When the Fluid is contained in a Vessel of the same Dimensions, from Top to Bottom, and held in an erect Position, as that represented Fig. 1. it is evident, the Pressure of the Fluid upon the Bottom will be in Proportion to its Magnitude, and the perpendicular Height of the Surface of the Fluid above it. For, conceiving it divided into Columns the Pressure upon the Bottom, by the fourth Proposition, will be as the Length or Height of the Columns: and it will also be as the Number of them, because the Quantity of Fluid, which presses upon the Bottom, is in that Proportion, that is, as the Magnitude of the Bottom pressed upon. But when the Vessel is inclined or irregular, the Truth of the Proposition is so far from being evident, that it has been commonly looked upon as a Paradox.

2. L

2. Let the Vessel ABCD (*Fig. 3.*) be filled with a Fluid to the Height EF, and held in an inclined Position, as there represented; I say, the Pressure of the Fluid is proportionable to the Magnitude of the Base CD, and FG or HD the perpendicular Height of the Surface of the Fluid above it.

For supposing the Fluid divided into the Strata EI, KM, LO, &c. so far, as the first Stratum EI is prevented from pressing upon KI the Surface of the next inferior Stratum, by being in some Measure supported by the Side of the Vessel FI, so far is its Pressure augmented by the Re-action of the opposite Side EK upon it, which is exactly equal to the Action of the former, because the Fluid pressing every Way alike, at the same Depths below the Surface, exerts an equal Force against both these Sides. The Surface therefore of the second Stratum is pressed with the same Degree of Force with which it would be, if the Quantity of Fluid contained in the former Stratum was included within the Space HKQI, which is exactly equal to it, as having the same Base KI, and the same perpendicular Height QI\*. Now, this being true of each Stratum, their Pressure upon CD the Base of the Vessel is the same, as if they were all placed perpendicularly over it, and filled the Space RHCD; which they would do, since

\* 31. *El.* 11.

the Sum of their perpendicular Height  $QI$ ,  $KS$ ,  $LT$  and  $NV$  are equal to  $HD$  the perpendicular Height of this Space, and each of their Bases  $KI$ ,  $LM$ , &c. is equal to  $CD$  its Base  $\dagger$ . But, by the foregoing Case, if the Space  $RHCD$  was filled with a Fluid, the Pressure of it would be proportionable to the Dimensions of the Base  $CD$ , and the perpendicular Height  $DH$ , therefore it is the same in the inclined Tube  $ABCD$   $\parallel$ .

3. Let the Vessel  $ABC$  be irregular, as represented *Figure* the fourth, and filled with Fluid to the Height  $D$ , I say, the Pressure of the Fluid upon the Base  $C$ , is proportionable to the Magnitude of the Base, and  $CD$  the perpendicular Height of the Surface of the Fluid above.

$\dagger$  31. *El.* 11.

$\parallel$  Perhaps it may be thought more Geometrical, to demonstrate this Proposition with the Generality of Authors from the Property of the inclined Plane. They consider  $AD$  the lower Side of the Tube, as an inclined Plane, on which the Fluid contained within it rests, and argue that it loses thereby a Part of its Weight in Proportion to the Length of the Plane, and therefore occasions no greater Pressure upon the Base, than if the Vessel was held erect, and filled only to the same perpendicular Height, as when inclined. But this Demonstration proves too much, for by this Way of Reasoning, one might shew, that the Pressure of the Fluid  $EFCD$  upon the Base  $CD$  is less than the Pressure of  $RHCD$  a Column of the same Fluid having the same Base and equal perpendicular Height with it. For both the inclined, and the perpendicular Column contain the same Quantity of Fluid, upon Account of the Equality of their Bases, and perpendicular Heights; but that rests upon an inclined Plane, which this does not, and therefore presses less upon the Base. But this is contrary both to Demonstration and Experience, the Argument therefore proves too much.

In order that the Proof of this Proposition may be the better understood, we must premise the following *Lemma*, viz.

That when a Fluid passes through a Pipe, as AB, (*Fig. 5.*) which in some Parts is larger than in others, the Moment, or Force with which it moves, is every where the same. For while the Fluid is passing through the Tube, its Velocity in every Part will be reciprocally as the Quantity of Matter; for Instance, it will be as much greater at C, than it is at D, as the Quantity passing through C at any Instant of Time, is less than at D, and so of the rest; because a less Quantity would be conveyed through the smaller Parts of the Tube in the same Time, unless it should move faster there in Proportion to the Smallness of them. Now the Momentum of Bodies is partly owing to the Quantity of Matter, and partly to the Velocity; (as explained Part I. Chap. 9. §. 1.) consequently what the Fluid, which is actually passing through the narrower Parts of the Tube, wants in Quantity, is compensated by its Velocity in those Parts, and what it wants in Point of Velocity in the other Parts, is made up by the Quantity passing through them; so that the Moment is the same in every Part of the Tube, whether larger or narrower\*. The same is true, whatever be the Position the Tube is held in.

Let

\* Thus, we may observe in a River or Canal, that by how much the Breadth or Depth is less in any Part, so much the more



Let us now conceive the Fluid in the Vessel ABC (*Fig. 4.*) to be distinguished into the Strata EF, GH, IK, &c. Let us also imagine the Bottom of the Vessel C to be moveable, that is, capable of sliding up and down the narrow Part of the Vessel, *v. g.* from C to GH, (without letting any of the Fluid run out.) Let it further be supposed that this moveable Bottom, is drawn up or let down with a given Velocity, while the Vessel itself is fixed and immoveable; it is evident the lowermost Stratum, which is contiguous to the Bottom, will be raised or let down with the same Velocity, and will thereby have a Moment proportionable to that Velocity, and the Quantity of Matter it contains: But by the Lemma, all the rest of the Strata will have the same Moment, consequently the Moment of all taken together, (that is, of the whole Fluid,) is the same, as if the Vessel had been no larger in any one Part, than it is at the Bottom, (for then the Moment of each Stratum would also have been as great as that of the lowermost;) the Pressure therefore, or Action of the Fluid, with which it endeavours to force the Bottom out of its Place, is as the Number of Strata, that is, the perpendicular Height of the

more rapid is the Stream in that Part; and on the contrary, where it is wider and deeper, the Motion of the Water is more gentle and languid. So that the Moment, with which it flows, is the same in every Part.

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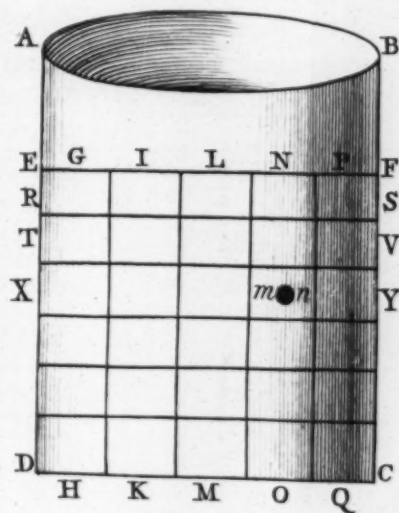
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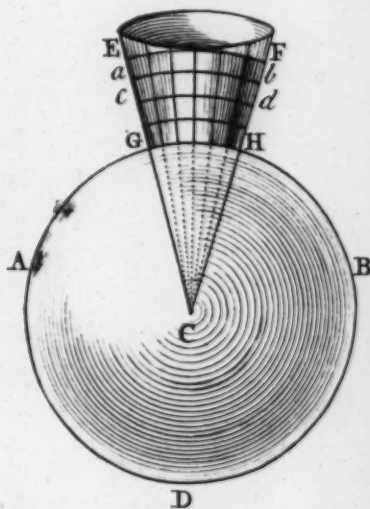
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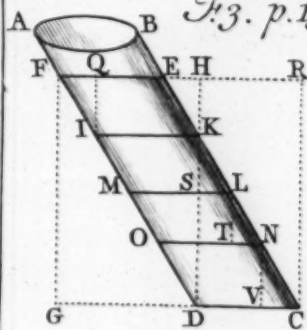
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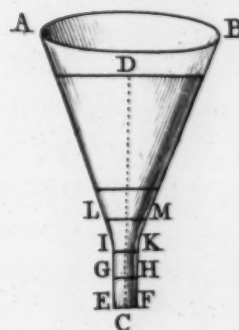
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Chap. I. *among themselves.* 17

Fluid, and the Magnitude of the lowermost Stratum, that is, the Bottom.

Again, suppose the Vessel ABC (*Fig. 6.*) fill'd with a Fluid to D; I say the Pressure upon the Bottom BC, is proportionable to the Dimensions of the Bottom, and to DE the perpendicular Height of the Fluid.

For, if we suppose the Bottom moveable, as before, and raised up or let down with a given Velocity, the Moment of every Stratum will be the same with that of the lowermost, by the Lemma; therefore the Moment of all taken together, is the same, as if the Vessel had been no less in any one Part, than it is at the Bottom; consequently the Pressure is proportionable to the perpendicular Height, and the Magnitude of the Bottom. This Case is the Converse of the former\*.

From

\* Upon this is founded the Practice of conveying Water through Pipes from Place to Place, &c. For from hence it follows, that if one End of a Pipe is laid in a Reservoir of Water, the Fluid will run into the Pipe, till it rises to a Level at the other End with its Surface in the Reservoir. Thus let ABC (*Fig. 8.*) represent a Reservoir or Bason of Water, DGE a Pipe laid from thence to E. If E the End of the Pipe is placed above the Line ABF, the Level of the Water in the Reservoir, the Water will run into the Pipe, till it rises in the other End to F the Level with AB; at which Time the Water in the Pipe will be in *Æquilibrium* with that in the Reservoir, and remain at Rest. But if the End of the Pipe is below the Surface of the Water in the Reservoir, it will continue to run out, till they are reduced to a Level. For, let GH be the lowest Part of the Pipe, then since F the perpendicular Height of the Fluid on one Side, is equal to B the perpendicular Height of the Fluid on the other, and GH, which (being the Place where

From hence it follows, that, if a Vessel is made of such a Form, as is represented (*Fig. 7.*) by ABCDEFG, and filled with a Fluid to the Height C, the Weight which the Bottom sustains, is as great as it would be, had the Vessel been IKFG (which is every where of the same Dimensions, that the other is of at the Bottom) and filled to the Top IK. Because the Pressure, by the Proposition, is proportionable to the Bottom and perpendicular Height, which in both Cases are the same \*.

## X. The

where the Fluids press one against another) may be considered as a Base to both, is common; it follows from this Proposition, that the Pressures on each Side are exactly equal; and therefore being in contrary Directions will necessarily destroy each other, and the Fluid will remain in *Æquilibrium*. But while the End E is below the Level, this *Æquilibrium* cannot be obtained, and therefore the Fluid will continue to run out.

For the same Reason, when two or more Tubes communicate with each other, the Surface of the Fluid they contain will stand at the same Level in all.

\* The Reason why the Vessel ABCDEFG, within the Fluid contained in it, does not weigh so much, as the Vessel IKFG, when full to the same Height, notwithstanding the Pressure upon the Bottom is the same in both, is, because ABDE the upper Part, or Cover of the former Vessel, is pressed upwards by the Fluid below it, with a Force equal to the Endeavour the Fluid in the small Tube BCD has to descend. Which Endeavour is the same that it would be, if the Tube BCD comprehended also the two Spaces ICBA and CKED, its Momentum being the same in both Cases by the Demonstration; the Cover therefore is pressed upwards with a Force equal to the Weight of as much Fluid as would fill the two Spaces ICBA and CKED; consequently the Vessel, whose Form is ABCDEFG is so much lighter than the other, that is, as much as the Fluid it contains is less.

From

X. The Velocity, with which a Fluid spouts out at an Hole in the Bottom of a Vessel, is equal to that, which a Body would acquire by falling freely from the Level of the Surface of the Fluid to the Hole.

Let there be a large cylindrical Tube ABCD (*Fig. 9.*) in the upper Part of which let us imagine a Cylinder of Ice FGHI exactly fitting it; let it further be supposed, that HI, the lower Surface of the Ice, is continually melting, so as to afford a Stream of Water running down the Middle of the Tube. Now the Form of this Stream of Water will necessarily be such, as is represented in the Figure by HLI; for the Water falling freely will descend faster and faster like other Bodies, causing thereby the Stream to become narrower and narrower. Now, let it be supposed, that the Tube has a Bottom, as CD, with an Hole in it at K, just sufficient to let the Stream pass freely; it is evident, there will be no Obstruction on this Account, but that the

From hence arises this Paradox, that the least Quantity of Fluid may be made to raise any Weight how great soever it be.

For since the Cover ABDE is pressed upwards with a Force equal to the Weight of as much Fluid, as would fill the two Spaces ICBA and CKED, and those Spaces may be enlarged at Pleasure in Height, by lengthening the Tube BCD (which at the same Time must be made proportionably smaller, otherwise the same Quantity of Fluid will not fill it;) it follows, that the same Quantity of Fluid may be made to press the Cover upwards with a given Force; if that Cover then is made moveable, any Weight that is laid upon it may be supported thereby.

C

Fluid



Fluid will pass through the Hole with such Velocity, as it naturally acquires by falling from HI, the lower Surface of the Ice. And if we suppose M and N, the empty Parts of the Tube, to be filled with Water, the Water will press equally upon the Sides of the Stream in every Direction (§. 6.) and therefore will be no Impediment to its Motion on that Account. Lastly, let us suppose the Ice taken away, and the Stream supplied from the Water at the Sides, as is the Case when a Fluid runs out through the Bottom of a Vessel; then will the Velocity, with which the Water flows through the Hole, continue the same; for so far as the Water coming from the Sides, endeavours to descend itself, so far it obstructs the Descent of the Stream, and no farther; and consequently causes no Alteration in the Velocity or Quantity of Fluid running out. The Velocity therefore, with which the Fluid passes through the Hole, is equal to that, which a Body would acquire by falling freely from the Level of the Surface of the Fluid to that Place.

If the Hole is made in the Side of the Vessel at the same Distance below the Surface, the Velocity will be the same, on Account of that equal Tendency Fluids have to move every Way alike \*.

## XI. The

\* Upon this Principle is founded the Practice of making artificial Fountains. For if to a Vessel or Reservoir ABCD, (Fig. 10.) filled with a Fluid to the Height EF, be fixed the

Pipe

XI. The Velocity, with which a Fluid spouts out from the Bottom or Side of a Vessel, is as the square Root of the Height of its Surface above the Hole \*.

The Cause, why a Fluid spouts out through an Hole made in the Bottom or Side of a Vessel, is the Pressure or Weight of the Fluid incumbent upon the Hole; from whence it should seem, that the Velocity ought to be as the Pressure; but if so, then the Quantity

Pipe CH, with a small Aperture at K, the Fluid will spout up from thence to FL, the Level of the Surface of the Fluid in the Vessel. For, by this Proposition, it will spout from K with such a Velocity, as a Body would acquire by falling from FL, the Level of the Surface, to the Aperture at K; that is, such as will carry it from the Aperture to the Level; because that Velocity which a Body acquires by falling from a certain Height, is sufficient to carry it back to the same Height from whence it fell.

But in Practice the Height the Fluid rises to, is less than that of the Level of its Surface in the Reservoir: this is owing to the Resistance it meets with from the Air, its Friction against the Sides of the Pipe, &c. It is found impossible to make it much exceed the Height of an hundred Feet: For, when it spouts out of the Aperture with a Velocity necessary to carry it higher, the Stream is immediately dashed to Pieces by the Resistance of the Air; whereby it loses its Force, and is prevented from rising to any considerable Height.

\* This Proposition may be otherwise demonstrated from the last, in the following Manner. For, since the Velocity with which a Fluid spouts out through an Hole in the Bottom or Side of a Vessel, is equal to that which a Body would acquire by falling from the Level of the Surface of the Fluid to the Hole, and the Velocities Bodies acquire by falling are as the square Roots of the Heights they fall from (Part I. Chap 5. §. 5) it follows, that the Velocity, with which a Fluid spouts out from an Hole in the Bottom or Side of a Vessel, is as the square Root of the Height of the Level of the Surface of the Fluid above the Hole.

run out would also be as the Pressure (for, the faster the Liquor flows, the greater is the Quantity thrown out in a given Time, and *vice versâ*) consequently, upon this Supposition we should have two Effects, each depending on the same Cause and equal to it, which is absurd. 'Tis not then the Quantity of Fluid run out, nor the Velocity with which it flows, but its Moment, or both these multiplied together, (Part I. Ch. 9. §. 1.) that is the true and adequate Effect of the Pressure. Now these being ever in the same Ratio with each other, 'tis necessary, in Order that the Effect may be proportionable to its Cause, that each of them be only as the square Root of the Pressure: For then, being multiplied together, their Product, or the Moment of the spouting Fluid, is adequately as the Pressure which occasions it; but the Pressure is as the perpendicular Height (§. 4.) therefore the Velocity, and also the Quantity of Fluid spouting out, is only as the square Root of the Height of its Surface above the Hole.

To give an Instance or two; suppose two Holes made in the Side of a Vessel, the one an Inch below the Surface of the Fluid it contains, the other four Inches; the Velocity with which the Liquor flows out of the lower Hole, will not be four Times as great, as that with which it flows through the upper, notwithstanding the Pressure is four Times greater: For if it should, the Quantity run out in a given Time

would

would also be four Times greater, consequently the Effect produced would be sixteen Times greater than it is at the upper Hole, that is, four Times greater than the Cause; which is absurd. Whereas the Velocity and Quantity of Matter will each be only twice as great as they are above, producing thereby a Force or Moment only four Times as great, which is proportionable to the Cause. So, if an Hole were made sixteen Times lower than the first, the Velocity and Quantity of Matter will not be each sixteen Times greater than at the other, but only four Times greater a-piece, and so the Moment sixteen Times greater, as the Pressure is\*.

## XII. When

\* From hence, we may see the Error some of the foreign Mathematicians have fallen into with Regard to the Forces of moving Bodies, who contend that they are as the Squares of the Velocities multiplied by the Quantities of Matter. For, from this Proposition it is, that one of the principal Arguments brought in Favour of this Opinion is derived. They argue thus, *Effects* are ever proportionable to their *Causes*, the *Pressure* of the incumbent Fluid is the *Cause* of its spouting out, the *Force* with which it spouts out, is the *Effect*; but by this Proposition the *Pressure* is as the *Square* of the *Velocity* it flows with, therefore the *Force* is likewise as the *Square* of the *Velocity*. True, it is so; but let us see the Consequence. The *Force*, with which the Fluid spouts out, is not only owing to the *Velocity*, but also to the *Quantity* run out in a given Time. They have each their Share in producing the *Force*, consequently the *Force* is in a Ratio compounded of both, or as the Product of one multiplied by the other, or, which comes to the same Thing, (since as was observed before, they are in the same Ratio with each other) as the *Square* of either of them. From hence it is that the Forces of Fluids in Motion are said to be as the Squares of their Velocities; not that they are so in Virtue of those Velocites

XII. When a Current of Water, or other Fluid falls, perpendicularly upon the Surface of a Plane, or flows against it, (as the Wind against the Sail of a Ship, or the like) the Force, which the Fluid exerts upon it, is equal to the Weight of a Column of the same Fluid, whose Base is equal to the Plane, and its Height such, that a Body falling freely through it would acquire a Degree of Velocity equal to that with which the Fluid moves \*.

In Order to demonstrate this Proposition, let us suppose the Vessel ABCD (*Fig. 11.*) filled with a Fluid, and having a large Hole EF in the Bottom, then will the Pressure of the Fluid cause a Stream to flow out, which in the Hole itself will have such a Degree of Velocity, as a Body would acquire by falling freely from the Surface of the Fluid in the Vessel to the Hole (as demonstrated §. 10.)

Velocities, as such, but in Virtue of them, and the Quantities of Matter taken together, or because the Squares of the Velocities is the same Thing with the simple Velocities multiplied by the Quantities of Matter. Therefore when it is said, the Forces of Fluids are as the Squares of the Velocities, that Part of the Force which arises from the Quantity of Matter is really taken into Consideration. How ridiculous then must it be in those Gentlemen to fetch an Argument from hence to prove, that the Forces of Bodies in Motion are as the Squares of the Velocities and Quantities of Matter too, when they are as the Squares of the Velocities, only because the Quantities of Matter are implied in them.

\* From this Proposition is deduced the Method of computing the Power of a Machine, which is to be moved by Wind or Water, &c. See an Instance of such a Calculation in the Memoirs of the Royal Academy of Sciences for the Year 1725.



In the midst of this Hole, and consequently in the Stream, let 'us suppose a Plane as PQ suspended, but somewhat less than is sufficient to fill the Hole, lest it stop the Current of the Water. Now, 'tis certain, this Plane supports a Column of the Fluid, equal to that which presses upon any other Part of the Bottom of the Vessel of equal Dimensions with itself (for being thus placed it may be looked upon as a Part of the Bottom) but every Part bears a Column, whose Base is equal to its own Dimensions, and its Height the same with that of the Surface of the Fluid in the Vessel: Consequently this Plane supports such a Column, that is, it is resisted by the Stream with a Force equal to the Weight of a Column, whose Base has the same Dimensions with itself, and whose Height is equal to that of the Surface of the Fluid in the Vessel, that is, such an Height as a Body by falling freely from, would acquire a Velocity equal to that with which the Fluid moves.

XIII. The Pressure of a Fluid against a perpendicular Bank or Sluice, &c. is equal to the Weight of a Column of the same Fluid, whose Base is equal to so much of the Bank as is below the Surface of the Fluid, and whose Height is equal to half the Depth of the Fluid\*.

If

\* From hence we see the Reason, why the Water of the Sea or great Lakes is as easily kept within their Banks (setting aside the Force which arises from the Motion of the Waves, &c.



If the Pressure upon every Part of the Bank from the Surface to the Bottom, was as great as it is at the Bottom, the Pressure against it would be equal to the Weight of a Column, whose Base is equal to so much of the Bank as is under the Surface of the Fluid, and which has the *whole* Depth of the Fluid for its Height; for the Pressure upon every Part of the Bank at the Bottom is equal to the Weight of a Column, whose Base corresponds to the Part pressed upon, and whose Height is that of the Depth of the Fluid; consequently if the Pressure was the same every where from Top to Bottom, it would be equal to the Weight of as many such Columns as would answer to all the Parts of the Bank: But the Pressure every where diminishes in Proportion as we approach the Top, where it is Nothing; it is therefore but *half* \* what it would be in the other Case; from whence the Proposition is clear.

*Exc.*) as that of the narrowest Canal, *viz.* because the Pressure of Fluids is not in Proportion to their Surfaces, but their Depths, and the Surfaces they press against.

\* Because the Sum of a Number of Terms in Arithmetical Progression beginning from Nothing, is half the Sum of an equal Number of Terms, each of which is equal to the last in the Progression.

Part II.

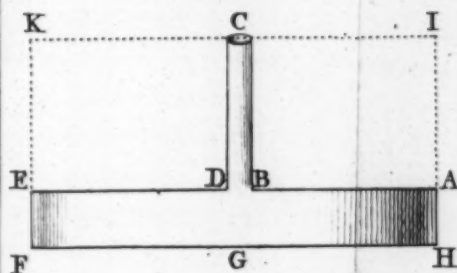
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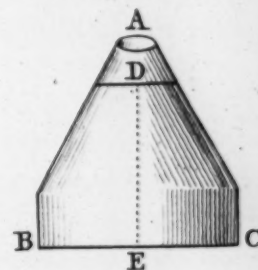
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Part II. Plate II. p. 28

*Fig. 7. p. 18.*



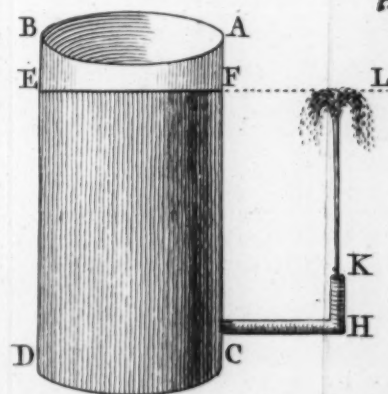
*Fig. 6. p. 17.*



*Fig. 8. p. 17.*



*Fig. 10. p. 21.*



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*Fig. 9. p. 20.*





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## C H A P. II.

*Of the Effects Fluids have on Solids  
immersed therein.*

**T**HE *Specific Gravity* of a Body is that by which it is said to be heavier, or lighter than another of a different Kind: Thus Lead is said to be specifically heavier than Cork; because supposing an equal Bulk of each, the one would be heavier than the other. From hence it follows, that a Body, specifically heavier than another, is also more dense, that is, contains a greater Quantity of Matter under the same Bulk, because Bodies weigh in Proportion to the Quantities of Matter they contain (Part I. Chap. 3. §. 7.)

II. If a Solid be immersed in a Fluid of the same specific Gravity with itself, it will remain suspended therein, in whatever Part of the Fluid it is put.

Let the Body FGHI (*Fig. 12.*) be immersed in the Fluid ABCD to the Depth MN, or any other whatever; I say, it will continue in the same Part of the Fluid, when left to itself, without either rising towards the Surface, or sinking towards the Bottom.

For, the Body being (by the Supposition) of equal Gravity with the Fluid, the Weight of the Column KLHI, which consists partly of  
D Fluid,

Fluid, and partly of the Body, is the same as if it had been all Fluid; consequently HI, that Part of the Surface of the Stratum MN, which lies immediately under the Body, is pressed with the same Degree of Force, that any other Part of the same Dimensions is, and therefore the whole Column KLHI will be supported in its Place. Now, the same being true of the Column KLHI, whatever be its Length, 'tis evident the Body will be suspended in its Place at any Depth.

III. But if the Body is specifically heavier than the Fluid, in which it is immersed, it will subside to the Bottom: For then, in whatever Part of the Fluid it is put, the Column KLHI will always be heavier than an equal Column, that consists all of Fluid; consequently HI, that Part of the Stratum MN, which lies immediately under the Body, will suffer a greater Pressure, than any other Part of the same Dimensions; and therefore will give way and permit the Body to subside continually till it reaches the Bottom.

IV. On the contrary, if the Body is specifically lighter than the Fluid, it will rise to the Top, in what Part of the Fluid soever it is put. For then, the Column KLHI will always be lighter than an equal Column which is all Fluid; consequently HI will be less pressed downwards, than any other Part of the same Stratum of equal Dimensions, and

will therefore continually rise up, carrying the Body with it, till it arrives at the Top.

V. A Body, being laid on the Surface of a Fluid specifically heavier than itself, sinks into it, till the immersed Part takes up the Place of a Quantity of Fluid, whose Weight is equal to that of the whole Body.

Let EFGH (*Fig. 13.*) be a Body, floating on a Liquor specifically heavier than itself, it will sink into it, till the immersed Part IKGH takes up the Place of so much Fluid as is equal to it in Weight. For, in that Case, GH, that Part of the Surface of the Stratum upon which the Body rests, is pressed with the same Degree of Force, as it would be, was the Space IKGH full of the Fluid; that is, all the Parts of that Stratum are pressed alike, and therefore the Body, after having sunk so far into the Fluid, is in *Æquilibrio* with it, and will remain at Rest.

From hence it follows, that a Body is as much specifically lighter, than the Fluid on which it floats, as the immersed Part is less than the Whole. For, by how much the less the immersed Part is, so much the less Fluid is equal in Weight to the whole Body; that is, the Body is so much the lighter in Respect of the Fluid. And, if the same Body be made to float successively in Fluids, whose specific Gravities differ among themselves, (but all exceed that of the Body,) the lighter the Fluids



are, so much greater will be the Part immersed \*.

VI. A Body, suspended in a Fluid specifically lighter than itself, loses a Part of its Weight (or rather communicates it to the Fluid) equal to that of a Quantity of Fluid of the same Bulk.

Let us, instead of supposing the Body suspended in the Fluid, imagine it to be away, and its Place filled with the Fluid; now 'tis evident, this being of the same specific Gravity with the circumjacent Fluid, will be entirely supported by it, or, if we suppose the Body to be of the same specific Gravity with the Fluid, it will be wholly suspended by it; we see therefore the Pressure of the circumambient Fluid, whereby it endeavours to buoy up the Body, is equivalent to the Weight of so much Fluid, as would fill the Place the Body takes up. But, since the Fluid presses only on the Surface of the Body, that Pressure is the same, whatever be the specific Gravity of the Body; the Body therefore loses so much of its Weight, as the Fluid would naturally buoy up; that is, so much, as is the Weight of a Quantity of Fluid of the same Bulk.

\* This Phenomenon is what gave rise to the Hydrometer, an Instrument of great Use in ascertaining the Genuine of Liquors; for it rarely happens, that the adulterated and the genuine Liquor (however they may agree in Appearance) are of the same specific Gravity.

T

This Proposition affords us a Method of determining the Relation, which the specific Gravities of Bodies, whether Fluid or Solid, bear to each other. For, whereas by weighing a Solid in a Fluid specifically lighter than itself, we find the absolute Weight of a Quantity of the Fluid equal to it in Bulk (*viz.* the Weight the Solid loses) the Relation, that Weight bears to the Weight of the Solid, is the Relation of their specific Gravities; because the Weights of Bodies, whose Bulks are equal, are as their specific Gravities: consequently, if the same Solid is weighed successively in different Fluids (all lighter than itself) we gain the Relation, which the specific Gravity of each bears to that of the Solid, and therefore to one another. Again, if different Solids are weighed in the same Fluid, the Relation, which the specific Gravity of that Fluid bears to each Solid, is had, and therefore also the specific Gravities of the Solids among themselves\*.

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\* Upon this is founded the Use of the Hydrostatical Balance for determining the specific Gravities both of Solids and Fluids. The Practice is thus. First, let the Solid be weighed in Air, that is, out of the Fluid; afterwards in it (this ought to be done by suspending it at one End of the Balance by a String, that is as nearly of the same specific Gravity with the Fluid made Use of as possible, and letting it sink into the Fluid till it is wholly immersed below the Surface; if the Fluid is Water, an Horse-Hair is most convenient to hang the Body at the End of the Balance by) then subtract its Weight in the Fluid from its Weight in Air, the Difference is what it loses in  
the

## C H A P. III.

## Of the Air.

I. **T**HAT Part of Natural Philosophy, which treats of the Properties of the *Air*, and the Effects of its *Pressure* and *Elasticity*, is called *Pneumatics*.

II. Air is a thin transparent elastic Fluid surrounding the Earth to a certain Height, and taken all together, is called the *Atmosphere*.

III. That Air is a Fluid, is evident from the easy Passage it affords to Bodies moving in it: For this shews it to be a Body, whose Parts easily yield to a Pressure, that is greater on one Side than on the other, which is the Definition of a Fluid.

IV. Air gravitates towards the Earth, or is heavy like other Bodies.

To prove this, we have Abundance of Arguments both from Sense and Experiment. Thus, when the Hand is applied to the Orifice of a Vessel, it readily perceives the Weight of the incumbent Atmosphere, as soon as the Fluid. This done, say, by the Rule of Proportion, as the Weight lost in the Fluid is to its Weight in Air, so is Unity, or any Number taken at Pleasure, to a Fourth, which by its Relation to the former, will express the Relation of specific Gravity of the Solid to that of the Fluid. Thus, the Relation, which the specific Gravity of the same Fluid bears to that of various Solids, or of the same Solid to that of various Fluids, and consequently the Relation of the specific Gravities of all among themselves may be obtained.

Air

Air included in the Vessel begins to be drawn out. Thus, Glass-Vessels exhausted of their Air (if not strong enough to sustain the Pressure of the incumbent Atmosphere) are crushed to Pieces by the Weight of the Air without. When the Air is exhausted out of a Vessel, the Vessel weighs less than before. With a great many more Experiments generally mentioned by Authors on this Subject \*.

V. The exact Weight of the incumbent Air is determined by filling a Tube with Mercury, and immersing the open End in a Vessel of the same Fluid: For then the Mercury will run down the Tube, till its Surface is fallen to the perpendicular Height of about twenty-nine or thirty Inches above the Surface of the Mercury in the Vessel, and no further: if the same Experiment is made with Water, the Surface of it will stand at about the Height of thirty-two Feet above the Surface of that in the Vessel; the Column of Mercury in one Case, and the Column of Water in the other, exactly balancing the Weight of a Column of Air, which reaches to the Top of the Atmosphere, and presses upon the Surface of the Fluid in the Vessels. This is what is called the Torricellian Experiment, from TORRICELLI the Inventor, and is the same with the common Barometer.

\* See Boyle's Tracts, or *Gravesande*, Lib. II. P. III.

From

From hence it follows (Chap. I. §. 9.) that all Bodies, at the Surface of the Earth, sustain as great a Weight from the Pressure of the Air, as is that of a Column of Water, whose Height is thirty-two Feet, and its Base equal to the Surface of the Body pressed upon\*.

VI. That the Suspension of the Mercury in the Barometer depends on the Pressure of the external Air, is beyond all doubt; for if the Barometer is included in the Air-Pump, the Mercury falls in the Tube, in Proportion as the Air is exhausted out of the Receiver; and if the Air is let in again gradually, the Mercury reascends proportionably, till it reaches its former Height.

VII. That the Atmosphere is extended to a determinate Height, appears from hence; viz. that when the Torricellian Tube is removed to a more elevated Place, the suspended Column of Mercury becomes shorter; which is, because a shorter Column of Air

\* The Pressure of the Atmosphere upon every square Inch, near the Surface of the Earth, is about fifteen Pounds, being equal to the Weight of a Column of Mercury, whose Height is thirty Inches, and its Base one square Inch. For, such a Column of Mercury would weigh about fifteen Pounds. The Weight of the Atmosphere therefore, which presses upon a Man's Body, is equal to so many Times fifteen Pounds, as the Surface of his Body contains square Inches.

The Reason why a Person suffers no Inconvenience from so great a Pressure, is owing to the Air included within the Pores and Fluids of the Body, which by its Reaction is a Counterpoise to the Pressure of the external Air.

presses

presses upon it; or, that the Tube, in this Situation, is nearer the Top of the Atmosphere.

VIII. The Elasticity of the Air is that Property by which it contracts itself into less Space, when an additional Pressure is laid upon it; and recovers its former Dimensions, when the Pressure is taken off. This is accounted its distinguishing Property, all the rest being common to it with other Fluids.

Of this we have numerous Proofs. Thus, a Bladder full of Air being compressed by the Hand, the included Air gives way; but when the Pressure is taken off, the Air expands itself, and readily fills up the Cavity, or Impression made in the Surface of the Bladder. And, if a larger Quantity of Air, than is naturally pressed into a Vessel by the Weight of the incumbent Atmosphere, is forced into it by the *Condenser* (an Engine for that purpose) and if that Air is afterwards let out by opening the Vessel, the Remainder is found to be of the same Weight as at first; from whence it follows, that the Air, by means of its Elasticity or Spring, drives out all that which was forced in by the Condenser, recovers its former Dimensions, and fills the Vessel as before.

IX. From hence, together with what has been observed about the Pressure of the Atmosphere, it follows, that the Air near the Surface of the Earth, is compressed into a much narrower Space by the Weight of the



Air above, than that which it would naturally take up, was it free from that Pressure; accordingly it is found by Experiment, that, when the Pressure of the Atmosphere is taken off from any Portion of the Air, it immediately expands itself into a vast Extent. Hence it is, that thin Glass Bubbles or Bladders filled with Air, being included in the Receiver of the Air-Pump, are broke in Pieces by the Spring of the Air, which they contain within them, when the Pressure of the external Air is taken off. Thus, a Bladder quite flaccid, containing only a small Quantity of Air in it, swells upon the Removal of the external Air, and appears distended, as if it contained as great a Quantity as possible. The same Effect is found in carrying a Bladder somewhat flaccid to a more elevated Place, for there the external Pressure being less, the Air included in the Bladder is in some Measure free from the Pressure of the Atmosphere; it therefore dilates itself, and distends the Bladder as in the former Case.

X. It is found by Experiment, that the Quantity of Space into which Air may be contracted by Pressure, is reciprocally proportionable to the compressing Force. From whence it follows, that the Density of the Air is proportionable to the Pressure which it sustains; because the less the Space is, into which a given Quantity of it is contracted, the denser it is. As to the utmost Degrees of Expansion

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sion and Contraction, which the Air is capable of, they are as yet unknown. In several Experiments made by Mr. BOYLE, Air in its natural State, that is, pressed only with the Weight of the incumbent Atmosphere, dilated itself, when that Pressure was taken off, into more than thirteen thousand Times the Space it took up before. And, he was able so far to compress it, that it should take up more than five hundred and twenty thousand Times less Space than that, into which it would dilate itself, when free from its Pressure\*.

XI. From this Property it follows, that the Air in the inferior Parts of the Atmosphere is more dense than that, which is at great Heights in the same; or, that the Density of the Air decreases continually, as we approach the Top of the Atmosphere. For the Density of the Air is proportionable to the Force

\* See *Boyle's Tracts and Experiments on the Spring and Pressure of the Air.*

Various have been the Opinions of Philosophers concerning the Cause of this prodigious Spring in the Particles of Air; some holding it to depend on their Figure, which they suppose to resemble in some Manner little Bundles of Twigs, or the Branches of Trees; some think them like Fleeces of Wool, others conceive them as rolled up like Hoops, or the Springs of Watches, and endeavouring to expand themselves by Virtue of their Texture. But Sir *Isaac Newton* is of Opinion, that such a Texture is by no Means sufficient to account for that vast Power of Expansion observed above; but that each Particle is endued with a repelling Force which encreases as they approach one another, and accordingly keeps them asunder at Distances reciprocally proportionable to the Pressure they sustain.

See *Hales's Statical Essays.* Vol. I. Chap. 6.

with which it is compressed, and that Force continually decreases, as we approach the Top.

Was the Density of the Atmosphere every where the same as it is near the Surface of the Earth, its Height (as is computed from the Quantity of Pressure it exerts in raising the Mercury in the Barometer) would be about five Miles. But whereas its Density continually decreases, as we approach the Top, and it is uncertain how far the Particles may expand themselves, where there is little or no Pressure, the true Height cannot be obtained. It is computed to continue of a sensible Density to the Height of about forty-five, or fifty Miles.

XII. The Elasticity of the Air produces the same Effects with its Pressure.

For, Action being equal to Reaction, the Force, which the Spring of the Air exerts in endeavouring to expand itself, is equal to the Force with which it is compressed; just as it is in the Spring of a Watch, which exerts no Force, but in Proportion as it is wound up; consequently a Quantity of Air in such a State of Contraction, as it would be compressed into by the Weight of the incumbent Atmosphere, exerts a Force equal to that Weight. If a Quantity of Air therefore is included in a Vessel, and is of the same Density with the circumambient Air, its Pressure against the Sides of the Vessel is equal to the Weight of the Atmosphere. Thus, Mercury is sustained to the same Height by the elastic Force of Air included

included in a Glass Vessel, no Way communicating with the external Air, as by the Weight of the Atmosphere itself.

XIII. The Elasticity of the Air is augmented by Heat, and diminished by Cold \*. For if a Bladder, which is about half filled with Air, is laid before the Fire, it will, when it is sufficiently heated, be distended and burst. Thus, Glass Bubbles being laid upon the Fire immediately burst with great Violence by the augmented Spring of the included Air.

XIV. The Density of the Air thus continually varying, according to the different Degrees of Heat and Cold, to which it is exposed, makes it difficult to ascertain its true specific Gravity. RICCIOLUS estimates it to be to that of Water, as one to a Thousand: MERSENNUS, as one to one Thousand, three Hundred: Mr.

\* This Property is found in all Bodies both Solid and Fluid, but in a much less Degree, than it is in Air. Thus, if a Flask be filled with Water, only to the lower Part of the Neck, and is then set upon the Fire, the Water, when it begins to grow warm, will rise into the Neck, and continue to ascend, as the Heat is increased. And when a Wire or Bar of Iron is heated, it is augmented both in Length and Diameter.

Upon this Property depends the Phenomena of the Thermometer, which is a Glass Bubble with a small hollow Stem rising from it. This Bubble and Part of the Stem is usually filled with Mercury, or Spirit of Wine, which will rise or fall in the Stem, as they are affected by the Heat or Cold of the external Air. If a sufficient Degree of Heat is suddenly applied to this Instrument, the Liquor is observed to descend a little before it rises, because the Glass distending itself, the Capacity of the Bubble is augmented, before the included Liquor is affected by the Heat.

BOYLE,

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BOYLE, by more accurate Experiments, found it to be, as one to nine Hundred and Thirty eight; and thinks, that all Things consider'd, the Proportion of one to a Thousand may be taken as a Medium; for there is no fixing any precise Proportion, because not only the specific Gravity of Air, but that of Water also, is continually varying. However, by some Experiments made since with more Accuracy before the *Royal Society*, the Proportion has been fixed at about one to eight Hundred and Eighty.

XV. Air is necessary for the Preservation of Animal and Vegetable Life: neither will Fire subsist without it. The Reason of this is as yet unknown to Philosophers. Mr. HALE by several curious Experiments in his Statical Essays makes it probable, that 'tis owing to its Elasticity \*.

C H A P. IV.

*Of the Resistance of Fluids.*

THE Resistance a Body meets with in moving through a Fluid, is of three Kinds. The first arises from the Friction of the Body against the Particles of the Fluid, the second from their Cohesion or Tenacity

\* See his *Analysis* of it. Statical Essays, Vol. I. Chap. 6.

among

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among themselves; the third, from their Inactivity, or the Tendency they have in common with other Bodies, to keep the Places they possess.

The first, *viz.* that which arises from the Friction of the Body against the Particles of the Fluid, is very inconsiderable; for whatever the Weight is, which presses the Particles of a Fluid together, the Freedom, with which a Body moves through it, is not sensibly diminished thereby. As was observed, Chap. I. §. 2. in the Notes.

The second, or that which arises from the Tenacity of the Particles of the Fluid, is as the Time the Body continues passing in it\*:

Thus,

\* We have a very curious Argument in Confirmation of this, and which at the same Time illustrates the Manner in which a Body makes its Way through a tenacious Fluid, by Sir *Isaac Newton* himself, in a Postscript to a Letter in the Philosophical Transactions, No. 371. It is as follows. 'Suppose Pieces of fine Silk, or the like thin Substance, extended in parallel Planes, and fixed at small Distances from each other. Suppose then a Globe to strike perpendicularly against the Middle of the outermost of the Silks, and by breaking through them to lose Part of its Motion. If the Pieces of Silk be of equal Strength, the same Degree of Force will be required to break each of them; but the Time, in which each Piece of Silk resists, will be so much shorter as the Globe is swifter; and the Loss of Motion in the Globe consequent upon its breaking through each Silk, and surmounting the Resistance thereof, will be proportional to the Time in which the Silk opposes itself to the Globe's Motion; insomuch that the Globe by the Resistance of any one Piece of Silk, will lose so much less of its Motion as it is swifter. But on the other Hand, by how much swifter the Globe moves, so many more of the Silks it will break through in a given Space of Time; whence the

\* Number



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Thus, suppose a Body continues to move within a Fluid for the Space of one Hour; if its Motion be very quick during that Time, it then moves a great Way in that Fluid, and consequently meets with a great deal of the Fluid to resist it; but is resisted the less in each Part thereof in Proportion to the Stay it makes there: whereas, if its Motion be slow, it then moves but a little Way in that Time, but is resisted so much the longer in each Part thereof. So that whether the Body moves fast or slow, the Resistance it suffers, is the same; and depends therefore on Nothing but the Time we suppose it to continue moving within the Fluid. This Species of Resistance is also very small, except in glutinous and viscid Fluids, whose Parts are not easily separated.

‘ Number of the Silks, which oppose themselves to the Motion of the Globe in a given Time, being reciprocally proportional to the Effect of each Silk upon the Globe, the Resistance made to the Globe by these Silks, or the Loss of Motion the Globe undergoes by them in a given Time, will be always the same.

‘ Now, if the Tenacity of the Parts of Fluids observes the same Rule, as the Cohesion of the Parts of these Silks; namely, that a certain Degree of Force, is required to separate and disunite the adhering Particles, the Resistance arising from the Tenacity of Fluids must observe the same Rule, as the Resistance of the Silks; and therefore in a given Time, the Loss of Motion a Body undergoes in a Fluid by the Tenacity of its Parts, will in all Degrees of Velocity be the same; or in fewer Words, that Part of the Resistance of Fluids, which arises from the Cohesion of their Parts, will be uniform.’

The

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The third Species is the principal Resistance, that Fluids give to Bodies, and arises from their Inactivity, or the Tendency the Particles, of which they consist, have to continue at Rest. The Quantity of this Resistance depends on the Velocity the Body moves with on a double Account: For, in the first Place, the Number of Particles put into Motion by the moving Body in any determinate Space of Time, is proportionable to the Velocity wherewith the Body moves; and in the next Place, the Velocity with which each of them is moved, is also proportionable to the Velocity of the Body; this Species therefore of Resistance is in a duplicate Proportion, or as the Square of the Velocity, with which the Body moves through the Fluid \*.

II. Farther, the Resistance a Body moving in a Fluid meets with from thence, may be considered with Regard to the Fluid; and then it will be found to be more or less, ac-

\* This may be otherwise demonstrated from the twelfth Section of the first Chapter; for from thence it follows, that the Resistance a Fluid gives to a Solid against which it moves, is proportionable to the Height a Body must fall from, to acquire such a Degree of Velocity as the Fluid moves with: But the Heights Bodies fall from are as the Squares of the Velocities they acquire by falling; consequently the Resistance a Fluid gives to a Solid, against which it moves, is also as the Square of its Velocity. Now it matters not, as to the Resistance, whether the Fluid moves against the Solid, or whether it be at Rest, and the Solid moves in it; the Resistance therefore which a Fluid gives to a Solid moving in it, is as the Square of the Velocity, with which it moves.

#### 44 *The Resistance of Fluids.* Part II.

cording to the Density of the Fluid. For by how much denser the Fluid is, so much the greater Number of Particles are to be put into Motion by the Body, in Order to make its Way through it.

III. The next Thing to be considered, is the Effects of the Resistance of Fluids upon Bodies moving in them; that is, the Retardation, which they cause in the Motion of a Body by their Resistance, or the Quantity of Motion they destroy.

And this, in similar Bodies of equal Magnitudes, is inversely as their Densities, or the Quantity of Matter they contain; for by how much the greater the Quantity of Matter in any Body is, so much the more easily does it overcome the Resistance it meets with from the Fluid. Thus we see the Resistance of the Air has a much less Effect in destroying the Motion of an heavy Body, than of a light one which has the same Dimensions.

IV. In similar Bodies of equal Densities but different Magnitudes, the Retardation is inversely as their homologous Sides. For the Resistance Bodies meet with in a Fluid, is inversely as the Quantities of Matter they contain (by the last) that is inversely as the Cubes of the homologous Sides; and it is also directly as their Surfaces, because 'tis by them that they move the Fluid out of its Place; that is, directly as the Squares of their homologous Sides consequent

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consequently the Retardation is inverfely as their homologous Sides \*.

Having given the fundamental Principles of *Hydrostatics*, and shewn how Fluids, both compreffible and incompreffible, are disposed to act upon each other, and upon Solids by their Prefsure, Motion, Elasticity, and Resistance; I proceed now to account for some of the more remarkable Phænomena of Nature, in which they are in Part, or altogether concerned: And this I design for the Subject of the following Differtations.

\* Because the inverfe Ratio of the Cubes of any Numbers being compounded with the direct Ratio of the Squares of the fame, gives the inverfe Ratio of the Numbers themselves.



## DISSERTATION I.

*Of Sound.*

**W**HEN the Parts of an elastic Body are put into a tremulous Motion, by Percussion, or the like; so long as the Tremors continue, so long is the Air, included in the Pores of that Body, and likewise that which presses upon its Surface, affected with the like Tremors and Agitations: Now, the Particles of Air being so far compressed together by the Weight of the incumbent Atmosphere, as their repulsive Forces permit, (as has been explained, Chap. 3.) it follows, that those, which are immediately agitated by the reciprocal Motions of the Particles of the elastic Body, will, in their Approach towards those which lie next them, impel them also towards each other; and thereby cause them to be more condensed, than they were by the Weight of the incumbent Atmosphere, and in their Return will suffer them to expand themselves again; whereby the like Tremors and Agitations will be propagated to them; and so on, till having arrived at a certain Distance from the Body, the Vibrations cease, being gradually destroyed by a continual successive Propagation of Motion to fresh Particles of Air throughout their Progress.

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Thus it is that Sound is communicated from a tremulous Body to the Organ of Hearing. Each Vibration of the Particles of the sounding Body is successively propagated to the Particles of the Air, till it reaches those which are contiguous to the *Tympanum* of the Ear, (a fine Membrane distended across it,) and these Particles in performing their Vibrations impinge upon the Tympanum, which agitates the Air included within it; and that being put into a like tremulous Motion, affects the auditory Nerve, and thus excites in the Mind the Sensation or Idea of what we call *Sound*.

Now since the repulsive Force of each Particle of Air is equally diffused around it every Way; it follows, that when any one approaches a Number of others, it not only repels those which lie before it, in a right Line; but the rest, laterally, according to their respective Situations: that is, it makes them recede every Way from itself, as from a Center: and, this being true of every Particle, it follows, that the aforesaid Tremors will be propagated from the sounding Body in all Directions, as from a Center: And farther, if they are confined for some Time from spreading themselves by passing through a Tube or the like, will, when they have passed through it, spread themselves from the End in every Direction. In like Manner, those which pass through an Hole in an Obstacle they meet with in their Way, will afterwards spread themselves



themselves from thence, as if that was the Place where they began; so that the Sound, which passes through an Hole in a Wall, or the like, is heard in any Situation whatever, that is not at too great a Distance from it. Something analogous to this we may observe in the Motion of Waves upon the Surface of a Fluid, which are propagated equally through all Parts of the Surface in a Circle, though occasioned not by a circular, but reciprocal Motion and Agitation of the Finger, or any other Body in a straight Line.

Since the repulsive Force, with which the Particles of Air act upon each other, is reciprocally as their Distances; (Chap. 3. §. 10.) it follows, that when any Particle is removed out of its Place by the Tremors of a sounding Body, or the Vibrations of those which are contiguous to it, it will be driven back again by the repulsive Force of those towards which it is impelled, with a Velocity proportionable to the Distance from its proper Place; because the Velocity will be as the repelling Force. The Consequence of this is, that, let the Distance be great or small, it will return to its Place in the same Time; (for the Time a Body takes up in moving from Place to Place will always be the same, so long as the Velocity it moves with is proportionable to the Distance between the Places.) The Time therefore in which each Vibration of the Air is performed, depends on the Degree of Repulsion

sion in its Particles: And so long as that is not altered, will be the same at all Distances from the tremulous Body; consequently, as the Motion of Sound is owing to the successive Propagation of the Tremors of a sounding Body through the Air, and as that Propagation depends on the Time each Tremor is performed in, it follows, that the Velocity of Sound varies as the Elasticity of the Air, but continues the same at all Distances from the sounding Body.

And as the Velocity, with which Sound is propagated, depends on the Elasticity of the Air, so it does also on its Density; for when the Density of the Air is augmented, while its Elasticity remains the same\*, a great Number of Particles will move forwards and backwards in each Vibration; now, since we suppose the Cause by which they put each other

\* Perhaps it will not appear to every one, how the Density of the Air can be augmented without a proportional Increase of its Elasticity; because, *cæteris paribus*, the nearer the Particles approach each other, the stronger is the Action of their repulsive Force.

But it is to be considered, that when the Air becomes colder, its Elasticity is diminished, and then the Particles are brought closer together by the Pressure of the Atmosphere, till they acquire an Elasticity equal to what they had before; viz. such as answers to the Pressure they sustain (Chap. 3. §. 12.) From hence we may infer, that the Propagation of Sound is slower in Winter than in Summer, when the Mercury in the Barometer is at the same Height; for the Pressure of the Air being the same, its Elasticity which depends upon it, is so too; but the Air is denser by Reason of the Cold, and therefore its Vibrations slower.

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into Motion (*viz.* their Elasticity,) the same, they will each receive a less Degree of Velocity; and so the Vibrations will be performed in a longer Time, whence the Succession of them will be slower, and the Progress of the Sound proportionably retarded \*.

Whereas the undulatory Motion of the Air, which constitutes Sound, is propagated in all Directions from the sounding Body; it will frequently happen, that the Air, in performing its Vibrations, will impinge against various Objects, which will reflect it back, and so cause new Vibrations the contrary Way; now, if the Objects are so situated, as to reflect a sufficient Number of Vibrations back (*viz.* such as proceed different Ways) to the

\* The Method of determining the Velocity with which Sound is propagated, is (by the Help of a short Pendulum) to estimate the Time, which passes between seeing the Fire of a Gun at a Distance, and hearing the Report. Its great Velocity makes it difficult to be determined exactly; accordingly Authors differ much in their Accounts. The most accurate Observers, Dr. Halley and Dr. Durham, have found it to be about one Thousand one Hundred and Forty-two Feet, which is almost a Quarter of a Mile, in a Second.

The usual Experiments to prove that the Air is necessary for the Propagation of Sound, are such as these. A small Bell being put into the Receiver of the Air-Pump, may be heard at a considerable Distance before the Air is exhausted out of it; but when the Air is much rarefied by exhausting, can scarcely be heard at all. When the Air is condensed, the Sound is augmented in Proportion to the Condensation. These Experiments do not only succeed in forced Rarefactions and Condensations, but in such also as are Natural; Sound being observed to be much weaker on the Tops of high Mountains, where the Air is less condensed by the Weight of the incumbent Atmosphere, than in the Valleys below.

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same Place, the Sound will be there repeated, and is called an *Echo* \*. And, the greater the Distance of the Objects is, the longer will be the Time, before the Repetition is heard. And when the Sound in its Progress meets with Objects, at different Distances, sufficient to produce an Echo, the same Sound will be repeated several Times successively, according to the different Distances of those Objects from the sounding Body; which makes what is called a *repeated Echo*.

If the Vibrations of the tremulous Body are propagated through a long Tube, they will be continually reverberated from the Sides of the Tube into its Axis, and by that means prevented from spreading, till they get out of it; whereby they will be exceedingly encreased, and the Sound rendered much louder than it would otherwise be †.

The Difference of *Musical Tones* depend on the different Number of Vibrations communicated to the Air, in a given Time, by the Tremors

\* In *Woodstock Park*, near *Oxford*, there is an Echo, which repeats distinctly seventeen Syllables by Day, and twenty by Night. See *Plot's Natural History of Oxfordshire*.

The Reason why it repeats more Syllables by Night, than by Day, is because the Air, being colder at that Time, is more dense; and therefore the Return of the first Vibrations being slower, gives Time for the Repetition of more Syllables.

† This is the Case in the *Stentorophonic Tube* or Speaking-Trumpet. See *Kircher de Re Musica. Lib 9. Par. 3. Ode Philos. Natur. Princip. p. 293.*

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Tremors of the sounding Body; and the quicker the Succession of the Vibrations is, the acuter is the Tone, and *è contra*.

A musical Chord performs all its Vibrations, whether great or small, in the same Time. For, if a String is stretched between two Pins, and a Force is applied to the middle Point, to draw it out of its rectilineal Situation; it is found by Experiment, that the Distance (if it be small) to which it is drawn, is as the Force applied; consequently the Velocity, with which it returns, when left to itself, will be as the Space it has to move over; it will therefore perform all its Vibrations in the same Time: This is the Reason why the same Chord, however struck, produces the same Note. It is also found by Experiment, that when Strings of equal Diameters, but different Lengths, are equally stretched, the longer they are, so much the less Weights will draw them from their rectilineal Situation to the same Distance;

Upon this Principle it is, that Sound is conveyed from one Side of a *Whispering* Gallery to the opposite one, without being perceived by those who stand in the Middle. The Form of a *Whispering* Gallery is that of a Segment of a Sphere, or the like arched Figure; and the Progress of the Sound through it may be illustrated in the following Manner.

Let ABC (*Fig. 14.*) represent the Segment of a Sphere; and suppose a low Voice uttered at D, the Vibrations expanding themselves every Way, some will impinge upon the Points E, E, &c. and from thence be reflected to the Points F, from thence to G, and so on, till they all meet in C; and by their Union there cause a much stronger Sound, than in any other Part of the Segment whatever, even at D the Point from whence they came.

the



but in whatever else may.

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the Forces therefore by which they return are less, and the Times of their Vibrations longer.

When two Chords perform their Vibrations in equal Times, the Tone produced is called an *Unison*. If one performs two, while the other one; 'tis an *Octave*. If one three, while the other two; 'tis a *Fifth*. If one three, while the other four; 'tis called a *Fourth*, &c.

To make an Unison Sound, it is not necessary, that the Vibrations of the two Strings should actually concur, but only that they should be performed in equal Times; so that they would always concur, if they began at the same Instant. For the Ear perceives not the single Vibrations distinctly, but only finds that Difference, which proceeds from the Intervals of Time, that pass between them \*.

\* Upon these Principles we may account for that remarkable Phenomenon in *Music*, that an intense Sound being raised, either with the Voice, or a sonorous Body, another sonorous Body near it, and in Unison with it, will thereby be made to sound. For the Vibrations of the Air, which correspond to the Tremors of the first sounding Body, agreeing exactly, in Point of Time, with those, which are capable of being given to the other Body at Unison with it; when they have by their first Impulse communicated a small Degree of Motion to it, will, by conspiring with it, as it moves forwards and backwards, continually increase its Motion, till it becomes sensible. The contrary happens in Strings which are in Discord with each other; for in this Case, though the first Vibration of one may give Motion to the other, yet their Vibrations not being performed in equal Times, the second will come *unseasonably*, i. e. when the other is moving the contrary Way, and obstruct its Motion. It is farther observable, that in two Strings, one of which vi-



brates twice, while the other once; if the first be sounded, the two Extrems of the other will each sound an Unison with it, while the middle Point remains at Rest. So if one vibrates thrice, while the other once, the last will be divided into three Parts, each of which will sound an Unison with it, and the two Points between those Parts will remain at Rest. For otherwise, that which vibrates twice, while the other once, must necessarily interfere with it at every second Vibration; and that which vibrates thrice, while the other once, would interfere with it at every third; so that it would not be put into a sufficient Motion to produce a Sound. But, when it is divided by the quiescent Points, it becomes so many Strings at Unison with the former, each of which easily receives its Vibrations from thence.

From hence likewise it is, that if we take two or three drinking Glasses, and put some Water or other Fluid into each of them, and place them near to each other, taking Care to fill them to such Heights, that (when struck) their Tones shall be in Unison; and then, if we slide the Finger along the Brim of one of the Glasses, pressing pretty strongly upon it, (which will cause it to sound) we shall see the Surface of the Fluids in the other Glasses begin to tremble; which shews that the Vibrations of the first Glas cause the like in the other at Unison with it; though not perhaps in a Degree sufficient to produce a Sound, strong enough to be heard distinctly from the former.

Thus it is that some Persons are able to break a drinking Glas by a Tone of their Voice at Unison with it. They first try the Tone of the Glas by striking it, then applying their Mouth near to the Brim of it, sound the same Note with their Voice; this sets the Glas a trembling; they then continually raise their Voice, sounding still the same Note; this encreases the Tremors of the Glas, which by that Means (if it is not too strong) is broke in Pieces.

The Effect of *Musick* upon Persons bit with the *Tarantula*, (if the Accounts we have of it from Abroad may be credited) is very surprizing. A Person bit with the *Tarantula* after some Time loses both Sense and Motion, and dies if destitute of Help. The most effectual Remedy is *Musick*. The Musician tries Variety of Airs, till he hits upon one that affects the Patient, who upon that begins to move by Degrees; first keeps Time with his Fingers, Arms, and Legs, afterwards is violently agitated in every Part of his Body; and then leaps up, begins to dance, and encreases in Activity every Moment; and  
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after five or six Hours, being very much fatigued, he is put to Bed, and left to sweat. The next Day the same Air brings him out of Bed for a new Dance. Which Exercise being thus continued, the Distemper is abated in the Space of four or five Days, the Effects of the Bite being in some Measure carried off by Sweat, and the Patient begins then to recover his Sense and Knowledge by little and little.

The Reason why the Patient is thus affected by the *Music*, is because the Nerves of his Body are so disposed in that Distemper, as easily to be agitated by the Vibrations which are occasioned by the Principle and stronger Notes of what is played.

See on the Subject of this Dissertation, Philosoph. Transact. No. 134, 234, 302, 313, 319, 337. Hist. de l'Acad. 1702, 1708. Grew's Cosmolog. Sacr. Book I. Chap. 5. Mead upon Poisons, p. 59. Keil's Anatomy, p. 214. Baglivi Prax. Medic. Dissert. I. Bellini de Urinis & Pulsibus, &c. sub fine Capitis de Mania.

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## DISSERTATION II.

## Of Capillary Tubes.

**B**Y a *Capillary Tube* is generally understood a Glass Pipe; the Diameter of whose Bore is at most but about one tenth of an Inch; though any Tube whose Cavity does not exceed that Magnitude, may be so called.

The Phænomena of Capillary Tubes being such as contradict a known Law in *Hydrostatics*, viz. that a Fluid rises in a Tube to the same Height with the Level of its Source\*, and likewise of Affinity with the Ascent of the Sap through the Stems of Plants for the Nourishment of their Fruit, and with divers other Operations of Nature: it has been thought of no small Moment in Philosophy to find out and establish their true Cause; which after numerous Experiments, and several Conjectures about it, is found to be no other than the Attraction of Cohesion; by which small Particles of Matter mutually adhere together, and form larger Bodies†. I shall now down

\* See Chap. I. §. 9. Case 4. in the Notes.

† See *Hauksbee's* and *Power's* Experiments. *Musschenbroeck's* 4to Edit. *Philosoph. Transact.* No. 355. *Mem. de l'Acad.* 1705, 1714, 1722, 1724. With others refer'd to in *Quæstiones Philosoph.*

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the several Phænomena, as so many Matters of Fact; and subjoin to each a Solution from that Cause. In Order to which, it may not be improper to premise the following Consideration by Way of *Lemma*.

Let us suppose the Vessel ABCD (*Fig. 15.*) filled with a Fluid to the Height LM, and let it be conceived as divided into the equal Portions EFGH, GHIK, IKLM, &c. Farther, let it be supposed, that each Particle of Matter in the inner Surface of the Vessel, has a Sphere of Attraction, whose Semidiameter is equal to the Breadth of three of those Portions; that is to say, that the Attraction of the Particle M reaches upwards as far as F, and downwards as far as S; and that of the Particle O, upwards as far as H, and downwards as far as U; and so of all the rest quite round the Tube. From hence it will follow, that every Particle of the inner Surface of the Vessel, which lies between EF and RS, conspires in endeavouring to raise the Fluid towards AB the Top of the Vessel; and that the Fluid is not affected by any other. For Instance, the Particle S, and all below it, will attract downwards three Strata of the Fluid (such as are contained in three equal Divisions of the Vessel) from above, and as many upwards from below; and therefore will have no Effect at all in raising or depressing the Fluid. But the Particle Q will attract only two Strata downwards; because there are no more above it,

it, and three upwards, and therefore will in some Measure tend to raise the Fluid; so the Particle O will attract but one downwards, and three upwards; the Particle M none downwards, and three upwards; the Particle K two upwards, and H only one: all which may clearly be seen by their Situations in the Figure with Respect to the Surface of the Fluid. Therefore in every Vessel, where there is a mutual Attraction between the Fluid it contains and the Particles of which it is composed, there will be a certain Number of Particles disposed around it in Form of a broad Periphery or Zone, as represented by AB; (*Fig. 16.*) half of which lies above the Surface of the Fluid, and half below it, and will tend to make it rise towards the Top. This being understood, the following Phænomena will not be difficult\*.

\* I have been the more particular in explaining this *Lemma* because it is not a bare *Periphery* of no Breadth, to which the Ascent of the Fluid is owing, but a *Zone* or *Cingulum* of Particles distended equally in Breadth both ways from the Surface of the Fluid; and because it is upon the Breadth of this that some of the following Solutions depend. As to the Thickness of it, that undoubtedly is equal to the Semidiameter of the Sphere of Attraction in the Particles of the Vessel; and therefore Vessels, whose Sides are of different Thicknesses (provided those Thicknesses be less than that Semidiameter) must have different Effects upon the same Fluid, though no one has as yet been so accurate as to observe it. The Reason why Fluid will not rise in a large Vessel, as well as in one that is Capillary, is because the Attraction of its Particles does not reach far enough into the Middle of the Vessel; and therefore it only rises about the Sides, standing higher than in the Middle: as may be seen in a Drinking-Glass, when a Quantity of Water is put into it, somewhat less than is sufficient to fill it.

Differt. 2. *Of Capillary Tubes.* 59

I. Let there be two Capillary Tubes, AB and CD (*Fig. 17.*) open at both Ends; and having their lower Orifices, A and C, immersed below the Surface of the Water contained in the Vessel FGHI: The Water will immediately rise up in each Tube above the Surface of that in the Vessel, beginning with a swift Motion, which will gradually decrease; till as much Water has entered the Tubes, as they are able to raise: And the Heights to which the Water will rise in them, will be reciprocally as their Diameters.

That the Water ought to rise in both Tubes is an immediate Consequence of the foregoing *Lemma*; because the Column of Water within the Tube is rendered lighter than an equal Column on the Outside, as being attracted upwards by a Portion of the interior Surface of the Vessel; and therefore will rise till it becomes as much longer than the external ones, as it is made lighter; that the *Æquilibrium*, which was destroyed by the Attraction of the Tube, may be restored by the Weight of the Column. The Reason that the Velocity with which it rises, ought constantly to decrease, is, because the heavier the Column is, the less is the Effect of the Attraction, which is always the same in a Tube of the same Diameter. And lastly, the Heights to which the Water rises in them, will be reciprocally as their Diameters; for then the Quantities raised will be

H directly



directly as the Diameters \*; but the Peripheries that raise them, (being always of the same Breadth, and having their Lengths equal to the Circumferences of the Tubes) are as those Diameters; the Quantities of Water therefore, being in the same Ratio, are as the Peripheries, *i. e.* as the Causes by which they are raised.

II. If the Tubes, before they are immersed in the Water, are filled to greater Heights, than those to which it would naturally rise in them; and then have their lower Orifices immersed in Water, the Water will subside till it stands in each at the same Height to which it would have risen; but, if they are held in a perpendicular Position without being immersed, the Water will not subside in the Tubes *quite* so far.

The Reason why the Water in the Tube, when its lower Orifice is immersed, subsides to the same Height it would have risen to, had the Tube been immersed when empty, is because the Column is suspended in one Case by the same Cause, by which it is raised in the other; but when the Tube full of Water is

\* The Heights to which the Water rises, being in a reciprocal Ratio of the Diameters; and the Contents of Cylindrical Tubes being in a direct Ratio of their Heights, and of the Squares of their Diameters; the Quantities of Water raised in this Case will be in a reciprocal Ratio of the Diameters, and direct one of the Squares of the same. Now these two Ratios being compounded together, give the direct one of the Diameters themselves; because the simple reciprocal Ratio destroys one of those, which are contained in the direct one of the Squares.

Differt. 2. *Of Capillary Tubes.* 61

held erect, without being immersed, it will not subside quite so far; because the lower End of the Tube, which the Water leaves behind it as it drops out, attracts it the contrary Way; so that the Column in this Case is suspended, not only by the inner Surface of the Tube at the Top, but also by its lower End; and therefore a greater Quantity of Water is suspended than in the former Case.

III. If a Tube, having its lower Orifice immersed in Water, be held obliquely, it will raise the Water to the same perpendicular Height, as when held erect.

For since Fluids press according to their perpendicular Heights, the Weight of the Column raised will not be proportionable to the attractive Force of the Tube, till it has arrived at the same perpendicular Height, to which it would have rose, if held erect.

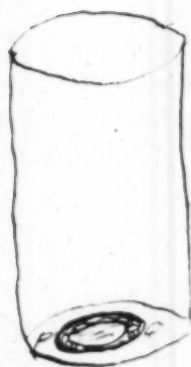
IV. If a Tube, when the Water is risen into it to its wonted Height, is laid in an Horizontal Situation, the Water will move towards the Middle of the Tube, leaving the End which was immersed a little behind.

The Solution of this Phænomenon depends on what was observed in the *Lemma* about the Breadth of the attracting Periphery, and its being equally situated on each Side the Surface of the Water; for from thence it follows, that if the Water should not run from the full End of the Tube, after it has laid in an Horizontal Situation, but remain contiguous

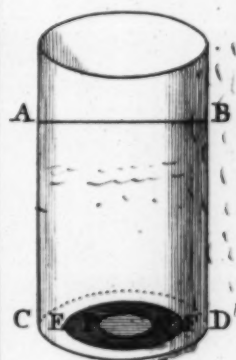
to it; that End of the Column of Water would be attracted only by such a Portion of a Periphery as lies within its Surface at that End; because the End of the Tube coinciding with the Surface, the other Half of the Periphery is wanting. Whereas, at the other End of the Column, there is a Periphery whose Breadth is entire; which overpowering the other, causes the Water to move towards the Middle of the Tube, till as much Periphery is left behind, as can at once affect the Water by the Attraction of its Particles; after which the Water being attracted equally each way, remains at Rest.

V. Let there be a Tube (*Fig. 18.*) consisting of two Parts, DR and RCK, of different Diameters; it follows from what has been said, that DR the smaller Part of the Tube, is able to raise Water higher than the other: Let then the Height to which the larger would raise it, be TF, and that to which it would rise in the lesser (was it continued down to the Surface of the Fluid) be XL. If this compound Tube be filled with Water, and the larger Orifice CK be immersed in the same Fluid, the Surface of the Water will sink no farther than XL; the Height to which the lesser Part of the Tube would have raised it.

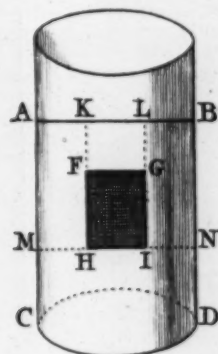
But if the Tube be inverted, as in *Fig. 19.* and the smaller Orifice XL be immersed, the Water will run out till the Surface falls to TF; the Height to which the larger Part of the  
Tube



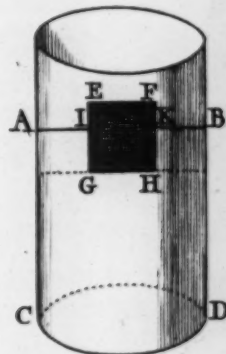
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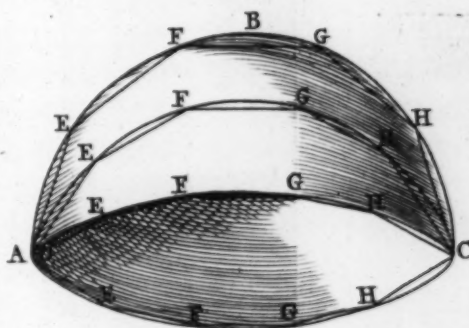
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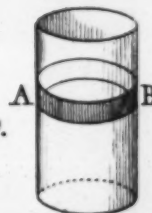
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Differt. 2. *Of Capillary Tubes.* 63

Tube would have raised it. The Size of the lower Part making no Alteration in the Height, at which the Fluid is suspended in either Case.

In order to account for these Phenomena, it must be considered, that when a Body is so disposed, that its different Parts shall move with different Degrees of Velocity, the greater Proportion the Velocity of that Part to which a moving Power is applied, bears to that of the rest; so much the more *effectual* is the Power in moving that Body: As is the known Case of the *Lever*, and all the other *Mechanical Powers*.

Now let us conceive the Tube, DR (*Fig.* 18.) continued down to HI, and let it be supposed at present that the Fluids contained in the Tube XLHI, and the compound one XLKC, are not suspended by the Periphery at L, but that they press upon their respective Bases, HI and CK. Let it farther be supposed that these Bases are each of them moveable, and that they are raised up or let down with equal Velocities; then will the Velocity with which XL the uppermost Stratum of the Fluid XLCK moves, exceed that of the same Stratum, considered as the uppermost of the Fluid in the Tube XLHI, as much as the Tube RCK is wider than DR (by the *Lemma*, Chap. 1. §. 9.) that is, as much as the Space MNKC exceeds XLIH; consequently by what was observed in the foregoing Paragraph, the  
Effect



Effect of the attracting Periphery XL, as it acts upon the Fluid contained in the Vessel XLCK, exceeds its Effect, as it acts upon that in XLHI, in the same Proportion. Since therefore it is able (*ex Hypoth.*) to sustain the Weight of the Fluid XLHI by its natural Power, it is able under this *Mechanical* Advantage, to sustain the Weight of as much as would fill the Space MNKC: But the Pressure of the Fluid XLCK is equal to that Weight, as having the same Base and an equal Height (Chap. I. §. 9.) its Pressure therefore, or the Tendency it has to descend in the Tube, is equivalent to the Power of the attracting Periphery XL, for which Reason it ought to be suspended by it.

Again, the Height (*Fig. 19.*) at which the attracting Periphery in the larger Part of the Tube is able to sustain the Fluid is not greater than NF, that to which it would have raised it, had the Tube been continued down to MN. For here the Power of the attracting Periphery acts under a like *Mechanical* Disadvantage; and is thereby diminished in Proportion to the Capacity of the Tube TFNM to that of HIXL; because, if the Bases of these Tubes are supposed to be moved with equal Velocities, the Rise or Fall of the Surface of the Fluid TFXL, would be so much less than that of TFMN. And, where the attracting Periphery TF is able by its natural Power to suspend the Fluid only to the Height

Differt. 2. *Of Capillary Tubes.* 65

Height NF in the Tube TFMN; it is in this Case able to sustain no greater Pressure than what is equal to the Weight of the Fluid in the Space HIXL: but the Pressure of the Fluid TFXL, which has equal Height, and the same Base with it, is equal to that Weight; and therefore is a just *Æquipondium* to the attracting Power.

VI. From hence we may clearly see the Reason, why a small Quantity of Water put into a Capillary Tube, which is of a Conical Form, and laid in an Horizontal Situation, will run towards the narrower End. For let AB (*Fig. 20.*) be the Tube, CD a Column of Water contained within it; when the Fluid moves, the Velocity of the End D will be to that of the End C reciprocally as the Cavity of the Tube at D, to that at C (by the *Lemma*, Chap. 1. §. 9.) that is, reciprocally as the Square of the Diameter at D, to the Square of the Diameter at C\*; but the attracting Periphery at D is to that at C, in the simple Ratio of the Diameter at D, to the Diameter at C. Now, since the Effect of the Attraction depends, as much upon the Velocity of that Part of the Fluid where it acts, as upon its natural Force, its Effect at D will be greater than at C; for though the Attraction at D be really less in itself than at C, yet its Loss of Force upon that Account, is more than

\* 12 El. 2.

compen-

compensated by the *mechanical* Advantage it has arising from hence, *viz.* that the Velocity of the Fluid in that Part is greater than it is at C, in a greater Proportion than the Force itself is less: The Fluid will therefore move towards B.

VII. From hence likewise it follows, that if a Vessel, as ABC (represented *Fig. 21.*) of any Form whatever, have its upper Part drawn out into a Capillary Tube as B; and if this Vessel is filled with Water, and have its lower Orifice placed on FD the Surface of the same Fluid; then the Water will remain suspended in the Vessel, provided the Capillary at the Top be small enough (was it continued down to the Bottom) to raise the Fluid to the Height B. Because, by Proposition the fifth, the lower Part of the Tube makes no Alteration in the Height, at which the Capillary B is able to sustain the Fluid.

VIII. And if the same Vessel be filled only to the Height DE (*Fig. 22.*) and a Drop of Water be put into the Capillary at B, the intermediate Part BDE being full of Air) the Water will continue suspended at the Height DE.

For, although the Fluid ACDE is not in Contact with the Drop of Water in the Capillary Tube, and therefore not immediately supported by it; yet the Pressure of the Atmosphere upon the Surface FG, and against the upper Part of the Drop in the Capillary B,

keep

Differt. 2. *Of Capillary Tubes.* 67

keeps the Fluid ADEC, and the Drop, and the intermediate Air from separating, just as in the former Case the Attraction of Cohesion in the Particles of the Water prevented a Separation between that in the Vessel and that in the Capillary. Consequently, as the Water in the Capillary was able in the former Case to sustain as much Fluid as the Vessel could contain, it is now sufficient to sustain the Fluid ADEC\*.

IX. Let there be a Capillary Siphon, as that represented, *Fig. 23, 24 or 25*, and let EF be the Height, to which Water might be raised by a Periphery equal to that at A. Now, since (as was observed, §. 2.) the lower End of a Tube when it is not immersed, causes a longer Column to be suspended than otherwise would be; that is, it supports a short Column besides that which is sustained by the attracting Periphery; let HI be the Height of such a Column as might be suspended by the End C: then, if any of those Tubes are filled with Water, and held as in the Figure (neither End being immersed) the Fluid will run out of the Tube at C, if CD the Difference of the

\* This happens quite otherwise in Vacuo, because the Pressure of the Air, which, as it were, connects the Drop with the Water ADEC, being wanting, it immediately falls for Want of a Support. Whereas the former Phenomenon equally succeeds in Vacuo; which shews that the Parts of the Fluid in the Vessel are connected with each other, and with that in the Capillary by their own mutual Attraction of Cohesion, there being nothing else whereby they can be supported.

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Legs exceeds EF and HI added together, otherwise not.

For the Column AB is a Counterpoise to BD, being of the same perpendicular Height; and therefore it is only the Weight of the Column CD, which determines the Fluid to move; unless that Weight therefore exceeds the Force of the attracting Periphery at A (which the Fluid AB must leave in rising up the Tube) and also what may be supported by the End C, that is, (*ex Hypoth.*) two Columns whose Heights are EF and HI, it cannot run out at C; otherwise it will, as being destitute of a sufficient Support.

X. If the End A is immersed in Water, and the other not (supposing the Tube full as before,) it will run out at C, though CD the Difference of the Legs, only exceeds HI. For then the Attraction at A ceases, and there is nothing to support the Column CD, but the Power the End C has to prevent Drops from falling off it.

XI. Again, if the End C is immersed in Water, (and the other not) it will run out at C, if CD exceeds EF, otherwise not.

For in this Case, there is nothing to support the Column CD, but the attracting Periphery at A, whose Power is supposed able to raise a Column as EF, and no more.

XII. And, if both Ends are immersed (the Tube being supposed full as before) the Water will run out at the lower, which ever it is.

For

## Differt. 2. *Of Capillary Tubes.* 69

For then the Attraction of both Ends ceases; and the longer Column over-balancing the shorter, the Fluid is determined thereby to run out at the lower End.

XIII. If either of the Tubes (*Fig. 23* or *24.*) are small enough to raise the Water from A to B, and if the Orifice A is immersed, the Fluid will rise to B, passing on to C, where it will run out or be suspended according to the foregoing Cases: but if the Periphery at G (*Fig. 25.*) is such as would support a Column of the Length of AM only, and if the Tube be filled from A to G (the Extremity A being still immersed as before) the Liquor will proceed no farther, but remain in the Position in which it is put.

For that Fluid which has passed B assists by its Weight the attracting Periphery in raising the Column AB, and therefore it runs down to C. But, if when the Tube is filled to G, the Periphery there is not able to support more than AM the Difference of the Legs AB and BG, the Fluid must necessarily stand there; since BG is no more than a Counterpoise to MB, and AM is supposed to be as much as the Periphery at G can sustain.

XIV. Tho' a Capillary Tube be shorter than the Height to which its attracting Periphery is able to raise a Fluid, v. g. tho' the Tube AB (*Fig. 26.*) be small enough (was it of sufficient Length) to raise Water as high as C; yet when the End A is immersed, the Fluid will



not run out at B, but only be suspended at that Height.

For when the Fluid is risen as high as B, it has then no more Periphery above it, to attract it any farther; and if it was forced up a little above the End, it would be attracted back again by it \*.

XV. The Ascent of different Fluids in the same Tube is various. *Musschenbroek* has found, that in a Tube in which Water will rise to the Height of twenty-six Lines, Oil of Wormwood will rise but eighteen or nineteen, whereas Urine will rise thirty-three or thirty-four.

\* Hence we see the Absurdity of supposing that a Fluid may be made continually to flow from a lower Place to an higher by a Capillary Tube, as such; for whether the Tube be of such Form, as is represented, *Fig. 23, 24, 25, or 26*, the Fluid will always stop when it comes at the higher End; because the Attraction is then in a Direction contrary to its Motion; and the Weight of the whole Fluid, if the Tube is straight, or of so much as is contained in DC, the Difference of the Legs, if it be crooked, is likewise an Impediment in it.

Hence also the Absurdity of supposing a Fluid to circulate by Virtue of a Capillary Tube, as such. For suppose the Ends of the Capillary Tube joined together, so that the Tube shall return into itself; if the Tube be not full of Liquor, the Liquor will place itself as near as possible to the smallest Part of the Tube, as may easily be inferred from what was observed in §. 6. and will rest there: if it be full, in which Case the Extremities of the Fluid join, then there is no Periphery or Zone of Particles lying part out of the Fluid, and part in it, and consequently all Ground or Cause of Motion, so far as it arises from the Tube being *capillary*, is entirely taken away. And if Part of the Tube be capillary, and Part not, the Case is still the same. How ridiculous then is it to affirm, as some have done, that the Circulation of the Animal Fluids is promoted by the Passages through which they flow, being of that Kind.

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## Differt. 2. *Of Capillary Tubes.* 71

The Reason of which is, because some Fluids are attracted more strongly by Glass, than others are. Mercury exhibits Phænomena just the Reverse of the former; for if the End of a Capillary Tube be immersed below the Surface of that Fluid, it will not rise in the Tube to a Level with that on the Outside. This is because the Particles of Mercury attract each other more forcibly than they are attracted by those of Glass \*.

\* See *Jurin's* Differt. Philosoph. Transf. No. 363.

According to *Musschenbroek* the Length of the upper Part of a Tube, which is above the Height to which it is able to raise a Fluid, conduces something towards the raising it; and therefore in a longer Tube a Fluid rises higher than in one of the same Dimensions that is shorter; and if a Tube, with so much Fluid contained in it, as it is able to raise, be laid in an Horizontal Situation, the Fluid will run to the Middle of it. But of this I have had no Experience: 'Tis possible that ingenious Professors, tho' very accurate in making Experiments, might herein be deceived. He acknowledges, (Experiment the fifteenth) that it sometimes happens otherwise.

Other Authors, besides those already referred to, that have treated on this Subject, are *Boyle* Exper. Phys. Mech. Exp. 9. *Marmius* Colleg. Cur. Tentam. 8. *Bernoulli* Gravit. Æth. *Hooke* Microg. Obser. 6. *Leeuwenboek* Continuat. Arcan. Nat. Epist. 31. *Sinclair* Art. Gravit.

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## DISSERTATION III.

*Of the Origin of Fountains.*

MANY have been the Conjectures of Philosophers concerning the Origin of Fountains; and great Pains have been taken both by the Members of the Royal Society, and those of the Academy of Sciences at Paris, in order to ascertain the true Cause of it. It was ARISTOTLE's Opinion, and held by most of the antient Philosophers after him, that the Air contained in the Caverns of the Earth, being condensed by Cold near its Surface, was thereby changed into Water; and that it made its Way through, where it could find a Passage. But we have no Experience of any such Transmutation of Air into Water.

Those who imagine that Fountains owe their Origin to Waters brought from the Sea by subterraneous Ducts, give a tolerable Account how they lose their Saltness by Percolation as they pass through the Earth; but find great Difficulty in explaining by what Power the Water rises above the Level of the Sea, to near the Tops of Mountains, where Springs generally abound; it being contrary to the Laws of *Hydrostatics*, that a Fluid should

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rise in a Tube above the Level of its Source. However, they have found two Ways, whereby they endeavour to extricate themselves from this Difficulty. The one is that of DES CARTES, who imagines, that after the Water is become fresh by Percolation, it is raised out of the Caverns of the Earth in Vapour towards its Surface; where meeting with Rocks near the Tops of Mountains in the Form of Arches or Vaults, it sticks to them, and runs down their Sides, (like Water in an Alembic) till it meets with proper Receptacles, from which it supplies the Fountains. Now, this is a mere *Hypothesis* without Foundation or Probability; for in the first Place we know of no internal Heat of the Earth to cause such an Evaporation; or if that were allowed, yet 'tis quite incredible, that there should be any Caverns so smooth, and void of Protuberances, as to answer the Ends of an Alembic, in collecting and condensing the Vapours together, in every Place where Fountains arise. There are others (as VARENIUS, &c.) who suppose that the Water may rise through the Pores of the Earth, as through Capillary Tubes by Attraction; but hereby they shew, that they are quite unacquainted with what relates to the Motion of a Fluid through such Tubes. For when a Capillary Tube opens into a Cavity at its upper End, or grows larger and

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and larger, so as to cease to be Capillary at that End; the Water will not ascend through that Tube into the Cavity, or beyond where the Tube is Capillary; because that Part of the Periphery of the Cavity, which is partly above the Surface of the Water, and partly below it, is not of the Capillary Kind. Nay, if the Cavity is continually supplied with Water, it will be attracted into the Capillary Tube, and run down it, as through a Funnel, if the lower End is immersed in the same Fluid, as in this Case it is supposed to be \*.

It has been a generally received Opinion, and much espoused by MARRIOTTE (a diligent Observer of Nature) that the Rise of Springs is owing to the Rains and melted Snow. According to him, the Rain-Water, which falls upon the Hills and Mountains, penetrating the Surface, meets with Clay or Rocks contiguous to each other, along which it runs, without being able to penetrate them, till being got to the Bottom of the Mountain, or to a considerable Distance from the Top, it breaks out of the Ground, and forms Springs.

In Order to examine this Opinion, Mr. PERRAULT, DE LA HIRE, and D. SIDÉ-LEAU endeavour'd to make an Estimate of the Quantity of Rain and Snow, that falls in

\* See the foregoing Dissertation, Sect. 14.

Differt. 3. *The Origin of Fountains.* 75

the Space of a Year, to see whether it would be sufficient to afford a Quantity of Water, equal to that which is annually discharged into the Sea by the Rivers. The Result of whose Inquiries was, that the Quantity of Rain and Snow which fell in a Year into a Cylindrical Vessel, would fill it (if secured from evaporating) to the Height of about nineteen Inches. Which Quantity D. SIDÉ-LEAU \* shewed, was not sufficient to supply the Rivers; for that those of *England, Ireland, and Spain* discharge a greater Quantity of Water annually, than the Rain, according to that Experiment, is able to supply. Besides which, another Observation was made by them at the same Time, viz. that the Quantity of Water raised in Vapour one Year with another, amounted to about Thirty-two Inches, which is thirteen more than falls in Rain: A plain Indication, that the Water of Fountains is not supplied by Rains and melted Snow.

Thus, the true Cause of the Origin of Fountains remained undiscovered, till Dr. HALLEY in making his Celestial Observations upon the Tops of the Mountains at *St. Helena*, about eight Hundred Yards above the Level of the Sea, found that the Quantity of Vapour which fell there (even when the Sky was clear) was

\* Memoirs of the Royal Academy of Sciences for the Year 1693.



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so great that it very much impeded his Observations, by covering his Glasses with Water every half Quarter of an Hour; and upon that attempted to determine by Experiment the Quantity of Vapour, exhaled from the Surface of the Sea, as far as it rises from Heat; in order to try, whether that might be a sufficient Supply for the Water continually discharged by Fountains. The Process of his Experiment was as follows. He took a Vessel of Water salted to the same Degree with that of Sea-Water, in which he placed a Thermometer; and by means of a Pan of Coals, brought the Water to the same Degree of Heat, which is observed to be that of the Air in our hottest Summer: This done, he affixed the Vessel of Water with the Thermometer in it, to one End of a Pair of Scales, and exactly counterpoised it with Weights on the other. Then, at the End of two Hours, he found by the Alteration made in the Weight of the Vessel, that about a sixtieth Part of an Inch of the Depth of the Water, was gone off in Vapour; and therefore in twelve Hours, one tenth of an Inch would have gone off. Now this accurate Observer allows the *Mediterranean Sea* to be forty Degrees long, and four broad; (the broader Parts compensating for the narrower; so that its whole Surface is one Hundred and sixty square Degrees;) which according to the Experiment must yield at least five Thousand, two Hundred and eighty Millions

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lions of Tons of Water. In which Account, no Regard is had to the Wind, and the Agitation of the Surface of the Sea; both which undoubtedly promote the Evaporation.

It remained now, to compare this Quantity of Water, with that, which is daily conveyed into the same Sea, by the Rivers. The only Way to do which, was to compare them with some known River; and accordingly he takes his Computation from the River *Thames*, and to avoid all Objection, makes Allowances, probably greater, than what were absolutely necessary.

The *Mediterranean* receives the following considerable Rivers, viz. the *Iberus*, the *Rhone*, the *Tibur*, the *Po*, the *Danube*, the *Niefter*, the *Boristhenes*, the *Tanais*, and the *Nile*. Each of these he supposes to bring down ten Times as much Water as the *Thames*; whereby he allows for smaller Rivers, which fall into the same Sea. The *Thames* then he finds by Mensuration to discharge about 20,300,000 Tons of Water a Day. If therefore the above-said nine Rivers yield ten Times as much Water as the *Thames* doth, it will follow, that all of them together yield but 1827 Millions of Tons in a Day; which is but little more than one Third of what is proved to be raised in Vapour out of the *Mediterranean* in the same Time. We have therefore from hence a Source abundantly sufficient for the Supply of Fountains.

78 *The Origin of Fountains.* Part II.

Now having found, that the Vapour exhaled from the Sea, is a sufficient Supply for the Fountains; he proceeds in the next Place to consider the Manner in which they are raised; and how they are condensed into Water again; and conveyed to the Sources of Springs.

In order to this he considers, that, if an Atom of Water was expanded into a Shell or Bubble, so as to be ten Times as big in Diameter as when it was Water, that Atom would become specifically lighter than Air; and therefore would rise so long as the Warmth which first separated it from the Surface of the Water should continue to distend it to the same Degree; and consequently, that Vapours may be raised from the Surface of the Sea in that Manner \*, till they arrive at a certain Height in the Atmosphere, at which they find the Air of equal specific Gravity with themselves. Here they will float, till being condensed by Cold, they become specifically heavier than the Air, and fall down in Dew, or being driven by the Winds against the Sides of Mountains, (many of which far surpass the usual Height to which the Vapours would of themselves ascend) are compelled by the Stream of the Air to mount up with it to the Tops of them; where being condensed into Water they presently precipitate, and

\* See this confuted in Dissertation 6.

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gleeting down by the Crannies of the Stone, Part of them enters into the Caverns of the Hills; which being once filled, all the Overplus of Water that comes thither, runs over by the lowest Place, and breaking out by the Sides of the Hills, forms single Springs. Many of these running down by the Valleys between the Ridges of the Hills, and coming to unite, form little Rivulets or Brooks: Many of these again meeting in one common Valley, and gaining the plain Ground, being grown less rapid, become a River; and many of these being united in one common Channel, make such Streams as the *Rhine* and the *Danube*; which latter, he observes, one would hardly think to be a Collection of Water condensed out of Vapour, unless we consider how vast a Tract of Ground that River drains, and that it is the Sum of all those Springs which break out on the South Side of the *Carpathian* Mountains, and on the North Side of the immense Ridge of the *Alps*; which is one continued Chain of Mountains from *Switzerland* to the *Black-Sea*.

Thus one Part of the Vapours, which are blown on to the Land, is returned by the Rivers into the Sea, from whence it came; another Part falls into the Sea before it reaches the Land; and this is the Reason, why the Rivers do not return so much Water into the *Mediterranean* as is raised in Vapour. A third Part falls on the Low-Lands, and is the

80 *The Origin of Fountains.* Part II.

the *Pabulum* of Plants, where yet it does not rest; but is again exhaled in Vapour by the Action of the Sun, and is either carried by the Winds to the Sea, to fall in Rain or Dew there, or else to the Mountains, to become the Sources of Springs.

However, it is not to be supposed, that all Fountains are owing to one and the same Cause; but that some proceed from Rain and melted Snow, which subsiding through the Surface of the Earth, makes its Way into certain Cavities, and thence issues out in the Form of Springs; because the Waters of several are found to increase and diminish in Proportion to the Rain which falls: That others again, especially such as are salt, and spring near the Sea-shore, owe their Origin to Sea-Water percolated through the Earth; and some to both these Causes: Though without doubt most of them, and especially such as spring near the Tops of high Mountains, receive their Waters from Vapours, as before explained\*.

\* There is a certain Species of Springs which ebb and flow alternately, and some that cease to flow for a Time, which from thence are called reciprocating or intermitting ones. Their Reciprocations may be accounted for in the following Manner.

Let ABC represent one Side of an Hill in which there is a Cavity DEF, and from this a subterraneous Duct IKL. Now as this Cavity fills with Water (suppose from Vapours percolating through the Surface of the Hill, or in any other Manner whatever) its Surface will rise in the Duct as it does in the Cavity, till it arrives at M, the Level with the upper Part of the Duct; at which Time it will run over at K, filling KLA the other Part of the Duct. Now, if the perpendicular Height of the Point K above L (or rather above A) be greater than its perpendicular

### Differt. 3. *The Origin of Fountains.* 81

pendicular Height above I, the Column KLA will overpoise the Column KI, and so cause the Water to run out at A, till its Surface in the Cavity sinks as far as I, (provided the Duct is large enough to convey the Water away faster than it enters the Cavity) at which Time the Fountain at A will cease to play, till the Surface of the Water in the Cavity rises again to M, and runs over at K as before. The Reason why the Water continues running (when the Duct is once full) till its Surface sinks to I, is because the Air pressing against it as it runs out at A, and also upon its Surface in the Cavity, keeps the Duct full, as long as the Water in the Cavity is high enough to feed its Orifice at I.

See more on this Subject in Philosoph. Transact. No. 119, 189, 192, 384, 424. Histoire de l'Acad. 1693, 1703, 1713. *Gulielmini della Natura de Fiumi.* Dale's History of *Harwich.* *Marriotte's Hydrostatics.* *Nieuwentyt Contempt.* 19. *Varenius Geograph.* Cap. 16. *Regnault*, Vol. 2. *Conversat.* 6. *Hales's Statical Essays*, Vol. 1. Experiment 19. *Micheletus* in *Append.* ad *J. Bernoullii de Effervesc.*

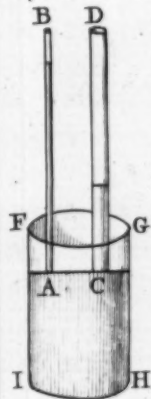


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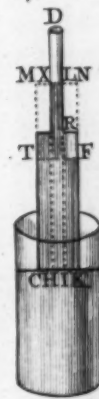


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p. 60.



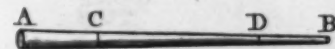
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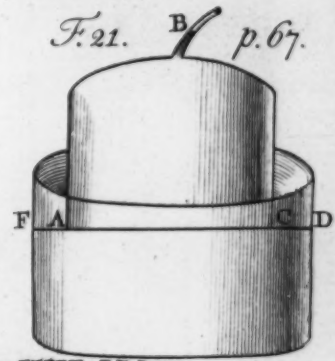
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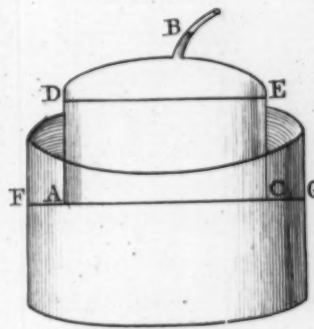
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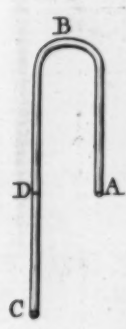
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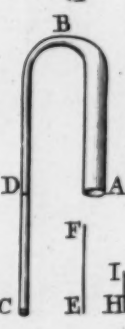
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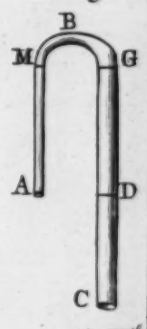
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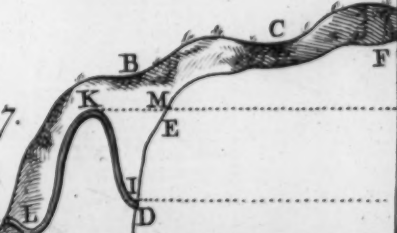
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## DISSERTATION IV.

## Of the Barometer.

**I**N treating of the Properties of the Air (Chap. III.) I have already taken Notice of the Construction of the common Barometer; and proved, that the Ascent and Suspension of the Mercury therein, is owing to the Pressure \* of the Air. I proceed now to a more particular Inquiry into the Original, and Use of this Instrument; and the different Forms under which it has appeared, since the Time of its Inventor *Torricelli*.

In

\* To say the Ascent and Suspension of the Mercury is owing to the Pressure and Elasticity of the Air, as is commonly done, is inaccurate. The Variation, indeed, in the Height of the Mercury, may be ascribed to the Elasticity of the Air, but no otherwise, than as to its *remote Cause*; viz. as it occasions an Alteration in the Quantity of Air, impending over the Place where the Variation happens; which alters its Weight, and so the Mercury is proportionably raised or depressed. To illustrate this, let it be supposed, that the Air is every where in *Equilibrio*, quite round the Globe, and at perfect Rest; and then, that its Elasticity, in some one Place near the Surface of the Earth, is augmented by the Heat of the Sun, all the rest of it remaining as before. The Consequence of this will be, that the superior Part of the Atmosphere, over this Place, will be raised higher by the Expansion of the inferior Air; and therefore, being unconfined, will spread itself, every Way, over the neighbouring Columns, which we suppose to retain their former State. The Quantity of Matter therefore in those Columns of Air, in whose lower Parts its Elasticity was increased, will be diminished, and that of the neighbouring

In the Beginning of the last Century it was a prevailing Opinion among Philosophers, that the Universe was full of Matter ; and that Nature (as they expressed it) abhorred a *Vacuum* : Accordingly they imagined, that if a Fluid was sucked up a Pipe with a sufficient Force, it would rise to any Height whatever ; since Nature would not suffer any Part of the Pipe to be empty. *Galileo*, who flourished about that Time, found upon Trial, that the common Pump would not raise Water, unless the Sucker reached within three and thirty Feet of its Surface in the Well † : From

hence ones augmented. A Barometer therefore placed in those Regions where the Air was rarefied, will subside ; while one in the neighbouring Countries will ascend ; and they will continue at different Heights, till the denser Air, rushing in upon the rarefied, restore the *Æquilibrium*. Thus we see, the Variation of the Air's Elasticity is not the *immediate* Cause of the Variation in the Barometer ; it first affects the Weight of the Air, by altering the Quantity incumbent over any Place, and that affects the Barometer. But, if we may have Recourse to *remote* Causes, we may, if we please, go one Step farther ; and say, the Ascent and Suspension of the Mercury is owing to the Heat of the Sun ; for, by the foregoing Instance, a Variation in the Heat of the Sun may sometimes be the Occasion of a Variation in the Height of the Mercury.

Neither is the Suspension of the Mercury, in a Tube, that is kept within Doors, to be ascribed to the Elasticity of the Air ; for that exerts no Force, but as the internal Air is pressed by the external, which endeavours to get in, where-ever it can find a Way.

† It is a common Notion, that a sucking Pump will not raise Water above thirty-three Feet, whereas it will raise it to any Height whatever, if the Sucker reaches within thirty-three Feet of the Surface of the Water ; as will be evident to any one that considers the Structure of the Pump : For all the Water, which has

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hence he judiciously inferred, that a Column of Water thirty-three Feet high was a Counterpoise to a Column of Air of an equal Base, whose Height extended to the Top of the Atmosphere; and that, for this Reason, the Water would not follow the Sucker any farther. *Torricelli*, observing this, took the Hint; and considered, that, if a Column of Water of about thirty-three Feet was equal in Weight to a Column of Air of the same Base §; a Co-

once passed through the Valve in the Sucker, is supported by that, as the Sucker is drawn up, and as the Sucker is let down it together with the Water between that and another placed in the Pump below, is supported by that other; so that the Height of the Water above the Sucker can be no Impediment to the rising of the Water below the Sucker, whatever the Length of the Column, which it forms, may be. The placing one Pump above another, where Water is to be raised from great Depths, is rather for Strength and Conveniency, than out of Necessity.

§ Perhaps it may be enquired here, how it comes to pass, that the Column of Air, which presses upon the stagnant Mercury in the Bason, is always supposed to have an equal Base with the suspended Column in the Tube; whereas, in Reality, its Base is equal to the Surface of the stagnant Mercury? The Reason is, that, as the Base of the Column of Air increases, in the same Proportion the Velocity, wherewith it descends, decreases, when it forces down the Surface of the Mercury in the Bason; consequently its Moment, or Pressure upon the Surface of the stagnant Mercury (so far as it relates to the suspending of it in the Tube) is no greater, than it would have been, had its Base been equal to that of the suspended Column; and therefore, in considering it as suspending a Fluid in a Tube, it is properly enough said to be a Column of such a Base.

Neither is this Supposition inconsistent with the ninth Proposition of the first Chapter, where it is demonstrated, that the Pressure of a Fluid is in Proportion to its perpendicular Height, and the Quantity of Surface, against which it presses. For, as the Surface of the Mercury may be considered as a Base on which the



Column of Mercury, no longer than about twenty-nine Inches and a half, would be so too; such a Column of Mercury being as heavy, as thirty-three Feet of Water. Accordingly he tried the Experiment in a Glass Tube (in the Manner laid down, Chap. III. §. 4.) and found it to succeed †. The Apparatus he made

Use

Column of Air rests, so the Base of the Column of Air may be consider'd as a Surface against which the Mercury presses. These two being equal, 'tis clear, that only the Relation of the Heights of the Columns are to be considered, and not that of their Bases.

† Notwithstanding this clear Proof of the Pressure of the Atmosphere, the Assertors of a *Plenum* would by no Means be prevailed upon to allow it to be such; but tried all Ways to account for this *Phænomenon* from some other Cause. The most chimerical Solution, and which at the same Time gave the adverse Party the greatest Difficulty to overthrow it, was that of *Linus*. He contended, that in the upper Part of the Tube, there is a Film, or Rope of Mercury, extended thro' the seeming Vacuity, and that the rest was suspended by it, and kept from falling into the Bason; and that this Film is able to support about twenty-nine Inches of Mercury. He confirms his *Hypothesis* by the following Experiment: Take, says he, a small Tube, open at both Ends, suppose about twenty Inches long; fill this Tube with Mercury, stopping the lower Orifice with your Thumb. Then closing the upper with your Finger, and immersing the lower in stagnant Mercury, you shall perceive, upon the Removal of your Thumb, a manifest Suction of your Finger into the Tube; and the Tube and Mercury will both stick so close to it, that you may carry them about the Room. Therefore, says he, the internal Cylinder of Mercury in the Tube is not held up by the preponderant Air without; for if so, whence comes so strong a Suction, and so firm an Adhesion of the Tube to your Finger?

Or if you fill the same Tube almost full of Mercury, leaving a little Space of Air within, and then immerse it in the stagnant Mercury, you will find, that, notwithstanding its Surface is at some Distance from your Finger, there will be a considerable Suc-

tion

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Use of, is now the common Barometer, or Weather Glas \*.

The Mercury standing at a less Height, the nearer it is carried to the Top of the Atmosphere,

tion of it, as before. From hence he infers, that the Finger supports the Mercury, by Means of the abovementioned Film, and that the Pressure of the Atmosphere is not concerned.

But, when it was found, that the Mercury would not stand so high in the Tube, on the Top of a Mountain, as below; and would quite fall, when the circumambient Air was extracted from it by the Pump, all Objections vanished; and *Linus's* funicular *Hypothesis* (as it was called) though it seemed to solve all other *Phænomena* relating to the Suspension of the Mercury, was with Justice rejected.

*Kircher*, when this new Doctrine of a *Vacuum* was first advanced at *Rome*, contended, that the Authors of it were establishing Principles not only repugnant to those of Nature, but such as would be prejudicial to the Orthodox Faith; as endeavouring to evince by this subtle Experiment, that there might be in Nature *locatum sine loco, accidentia sine subiecto*, and therefore made the Experiment with Water, in the following Manner. He caused a small Bell to be fixed in the upper Part of the Tube, imagining, that, if there should be a *Vacuum*, the Bell would not be made to sound: But in making the Experiment, some Air got into the Tube (for he tells us, that but ten Feet of Water remained in the Tube, after it was inverted) the Bell therefore was heard to sound; and so the Notion of a *Vacuum*, till more accurate Experiments evinced the contrary, was exploded with Contempt.

\* *Huygens* observed, that, if a Tube seventy-five Inches long, was filled with Mercury well purged of its Air, the whole Quantity of Mercury would remain suspended; whereas, according to the *Torricellian* Experiment, the Mercury ought to have subsided to the Height of about twenty-nine Inches.

The Cause of this *Phænomena* seems to be, that, by the great Weight of so long a Column of Mercury, it was pressed into so close Contact with the Glas in pouring in, that by the mutual Attraction of Cohesion between the Mercury and the Glas, the whole Column was sustained, after the Tube was inverted.

(Chap. III. §. 7.) renders it useful in determining the Height of Mountains ; and finding out the different Elevation of one Place above another. Accordingly, Dr. *Halley* has given us a Table for that Purpose, in the Philosophical Transactions N°. 181, shewing how many Feet each Inch in the Descent of the Mercury answers to, as it is conveyed to the Top of a Mountain, or other elevated Place. And Dr. *Nettleton* has done the like in the Philosophical Transactions N°. 388, shewing what Number of Feet answers to each tenth Part of an Inch, from twenty-six to thirty-one Inches of Mercury.

But the principal Use of it is, to estimate the Gravity of the Air at different Times, in Order to foresee the Alterations of the Weather, which are consequent thereon. To this End, Dr. *Halley* in the same Transaction has also laid down the more remarkable *Phænomena*, relating to the different Heights of the Mercury at different Times, together with the Solution of each ; which are so just, and so agreeable to true Philosophy, that I doubt not but the Reader will excuse me for giving his Account in his own Words, rather than to render it imperfect, by endeavouring to vary from it, or abridge it.

“ 1. In calm Weather, when the Air is inclined to Rain, the Mercury is commonly low.

“ 2. In

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" 2. In serene, good, and settled Weather,  
" the Mercury is generally high.

" 3. Upon very great Winds, though they  
" be not accompanied with Rain, the Mercury  
" sinks lowest of all, with Relation to the  
" Point of the Compass the Wind blows upon.

" 4. *Cæteris paribus*, the greatest Heights  
" of the Mercury are found upon easterly and  
" north-easterly Winds.

" 5. In calm frosty Weather, the Mercury  
" generally stands high.

" 6. After very great Storms of Wind,  
" when the Mercury has been low, it gene-  
" rally rises again very fast.

" 7. The more northerly Places have greater  
" Alterations of the Barometer, than the  
" more southerly.

" 8. Within the Tropics, and near them,  
" those Accounts we have had from others,  
" and my own Observations at *St. Helena*,  
" make very little or no Variation of the  
" Height of the Mercury in all Weathers.

" Hence I conceive that the principal Cause  
" of the Rise and Fall of the Mercury, is from  
" the variable Winds, which are found in the  
" temperate Zone, and whose great Uncon-  
" stancy, here in *England*, is most notorious.

" A second Cause is the uncertain Exhalation  
" and Precipitation of the Vapours lodging in  
" the Air, whereby it comes to be at one Time  
" much more crouded than at another, and

“ consequently heavier, but this latter in a great  
 “ Measure depends upon the former. Now,  
 “ from these Principles, I shall endeavour to  
 “ explicate the several *Phænomena* of the Ba-  
 “ rometer, taking them in the same Order I  
 “ laid them down. Thus,

“ 1. The Mercury's being low inclines it  
 “ to rain, because, the Air being light, the  
 “ Vapours are no longer supported thereby,  
 “ being become specifically heavier than the  
 “ Medium wherein they floated, so that they  
 “ descend towards the Earth, and in their Fall,  
 “ meeting with other aqueous Particles, they  
 “ incorporate together, and form little Drops  
 “ of Rain ; but the Mercury's being at one  
 “ Time lower than at another, is the Effect of  
 “ two contrary Winds blowing from the Place  
 “ where the Barometer stands ; whereby the  
 “ Air of that Place is carried both Ways from  
 “ it, and, consequently, the incumbent Cylin-  
 “ der of Air is diminished, and accordingly the  
 “ Mercury sinks : As for Instance, if in the  
 “ *German* Ocean it should blow a Gale of  
 “ westerly Wind, and at the same Time an  
 “ easterly Wind in the *Irish* Sea ; or if in  
 “ *France* it should blow a northerly Wind,  
 “ and in *Scotland* a southerly ; it must be  
 “ granted, that that Part of the Atmosphere,  
 “ impendant over *England*, would thereby be  
 “ exhausted and attenuated, and the Mercury  
 “ would subside, and the Vapours, which be-

“ fore

“ fore floated in those Parts of the Air of  
 “ equal Gravity with themselves, would sink  
 “ to the Earth.

“ 2. The greater Height of the Barometer  
 “ is occasioned by two contrary Winds blow-  
 “ ing towards the Place of Observation, where-  
 “ by the Air of other Places is brought thither  
 “ and accumulated ; so that the incumbent  
 “ Cylinder of Air, being encreased both in  
 “ Height and Weight, the Mercury pressed  
 “ thereby must needs stand high, as long as  
 “ the Winds continue so to blow ; and then  
 “ the Air being specifically heavier, the Va-  
 “ pours are better kept suspended, so that they  
 “ have no Inclination to precipitate and fall  
 “ down in Drops, which is the Reason of the  
 “ serene good Weather which attends the  
 “ greater Heights of the Mercury.

“ 3. The Mercury sinks the lowest of all by  
 “ the very rapid Motion of the Air in Storms  
 “ of Wind. For the Tract or Region of the  
 “ Earth’s Surface, wherein the Winds rage,  
 “ not extending all round the Globe, that  
 “ stagnant Air which is left behind, as like-  
 “ wise that on the Sides, cannot come in so  
 “ fast as to supply the Evacuation made by so  
 “ swift a Current, so that the Air must neces-  
 “ sarily be attenuated, when and where the  
 “ said Winds continue to blow, and that more  
 “ or less, according to their Violence : add  
 “ to which, that the horizontal Motion of the  
 “ Air



“ Air being so quick as it is, may, in all Probability, take off some Part of the perpendicular Pressure thereof; and the great Agitation of its Particles is the Reason why the Vapours are dissipated, and do not condense into Drops, so as to form Rain, otherwise the natural Consequence of the Air's Rarefaction †.

“ 4. The Mercury stands the highest upon the easterly and north-easterly Wind, because in the great *Atlantic Ocean*, on this Side the thirty-fifth Degree of north Latitude, the Winds are almost always westerly or south-westerly; so that, whenever here the Wind comes up at east and north-east, 'tis sure to be checked by a contrary Gale as soon as it reaches the Ocean; wherefore, according to what is made out in our second Remark, the Air must needs be heap-

† The Reason the Doctor assigns for the sinking of the Mercury the lowest of all in violent Storms of Wind, seems not sufficient. Perhaps it may be better accounted for thus; the Cause why the Wind blows at all, is in order to restore the *Æquilibrium* of the Atmosphere, when lost (as may be inferred from what was said in the first Note of this, and will be more largely explained in the following Dissertation;) it therefore always blows towards that Point, where the Air is most rarefied and lightest. Now the Air, in its Progress to that Point, must certainly move faster and faster; for the Cause, which gave it Motion at first, continues to act upon it all the Way. Consequently, in whatever Place the Wind blows with great Rapidity, that Place is at, or near the Point, where the Air is most rarefied, and lightest; which is a sufficient Reason for the Mercury's standing low at that Place.

“ ed

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ed over this Island, and consequently the Mercury must stand high, as often as these Winds blow. This holds true in this Country, but is not a general Rule for others, where the Winds are under different Circumstances; and I have sometimes seen the Mercury here as low as twenty-nine Inches upon an easterly Wind, but then it blew exceeding hard, and so comes to be accounted for, by what was observed upon the third Remark.

“ 5. In calm frosty Weather the Mercury generally stands high, because (as I conceive) it seldom freezes, but when the Winds come out of the northern, and north-eastern Quarters; or, at least, unless those Winds blow at no great Distance off: For the north Parts of *Germany, Denmark, Sweden, Norway*, and all that Tract from whence north-eastern Winds come, are subject to almost continual Frost all the Winter; and thereby the lower Air is very much condensed, and in that State is brought hitherwards by those Winds, and being accumulated by the Opposition of the westerly Wind blowing in the *Ocean*, the Mercury must needs be pressed to a more than ordinary Height; and, as a concurring Cause, the shrinking of the lower Parts of the Air into lesser Room by Cold, must

“ needs

“ needs cause a Descent of the upper Parts of  
 “ the Atmosphere, to reduce the Cavity made  
 “ by this Contraction to an *Æquilibrium*.

“ 6. After great Storms, when the Mer-  
 “ cury has been very low, it generally rises  
 “ again very fast: I once observed it to rise  
 “ one Inch and a half in less than six Hours,  
 “ after a long continued Storm of south-west  
 “ Wind. The Reason is, because the Air be-  
 “ ing very much rarefied, by the great Eva-  
 “ cuations which such continued Storms make  
 “ thereof, the neighbouring Air runs in the  
 “ more swiftly, to bring it to an *Æquilibri-*  
 “ *um*; as we see Water runs the faster for  
 “ having a greater Declivity.

“ 7. The Variations are greater in the more  
 “ northerly Places, as at *Stockholm*, greater  
 “ than that at *Paris* (compar'd by M. *Pas-*  
 “ *chal*;) because the more northerly Parts  
 “ have usually greater Storms of Wind than  
 “ the more southerly, whereby the Mercury  
 “ should sink lower in that Extream; and  
 “ then the northerly Winds bringing the more  
 “ dense and ponderous Air from the Neigh-  
 “ bourhood of the Pole, and that again being  
 “ checked by a southerly Wind at no great  
 “ Distance, and so heaped, must of Necessity  
 “ make the Mercury in such Case stand higher  
 “ in the other Extream.

“ 8. Lastly

\* See

† M

“ 8. Lastly, this Remark, That there is little or no Variation near the *Equinoctial*, does above all others confirm the *Hypothesis* of the variable Winds being the Cause of these Variations of the Height of the Mercury; for, in the Places above-named, there is always an easy Gale of Wind blowing nearly upon the same Point, *viz.* east-north-east at *Barbadoes*, and east-south-east at *St. Helena* \*, so that, there being no contrary Currents of Air to exhaust or accumulate it, the Atmosphere continues much in the same State: However, upon Hurricanes (the most violent of Storms) the Mercury has been observed very low, but this is but once in two or three Years, and it soon recovers its settled State, about  $29 \frac{1}{2}$  Inches.”

Monfieur *Leibnitz* accounted for the Descent of the Mercury before Rain, upon another Principle †, *viz.* as a Body specifically lighter than a Fluid, while it is suspended by it, adds more Weight to that Fluid, than when, by being reduced in its Bulk, it becomes specifically heavier, and descends; so the Vapour, after it is reduced into the Form of Clouds, and descends, adds less Weight to the Air, than before; and therefore the Mer-

\* See the Cause of this assigned in the following Dissertation.

† *Memoir. de l'Acad.* 1711.

cury falls. To which it is answered, 1<sup>st</sup> That, when a Body descends in a Fluid, its Motion, in a very little Time, becomes uniform (or nearly so) a farther Acceleration of it being prevented by the Resistance of the Fluid; and then, by the third Law of Nature, it presses the Fluid downwards, with a Force equal to that whereby it tends to be farther accelerated, that is, with a Force equal to its whole Weight. 2<sup>dly</sup>. The Mercury, by its Descent foretells Rain a much longer Time before it comes, than the Vapour, after it is condensed into Clouds, can be supposed to take up in falling. 3<sup>dly</sup>. Supposing that as many Vapours as fall in Rain, during the Space of a whole Year, were at once to be condensed into Clouds, and even quite cease to gravitate upon the Air, its Gravity would scarce be diminished thereby, so much as is equivalent to the Descent of two Inches of Mercury in the Barometer. Farther, in many Places between the Tropics, the Rains fall at certain Seasons, in very great Quantities \*, and yet the Barometer shews there very little or no Alteration in the Weight of the Air.

The following are Mr. *Patrick's* Observations on the rising and falling of the Mercury. They are very just, and are to be accounted

\* See Dissertation the VIth.

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for on the same Principles with those of Dr. Halley.

" 1. The rising of the Mercury presages in general fair Weather ; and its falling, foul Weather ; as Rain, Snow, high Winds and Storms.

" 2. In very hot Weather, the falling of the Mercury foreshews Thunder.

" 3. In Winter the rising presages Frost ; and in frosty Weather, if the Mercury falls three or four Divisions, there will certainly follow a Thaw. But in a continued Frost, if the Mercury rises, it will certainly snow.

" 4. When foul Weather happens soon after the falling of the Mercury, expect but little of it. And, on the contrary, expect but little fair Weather, when it proves fair shortly after the Mercury has risen.

" 5. In foul Weather, when the Mercury rises much and high, and so continues for two or three Days before the foul Weather is quite over, then expect a Continuance of fair Weather to follow.

" 6. In fair Weather, when the Mercury falls much and low, and thus continues for two or three Days before the Rain comes ; then expect a great deal of wet, and probably high Winds.

" 7. The



“ 7. The unsettled Motion of the Mercury denotes uncertain and changeable Weather.

“ 8. You are not so strictly to observe the Words engraved on the Plates (though, for the most Part, it will agree with them) as the Mercury’s *Rising* and *Falling*: For if it stands at *Much Rain*, and then rises up to *Changeable*, it presages fair Weather, although not to continue so long, as it would have done, if the Mercury were higher: And so on the contrary, if the Mercury stood at *Fair*, and falls to *Changeable*, it presages foul Weather; though not so much of it, as if it had sunk down lower.”

From these Observations, it appears, That it is not so much the Height of the Mercury in the Tube, that indicates the Weather, as the Motion of it up and down; wherefore in Order to pass a right Judgment of what Weather is to be expected, we ought to know, whether the Mercury is actually *Rising* or *Falling*; to which End, the following Rules are of Use.

1. If the Surface of the Mercury is convex, standing higher in the Middle of the Tube than at the Sides, it is generally a Sign that the Mercury is then rising.

2. If the Surface is concave, or hollow in the Middle, it is sinking. And,

3. If

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3. If it is plain, the Mercury is stationary, or rather, if it is a little convex; for Mercury being put into a Glass Tube, especially a small one, will naturally have its Surface a little convex; because the Particles of Mercury attract each other more forcibly than they are attracted by Glass. Further,

4. If the Glass is small, shake the Tube; and if the Air is grown heavier, the Mercury will rise about half the tenth of an Inch higher, than it stood before; if it is growing lighter, it will sink as much. This proceeds from the Mercury's sticking to the Sides of the Tube, which prevents the free Motion of it, till it is disengaged by the Shock. And therefore, when an Observation is to be made with such a Tube, it ought always to be shaken first; for sometimes the Mercury will not vary of its own Accord, till the Weather, it ought to have indicated, is present.

The Usefulness of knowing whether the Mercury is actually rising or falling, and the Advantage that would arise from perceiving the most minute Variations in estimating the Heights of Places, have given Occasion to the Invention of several Kinds of Barometers different from the *Torricellian*, though founded on the same Principle; wherein the Scale of Variation, which in that is not above three Inches,

N                      should

should be considerably larger. Of which I am now to give some Account.

1. The first is that of *Des Cartes*, which was made in the Form expressed, *Fig. 28.* where *AB* is a Tube hermetically \* sealed at *A*, and having its lower Orifice immersed in stagnant Mercury *EF*, and filled with the same Fluid to *G*, from thence to *H* with Water, and empty from thence to the Top. Now, when the Mercury rises in this Tube, suppose from *G* to *L*, the Water will be raised in the small Tube, perhaps from *H* to *M*, viz. as many Times further, as the Tube *CA* is smaller than *CD*; by which Means the Variations become much more sensible, than they are in the common Barometer. The Inconvenience of this was, that the Air, included in the Water, getting loose by Degrees, filled the void Space at the Top, and so spoiled the Machine.

2. He then contrived it thus, *ABC (Fig. 29.)* is a bent Tube hermetically sealed at *A*, filled with Water from *F* to *D* (tinged with *Aqua Regia* to prevent its freezing) from *D* to *E* with Mercury, and empty from thence to the Top. Then, upon the Mercury's rising, suppose from

\* A Tube is said to be *hermetically* sealed, when the End is closed, that nothing can possibly evaporate through it. And, because this is best done, when it is closed up with its own Substance; or when its Bore does not reach quite to the End of it, it is then said to be *hermetically* sealed.

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E to M, and falling as much at D, the Surface of the Water at F would sink so many Times farther than the Surface of the Mercury at D, as the Tube CG was smaller than GH. The Water here is liable to evaporate, though that may, in some Measure, be prevented, by pouring a few Drops of Oil of sweet Almonds upon it. Others have contrived,

3. The Horizontal or Rectangular Barometer (Fig. 30.) hermetically sealed at A, and filled with Mercury from D to E; then, as the upper Surface of it rises in the Tube, suppose from E to F, the lower will be driven from D to G, as many Times farther, as this Part of the Tube is less than that at E. But it often happens, that some Parts of the Mercury break off from the rest in the Leg BC, and are left behind. This Inconvenience is remedied in,

4. The Diagonal Barometer ABC (Fig. 31.) wherein the Mercury, instead of rising from B to D (suppose that Space to correspond to the Scale of Variation in a strait Tube) will rise from B to A; for it will always stand at the same perpendicular Height, whatever be the Inclination of the Tube; because Fluids press only according to their perpendicular Altitude \*. But AB the upper Part of the Tube must not be too much inclined, lest the Mercury break in it, as in the former.

\* Chapter I. §. 9.

5. AB (*Fig. 32.*) is Dr. Rook's Wheel-Barometer, wherein ABD is a Tube filled with Mercury from *a* to E; *a* is an Iron Ball, swimming on the Surface of the Mercury; this, as it subsides with the Surface of the Mercury, draws the little Wheel *mn* round, to whose Circumference it is fixed by Means of the String *ac*\*: This Wheel carries the Index PQ, which points to the graduated Edge of the Circle KL, and by its Motion shews the most minute Variations of the Mercury. When the Ball *a* is raised by the Mercury on which it swims, the Index is drawn the contrary Way by a lesser Ball *b*, which hangs on the other Side the Wheel. The Friction in this Machine, unless it be made with great Accuracy indeed, renders it useless.

6. The pendant Barometer is another Contrivance to render the Variations more sensible. It consists of a small conical Tube, (represented *Fig. 33.*) hermetically sealed at A, and filled with Mercury from C to D, and empty from thence to A. Now, supposing the Gravity of the Air encreased, it will raise the Mercury higher in the Tube, and so force it into a narrower Part; by which Means the Column becoming longer, its perpendicular Pressure upon the Air below will be proportionably encreased.

\* The Tube is smaller at *a* than at E, that the greatest Variation may be at that Surface of the Mercury on which the Ball rests.

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On the contrary, when the Air becomes lighter, the Mercury descends into a larger Part of the Tube, and by that Means has the Length of its Column proportionably contracted. The Inconvenience that attends this Barometer, is that the Tube must be very small, otherwise the Mercury will fall out; or the Air will be apt to get into it, and divide the Column in several Places; and when the Tube is very small, the Friction of the Mercury against the Sides of it, will hinder it from rising and falling freely.

7. Dr. *Hook*, observing how unfit the common Barometer was to be used on Board of Ship, by Reason its Position ought to be steady, contrived the following one, called, from its Use, a *Marine* Barometer, consisting of two Parts, the one AB (*Fig. 34.*) the common Spirit Thermometer, the other CD, a Tube filled with Air from C to E, and from thence to the End D with tinged Water. This End is immersed in the same Fluid contained in the Vessel GF; and having its Surface exposed to the Pressure of the external Air. Now, the last of these Machines will be affected both by the Warmth of the external Air, and also by its Pressure: The former dilating the Air included in CE, and by that Means driving the Water downwards; the latter pressing it up higher in the Tube: Whereas the other, *viz.* AB, is affected by the Warmth of the Air alone. Consequently,



sequently, were these Instruments graduated in such a Manner, that, if the Gravity of the external Air should always remain the same it was when the Instruments were made, their Variations (then only depending on its Warmth) should exactly correspond with each other; that is, when the Spirit in the Tube A B should ascend to 1, the Water in CD should descend to 1, &c. Then, whenever their Variations should be observed to differ from each other, the Difference could only be ascribed to some Alteration in the Pressure of the Air upon the Surface of the Water in the Vessel G F. In Proportion therefore as the Difference is greater, or less, so is the Alteration in the Gravity of the Air, from what it was when the Instruments were adjusted. For Instance, when the Water stands above the Division, which corresponds to that, which the Spirit points to in the other Machine, it is an Indication, that the Pressure of the Air is greater at that Time, than when the Instruments were graduated, and *vice versa*.

This Machine is not only more useful at Sea, than the common one, as not requiring a steady Position; but may have its Scale of Variation considerably enlarged, by making the Bore of the Tube C D very small, in Proportion to the Capacity of its Head C.

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But it is observed, that, in long keeping the Instrument, the included Air loses somewhat of its Elasticity; whereby, in Process of Time, the Water stands higher than it ought, and therefore indicates the Gravity of the Air to be greater than what it is.

In the Philosophical Transactions, N<sup>o</sup>. 427. I have given an Account of a Barometer, wherein the Scale of Variation may be encreased *ad Infinitum*. The Description of it is as follows: ABCD (*Fig. 35.*) is a cylindrical Vessel, filled with a Fluid to the Height W, in which is immersed the Barometer SV, consisting of the following Parts: The principal of which is a Glass Tube TP, (represented separately at *tp*) whose upper End T is hermetically sealed: This End does not appear to the Eye, being received into the lower End of a Tin Pipe GH, which in its other End G receives a cylindrical Rod, or Tube ST, and thereby fixes it to the Tube TP. This Rod ST may be taken off, in order to put in its stead a larger, or lesser, as Occasion requires. S is a Star at the Top of the Rod ST, and serves as an Index, by pointing to the graduated Scale LA, which is fixed to the Cover of the Vessel ABCD. MN is a large cylindrical Tube made of Tin (represented separately at *mn*) which receives in its Cavity the smaller Part of the Tube TP, and is well cemented to it

at both Ends, that none of the Fluid may get in.

The Tube TP, with this Apparatus, being filled with Mercury, and plunged into the Bason V, which hangs by two, or more Wires, upon the lower End of the Tube MN, must be so poized, as to float in the Liquor contained in the Vessel ABCD, and then the whole Machine will rise, when the Atmosphere becomes lighter, and *vice versa*.

I shall here add a Computation, in order to shew the Possibility of the Variation being infinite, upon a given finite Variation of the Weight of the Atmosphere, and withal the Reason why it may be so. Those who would see a Mathematical Proof of it, may consult the Note below \*.

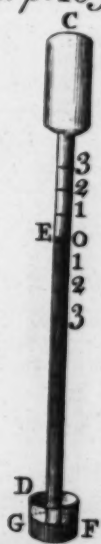
Let

\* Let the specific Gravity of Quicksilver be to that of Water, or to the Liquor the Barometer floats in, as  $s$  to  $1$ ; and if it be proposed, that the Variations in this compound Barometer shall be to the contemporary Variations of the common Barometer in the given Ratio of  $n$  to  $1$ , this Effect will be obtained, by making the Diameter of the Rod ST to the Diameter of the Cavity of the

Tube HI, as  $\sqrt{\frac{n+s}{ns}}$  to  $1$ , which may be thus demonstrated.

Let us suppose, that the Variation in the Height of the Quicksilver in the common Barometer, which we will call  $w$ , is such, that a cubic Inch of Quicksilver shall rise into the Vacuum XT; in order to which, a cubic Inch of Quicksilver must rise from the Vessel V; that is, the Surface P must subside so far, that a cubic Inch of Water (if that be the Fluid made Use of) shall enter the Vessel V, by which Means the Barometer with the Parts annexed will be heavier by a cubic Inch of the Fluid.

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Let it be supposed, that the Fluid made use of is Water, and that the given Variation in the Weight of the Atmosphere is such, that,  
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Now this additional Weight of a cubic Inch of Fluid, will make the whole Barometer subside (according to the Law of Hydrostatics) till a cubic Inch of the Rod HS, immediately extant above the Surface at W, shall come under it; but the Length of such a Magnitude of HS will exceed the Length of an equal Magnitude of Quicksilver in the larger Tube X, as many Times as the Square of the Diameter at X exceeds the Square of the Diameter at H (the Lengths of equal Cylinders being reciprocal to their Bases.) That is, the perpendicular Descent of the compound Barometer will be to  $v$ , the perpendicular Ascent of the Mercury in the common Barometer, as  $d$  to 1 (supposing this the Ratio of their Bases) and consequently will be equal to  $dv$ .

But, by this Descent, the Distance PW, between the Surface of the stagnant Quicksilver and the Top of the Fluid, will be augmented by a Column, whose Height is  $dv$ , the Descent of the compound Barometer; and consequently the Weight of the whole Column of the Fluid pressing on the lower Surface of the Quicksilver (to which the Height X is partly owing) will be encreased by a Column of that Length; and this Increase would produce a second Ascent of the Mercury at X equal to itself, namely,  $dv$ , were the Fluid as heavy as Quicksilver; but since it is supposed to be lighter in the Ratio of  $s$  to 1, the Ascent of the Quicksilver,

on this Account, will only be  $\frac{dv}{s}$ .

But now, as in the former Case, when the Ascent of the Mercury was  $v$ , the Descent of the compound Barometer was shewn

to be  $dv$ ; so here, the Ascent of the Mercury being  $\frac{dv}{s}$ , the Descent of the compound Barometer will be  $\frac{d^2v}{s}$ , and the next Descent  $\frac{d^3v}{s^2}$ , and the next  $\frac{d^4v}{s^3}$ , and so on to Infinity. There-

fore the whole Descent of the compound Barometer, is to the Ascent of the Mercury in the common Barometer, that is,  $v$  is to 1

as  $d + \frac{d^2}{s} + \frac{d^3}{s^2} + \frac{d^4}{s^3} + \dots$  to 1, or as  $\frac{d}{s-d}$  to 1; be-

cause, the Terms of the Series being in geometrical Proportion, the



by pressing upon the Surface of it at W, the Surface of the Mercury at X may be raised an Inch higher (measuring from its Surface at P) than before; and that the Breadth of the Cavity of the Tube at X, and of the Bason at P are such, that, by this Ascent of the Mercury, there may be a cubic Inch of it in the Cavity X more than before, and consequently in the Bason a cubic Inch less. Now, upon this Supposition, there will be a cubic Inch of Water in the Bason more than there was before; because the Water will succeed the Mercury to fill up its Place. Upon this Account the whole Machine will be render'd heavier than it was before, by the Weight of a cubic Inch of Water, and therefore will sink, according to the Laws of Hydrostatics (Chap. II. §. 5.) till a cubic Inch of that Part of the Rod WS, which

the Sum of them all is  $\frac{ds}{s-d}$ . Hence we have  $n = \frac{ds}{s-d}$ , and therefore  $ns = ds + dn$ ; that is,  $1 : d :: n + s : ns :: \frac{ns}{n+s} : 1$

and therefore, by extracting the square Roots of each Term in the Proportion,  $1 : \sqrt{d}$  (that is, the Diameter of ST to the Diameter of HI) as  $\sqrt{\frac{n+s}{ns}}$  to 1. Q. E. D.

*Example 1.* Putting  $s = 14$  and  $n = 1$ , the Variation in each Barometer will be equal, by taking the Diameter of ST to the Diameter of HI, as  $\sqrt{\frac{15}{14}}$  to 1, that is, as 30 to 29 nearly.

*Example 2.* If  $n$  be put infinite, the Diameter of ST will be to the Diameter of HI, as  $\sqrt{\frac{1}{s}}$  to 1, or 1 to  $\sqrt{14}$ ; that is, as 1 to  $3\frac{1}{4}$  nearly.

was

II. Differt. IV. *Of the Barometer.* 109

was above the Surface of the Water at W, comes under it. Then, if we suppose this Rod so small, that a cubic Inch of it shall be fourteen Inches in Length, the whole Machine will sink fourteen Inches lower into the Fluid than before, and consequently the Surface of the Mercury in the Bason will be pressed more than it was before, by a Column of Water fourteen Inches high. But the Pressure of fourteen Inches of Water is equivalent to one of Mercury (because Water is about fourteen Times lighter than Mercury); this additional Pressure therefore will make the Mercury ascend at X, as much as the supposed Variation in the Weight of the Air did at first. This Ascend will give Room for a second cubic Inch of Water to enter the Bason; the Machine will therefore be again rendered heavier by the Weight of so much Water, and accordingly will subside fourteen Inches farther. This will occasion another additional Pressure of Water, which will raise another Inch of Mercury, and make the Machine sink fourteen Inches more, and so on, without ever approaching nearer to an *Equilibrium* with the external Air: And therefore a Scale, answering to the Variation of this Barometer, ought strictly and properly to be of an infinite Length; because, after this Barometer has sunk or risen thousands of Miles (if that were possible) it would still have the same Tendency to sink or rise on, as when it first set out.

Now

Now, was the Rod WS so small, that a cubic Inch of it should be more than fourteen Inches long (the other Parts remaining as was supposed above) the Variation in this Barometer would be more than infinite, or negative with Respect to those of the common Barometer. The Meaning of which is, that whereas, in the common Barometer, the suspended Column of Mercury, by its rising or falling, approaches nearer to an *Æquilibrium* with the external Air, this Barometer would continually recede from an *Æquilibrium* with it; so that the farther it should move up or down, instead of acquiring by that Means a less Tendency to move on, as the Mercury in the common Barometer does, it would acquire a greater.

On the contrary, when a cubic Inch of the Rod is less than fourteen Inches in Length, the Variation will be finite; and may be made to bear any Proportion to those of the common Barometer whatever, as demonstrated in the foregoing Note.

While I am writing this, another Method occurs to me of making a Barometer, where the Scale of Variation shall bear any Proportion to that of the common one. It is this. Let there be a compound Tube, as ABC (Fig. 36.) hermetically sealed at A, and open at C empty from A to D, filled with Mercury from thence to B, and from thence to E with Water.

# Differt. IV. *Of the Barometer.* - III

ter: Then, if the Tube FC be a little more than five Times less in Diameter than the Tube FA, the Variation in the lower Surface of the Water at E will be infinite; if it be above so many Times less, it will be more than infinite, otherwise it will be finite. See the Demonstration in the Note \*.

That

\* Let  $v$  denote a given Variation in the common Barometer,  $x$  the correspondent Variation at E sought. Let the Ratio of  $m$  to 1, express that of the specific Gravity of Mercury, to that of Water; and  $d$  to 1, that of the Diameter of the Tube FA to FC. Then the Variation at E, the lower Surface of the Water, being supposed  $x$ , the Variation of it at B, the upper Surface of it, will

be  $x + \frac{x}{dd}$ , and consequently GE, the Difference of the Legs EK and

EB, will vary  $x + \frac{x}{dd}$ . Again, the Variation of the Surface of

the Mercury at B will be the same with that of the Water in the

same Place, *viz.*  $\frac{x}{dd}$ ; and, if the Tube is of the same Diameter

at D, as at B, the Variation of the Surface at D will also be the

same, that is,  $\frac{x}{dd}$ : The Sum of both Variations, or the Variation

of HD the Difference of the Legs, will therefore be  $\frac{2x}{dd}$ . Now the

Pressure of the Mercury and Water together upon the Air at E, is owing to the Lengths of HD and GE; and since one of these will always shorten, when the other lengthens, the Variation in their Pressure will depend on the Variation of the Difference of their Weights, that is, of the Difference between the Weight of  $x + \frac{x}{dd}$

and of  $\frac{x}{dd}$ . But the Weight of  $x + \frac{x}{dd}$  (being the Weight of a Column of Water) compared to that of a Column of Mercury of

$$x + \frac{x}{dd}$$

the same Length, is only  $\frac{1}{m}$ . The Difference therefore between

That the Variation in this Barometer may be infinite, will appear from the following Computation.

Let the Proportion between the Bores of the Tube A F and F C be such, that when H D the Difference of the Legs wherein the Mercury is contained, is augmented one Inch, G E the Difference of the Legs wherein the Water is contained, shall be diminished fourteen then, as much as the Pressure of the Mercury is augmented, that of the Water will be diminished, and so the Pressure of both taken together will remain as it was. And consequently after it has began to rise, it will always have the same Tendency to rise on, without ever coming to an *Æquilibrium* with the Air.

How far this Barometer will succeed in Practice, must be left to Experience to determine

between  $x + \frac{x}{dd}$  and  $\frac{2x}{m}$  will be always equal to the Variation in

the common Barometer, and therefore  $\frac{x + \frac{x}{dd}}{2x} = \frac{m}{dd}$  and  $\frac{m}{dd} = v$ , and by the common Method of Reduction,  $x = \frac{v m d d}{2m - dd - 1}$ : That is,  $x$

$v :: m d d : 2m - dd - 1$ . Now, if we put  $m = 14$ , and  $d = 5$ ,  $2m - dd - 1$  will be as much as  $2m$ , and therefore  $d = \sqrt{-27}$  will be equal to nothing; and so  $x$  being by the Proportion as many Times more than  $v$ , as  $m d d$  is than nothing, 'tis infinite. And if  $m$  be put  $= 14$ , and  $d = 5$ ,  $m d d$  will be equal to 350, and  $2m - dd - 1 = 2$ ; and therefore the Variations in this Case will be to those in the common Barometer, as 175 to one.

Pro-

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Probably, if the Bore of the Tube FC be made very small, viz. about the twentieth Part of an Inch Diameter, the Air will not ascend through the Water, as it is apt to do through the Mercury in the pendant Barometer; and the Smallness of the Bore will not prevent the Water from moving, near so much as it does the Mercury in that Barometer.

There is an Improvement of another Kind in the common Barometer, whereby it is rendered *portable*. The Tube containing the Mercury, instead of having its lower End immersed in a Vessel of that Fluid, has it tied up in a leathern Bag, not quite full of Mercury. And though the external Air cannot get into the Bag to suspend the Mercury in the Tube, by pressing on its Surface, as in the common one; yet it has the same Effect by pressing on the Outside of the Bag, which, being pliant, yields to the Pressure, and keeps the Mercury suspended in the Tube at its proper Height. This Bag is generally inclosed in a little Box, through the Bottom of which passes a Screw; with this Screw the Bag may be compressed, so as to force the Mercury up to the Top of the Tube; which keeps it steady, and hinders it from breaking the Tube by dashing against the Top when it is carried about, as it otherwise would be apt to do.

See



See more on the Subject of this Dissertation,  
*Weidleri Institutiones Mathematicæ*. p. 608. *Melchior Verdries Phys. Pars specialis*, Cap. IV.  
 §. 15. *Mr. Paschal's Traité de l'Equilibre des Liqueurs*. *Sinclair's Ars magna Gravitatis & Levitatis*. *Mariotte's Second Essay de la Nature de l'Air*. *Philosoph. Burgund. Tom. II*.  
 p. 850. *Saul's Treatise on the Barometer*. *Regnault's Philosoph. Conversat.* 22. *Clare's Motion of Fluids*, p. 141. *Mem. de l'Acad.*  
 1705, 1711. *Philosophical Transactions* N<sup>o</sup>.  
 9, 10, 11, 55, 86, 91, 165, 181, 185, 208,  
 229, 236, 237, 240, 243, 269, 351, 366,  
 385, 388, 405, 406, 427. *Cotes's Hydrostatical and Pneumatical Lecture*. *Lect.* 7.  
 With several other Authors referred to in *Mr. Johnson's Quæstiones Philosophicæ*, Cap. VI.  
*Quæst.* 36, 37.



DISSERTATION V.

*Of the Origin of the Winds.*

THE Wind is no other than the Motion of the Air upon the Surface of the Globe. Some of the Ancients took it to be Air, rushing out of the Bowels and Cavities of the Earth: And others thought it an Exhalation from its Surface. But these are *Hypotheses* too chimerical to stand in Need of a particular Confutation. Some of the Moderns, who held a *Plenum*, have accounted for it thus: They imagined, that the Air being confined above, as it must be, if we suppose a *Plenum*, would, when more than ordinarily rarefied, or stocked with Vapours, drive away the neighbouring Air, in order to make Room for itself; and by this Means occasion a Wind. Others, observing a constant and perpetual easterly Wind to blow at the *Equator*, ascribed its Origin to the diurnal Rotation of the Earth about its Axis from West to East; which they thought would occasion the Air upon its Surface to seem to move the contrary Way, being in some Measure left behind. But, whereas there are Winds, in some Places near the *Equator*, that blow on other Points of the Compass

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(as we shall see hereafter) this *Hypothesis* is insufficient. Besides, the Air pressing upon the Surface of the Earth by its Gravity, like other Bodies, and having nothing to hinder it from moving freely along with it, must necessarily, in Time, acquire an equal Degree of Velocity, and so keep Pace with it, all the Way round.

The principal Cause of the Wind, or, in other Words, of the Air's moving from Place to Place, upon the Surface of the Earth, is the Atmosphere's being heated over one Part more than over another. For, in this Case, the warmer Air being rarefied, becomes specifically lighter than the rest; it is therefore over-poised by it, and raised upwards, the upper Parts of it diffusing themselves every Way over the Top of the Atmosphere; while the neighbouring inferior Air rushes in from all Parts at the Bottom; which it continues to do, till the *Æquilibrium* is restored.

Upon this Principle it is, that most of the Winds may be accounted for.

To begin with those which blow under the *Equator*.

1. Under the *Equator*, the Wind is always observed to blow from the East Point \*.

For

\* For the Reader's Ease (who perhaps is not furnished with the *Philosophical Transactions*) I shall here insert by Way of Note, from Dr. *Halley's* Account, so much of the History of the Winds, as may be necessary for the understanding this Theory.

“ The

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For, supposing the Sun to continue vertical over some one Place, the Air will be most rarefied there ; and consequently, the neighbouring

“ The universal Ocean, says he, may most properly be divided into three Parts, viz. 1. The *Atlantic* and *Æthiopic* Seas. 2. The *Indian* Ocean. 3. The great *South* Sea, or the *Pacific* Ocean.

“ I. In the *Atlantic* and *Æthiopic* Seas, between the *Tropics*, there is a general easterly Wind all the Year long, without any considerable Variation ; excepting, that it is subject to be deflected therefrom, some few Points of the Compass, towards the North, or South, according to the Position of the Place.

“ 1. Near the Coast of *Africa*, as soon as you have passed the *Canary* Isles, you are sure to meet a fresh Gale of North-east Wind, about the Latitude of twenty-eight Degrees North ; which seldom comes to the Eastwards of the East-north-east, or passes the North-north-east. This Wind accompanies those bound to the Southward, to the Latitude of ten Degrees North, and about an hundred Leagues from the *Guinea* Coast ; where, till the fourth Degree of North Latitude, they fall into *Calm*s and *Tornadoes*, or sudden Storms.

“ 2. Those bound to the *Caribbee* Isles, find, as they approach the *American* Side, that the aforesaid North-east Wind becomes still more and more easterly, so as sometimes to be East, sometimes East by South, but yet most commonly to the Northward of the East, a Point or two, seldom more. 'Tis likewise observed, that the Strength of these does gradually decrease, as you sail to the Westward.

“ 3. That the Limits of the *Trade* and *variable* Winds in this Ocean, are farther extended on the *American* Side, than the *African* ; for, whereas you meet not with this certain Wind, till after you have pass'd the Latitude of twenty-eight Degrees on this Side ; on the contrary Side it commonly holds to thirty, thirty-one, or thirty-two Degrees of Latitude ; and this is verified likewise to the Southward of the *Equinoctial* ; for near the *Cape of Good Hope*, the Limits of the *Trade* Wind are three or four Degrees nearer the *Line*, than on the Coast of *Brazil*.

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ing Air will rush in from every Quarter with equal Force. But, as the Sun is continually shifting to the Westward, the Part where the Air

“ 4. That from the Latitude of four Degrees North, to the  
“ aforesaid Limits on the South Side of the *Equator*, the Winds  
“ are generally and perpetually between the South and East,  
“ and most commonly between the South-east and East; ob-  
“ serving always this Rule, that on the *African* Side they are  
“ more southerly, on the *Brasilian* more easterly, so as to be-  
“ come almost due East, the little Deflection they have being  
“ still to the Southward. In this Part of the Ocean, it has  
“ been my Fortune to pass a full Year, in an Employment that  
“ obliged me to regard more than ordinarily the Weather; and  
“ I found the Winds constantly about the South-east, the most  
“ usual Point South-east by East: When it was easterly, it ge-  
“ nerally blew hard, and was gloomy, dark, and sometimes  
“ rainy Weather: If it came to the Southwards, it was gene-  
“ rally serene, and a small Gale next to a Calm; but this not  
“ very common. But I never saw it to the Westwards of the  
“ South, or Northwards of the East.

“ 5. That the Season of the Year has some small Effect on  
“ these *Trade Winds*; for that, when the Sun is considerably to  
“ the Northward of the *Equator*, the South-east Winds, espe-  
“ cially in the Streight of this Ocean (if I may so call it) be-  
“ tween *Brasil*, and the Coast of *Guinea*, do vary a Point or  
“ two to the Southward, and the North-east become more  
“ easterly; and, on the contrary, when the Sun is towards the  
“ *Tropic of Capricorn*, the South-easterly Winds become more  
“ easterly; and the North-easterly Winds, on this Side the  
“ *Line*, veer more to the Northward.

“ 6. That as there is no general Rule, that admits not of some  
“ Exception, so there is in this Ocean a Tract of Sea, wherein  
“ the southerly and south-west Winds are perpetual, viz. all  
“ along the Coast of *Guinea*, for above five hundred Leagues  
“ together, from *Sierra Leona*, to the Isle of *St. Thomas*: For  
“ the South-east *Trade Wind* having pass'd the *Line*, and ap-  
“ proaching the Coast of *Guinea* within eighty or an hundred  
“ Leagues, inclines towards the Shore, and becomes South-south-  
“ east; and by Degrees, as you come nearer, it veers about to  
“ South, South-south-west, and in with the Land South-west,

“ and

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Air is most rarefied, is carried the same Way; and therefore the Tendency of all the lower Air, taken together, is greater that Way, than any other.

" and sometimes West-south-west. These are the Winds, which  
" are observed on this Coast when it blows true; but there are  
" frequent Calms, violent sudden Gusts, called *Tornados*, from  
" all Points of the Compass, and sometimes unwholesome foggy  
" easterly Winds, called *Hermitæ*, by the Natives, which too  
" often infest the Navigation of these Parts.

" 7. That to the Northwards of the *Line*, between four and  
" ten Degrees of Latitude, and between the Meridians of *Cape*  
" *Verde*, and of the eastermost Islands that bear that Name, there  
" is a Tract of Sea, wherein it were improper to say, there is  
" any *Trade Wind*, or yet a *variable*; for it seems condemned to  
" perpetual Calms, attended with terrible Thunder and Light-  
" ning, and Rains so frequent, that our Navigators from thence  
" call this Part of the Sea, the *Rains*: The little Winds that  
" are, being only some certain Gusts, of very little Continu-  
" ance, and less Extent; so that sometimes each Hour you  
" shall have a different Gale, which dies away into a Calm be-  
" fore another succeeds: And in a Fleet of Ships in Sight of  
" one another, each shall have the Wind from a several Point of  
" the Compass: With these weak Breezes, Ships are obliged to  
" make the best of their Way to the Southward, through the  
" aforesaid six Degrees; wherein it is reported some have been  
" detain'd whole Months for want of Wind. —

" II. In the *Indian Ocean*, the Winds are partly general,  
" as in the *Æthiopic Ocean*; partly periodical, that is, half  
" the Year they blow one Way, and the other half near upon  
" the opposite Points; and these Points, and Times of shifting,  
" are different in different Parts of this Ocean.

" 1. Between the Latitudes of ten Degrees and thirty De-  
" grees South, between *Madagascar* and *Hollandia nova*, the  
" General Trade-Winds about the South-east and by East, are  
" found to blow all the Year long, to all Intents and Purposes,  
" after the same Manner, as in the same Latitudes in the  
" *Æthiopic Ocean*, as it is described in the fourth Remark  
" foregoing.



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other, Thus the Tendency of the Air towards the West, becomes general, and its Parts impelling one another, and continuing to move

“ 2. That the aforesaid South-east Winds extend to within two Degrees of the *Equator*, during the Months of *June, July, and August*, &c. to *November*; at which Time, between the South Latitude of three and ten Degrees, being near the *Midian* of the North End of *Madagascar*, and between two and twelve South Latitude, being near *Sumatra* and *Jawa*; the contrary Winds from the North-west, or between the North and West, set in, and blow for half a Year, viz. from the Beginning of *December* till *May*: And this *Monsoon* is observed as far as the *Molucca* Isles.

“ 3. That to the Northward of three Degrees South Latitude, over the whole *Arabian* and *Indian* Sea, and Gulf of *Bengal*, from *Sumatra* to the Coast of *Africa*, there is another *Monsoon*, blowing from *October* to *April*, upon the North-east Points: But in the other half Year, from *April* to *October*, upon the opposite Points of South-west and West-south-west, and that with rather more Force than the other, accompanied with dark, rainy Weather; whereas the North-east blows clear. 'Tis likewise to be noted, that the Winds are not so constant, either in Strength or Point, in the Gulf of *Bengal*, as they are in the *Indian* Sea, where a certain steady Gale scarce ever fails. 'Tis also remarkable, that the South-west Winds, in these Seas, are generally more southerly on the *African* Side, and more westerly on the *Indian*.

“ 4. There is a Tract of Sea to the Southward of the *Equator*, subject to the same Changes of the Winds, viz. near the *African* Coast, between it and the Island *Madagascar* or *St. Laurence*, and from thence Northwards, as far as the *Line*; wherein, from *April* to *October*, there is found a constant fresh South-south-west Wind, which, as you go more northerly, becomes still more and more westerly, so as to fall in with the West-south-west Winds, mentioned before in those Months of the Year to be certain to the Northward of the *Equator*. What Winds blow in those Seas, for the other half Year, I have not yet been able to obtain to my full Satisfaction: The Account which has been given me, is only this

“ That

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move till the next Return of the Sun, so much of its Motion, as was lost by his Absence, is again restored, and therefore the easterly Wind becomes *Perpetual*.

Some

" That the Winds are much easterly hereabouts, and as often to the North of the true East, as to the Southward thereof.

" 5. That to the Eastward of *Sumatra* and *Malacca*, to the Northwards of the *Line*, and along the Coast of *Camboja* and *China*, the *Monsoons* blow North and South; that is to say, the North-east Winds are much northerly, and the South-west much southerly. This Constitution reaches to the Eastward of the *Philippine* Isles, and as far northerly as *Japan*; the northern *Monsoon* setting in, in these Seas, in *October* or *November*; and the southern in *May*, blowing all the Summer Months. Here it is to be noted, that the Points of the Compass from whence the Winds come, in these Parts of the World, are not so fixed, as those lately described; for the southerly will frequently pass a Point or two to the Eastwards of the South, and the northerly as much to the Westwards of the North, which seems occasioned by the great Quantity of Land which is interspersed in these Seas.

" 6. That in the same Meridians, but to the Southwards of the *Equator*, being that Tract lying between *Sumatra* and *Java* to the West, and *New Guinea* to the East, the same northerly and southerly *Monsoons* are observed; but with this Difference, that the Inclination of the northerly is towards the North-west, and of the southerly towards the South-east: But the *Plagæ Venti* are not more constant here than in the former, viz. variable five or six Points. Besides, the Times of the Change of these Winds are not the same, as in the *Chinese* Seas, but about a Month, or six Weeks later.

" 7. That the contrary Winds do not shift all at once, but in some Places the Time of the Change is attended with *Calms*, in others with *variable* Winds; and it is particularly remarkable, that the End of the westerly *Monsoon*, in the Seas of *China*, are very subject to be tempestuous. The Violence of these Storms is such, that they seem to be of the Nature of the *West-India* Hurricanes, and render the Navigation of these Parts very unsafe about that Time of Year. These

" Tempest

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Some are inclined to think, that the continual shifting of the Sun to the Westward, should produce a westerly Wind under the *Equator*, by causing the Current of the Air from the West to exceed and over-balance that,

“ Tempests are by our Seamen usually term’d the *Breaking up*  
“ of the *Monsoons*.

“ III. The Third Ocean, called *Mare Pacificum*, whose Extent is equal to that of the other two (it being from the West Coast of *America* to the *Philippine* Islands, not less than an hundred and fifty Degrees of Longitude) is that which is least known to our own, or the neighbouring Nations : That Navigation that there is on it, is by the *Spaniards*, who go yearly from the Coast of *New Spain* to the *Manilla’s* : But that but by one beaten Tract ; so that I cannot be so particular here, as in the other Two. What the *Spanish* Authors say of the Winds, they find in their Courses ; and what is confirmed by the old Accounts of *Drake* and *Candish*, and since by *Schooten*, who sailed the whole Breadth of this Sea, in the southern Latitude of fifteen or sixteen Degrees, is, that there is a great Conformity between the Winds of this Sea, and those of the *Atlantic* and *Æthiopic* ; that is to say, that to the Northward of the *Equator*, the predominant Wind is between the East and North-east ; and to the Southwards thereof, there is a constant, steady Gale, between the East and South-east, and that on both Sides the *Line*, with so much Constancy, that they scarce ever need to attend the Sails ; and Strength, that it is rare to fail of crossing this vast Ocean in ten Weeks Time ; which is about an hundred and thirty Miles a Day : Besides, ’tis said, that Storms and Tempests are never known in these Parts ; wherefore some have thought it might be as short a Voyage to *Japan* and *China*, to go by the Streights of *Magellan*, as by the Cape of *Good Hope*.

“ The Limits of these *General* Winds are much the same as in the *Atlantic* Sea, viz. about the thirtieth Degree of Latitude on both Sides. Besides, a farther Analogy between the Winds of this Ocean, and the *Æthiopic*, appears in that, that upon the Coasts of *Peru* they are always much southerly, like as they are found near the Shores of *Angola*.”

which

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which opposes it from the East. For, whereas the eastern Air retains its Heat some time after the Sun is removed from it, it must always be rarefied to a greater Degree, and also to a greater Distance from the Place to which the Sun is vertical, than the western Air is; and therefore the western Air, being more ponderous, should be an Over-balance for the eastern, and drive the Current before it.

But it is to be observed, that we are not to consider the Point to which the Sun is vertical, but the Point of greatest Rarefaction (which, upon Account of the Sun's Motion to the Westward, lies always to the Eastward); and then see, which Side of the Column of Air, incumbent over that Point, sustains the greater Pressure from the neighbouring Columns. Now, although the Air is rarefied even farther to the East of this Point, than to the West, yet still, if we suppose the Point to keep its Place, the Air over it will sustain an equal Degree of Pressure on each Side. For, since no Column can be assigned on the western Side, but one also on the eastern may be found under an equal Degree of Rarefaction, and therefore of the same specific Gravity; and since Fluids of equal Heights, and the same specific Gravities (whatever be the Breadth of their Columns) press equally against equal Objects, (Chap. I. §. 9.) 'tis very evident, that the Column of Air, over the  
Point

Point of greatest Rarefaction, is pressed equally both Ways; and so, upon this Supposition, each Wind will blow towards that Point with equal Force. But, if we suppose the Point of greatest Rarefaction to shift towards the West, we shall find, that this *Æquilibrium* will by that Means be destroyed, and the Motion of the Air (upon the whole) determined that Way. For let us consider any Portion of the western Air approaching towards the Point of greatest Rarefaction, if that Point shifts, in the mean Time, towards the West, then will that Portion reach the said Point sooner than it otherwise would have done; thereby losing a Part of its Motion, by which Means the westerly Current will be diminished. Again, if, while the East Wind blows towards the Point of greatest Rarefaction, that Point moves on before it, then will the eastern Air have a greater Quantity of Motion, than it otherwise would have had; that, which should have been an Impediment to it, being, upon this Supposition, in some Measure withdrawn; and so the East Wind will be augmented. Thus, the West Wind having its Force lessen'd by the Motion of the Sun, and the East one being increased, the latter at length as it were absorbs the former, and carries it away in its own Direction.

2. On each Side of the *Equator*, to about the thirtieth Degree of Latitude, the Wind is found

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found to vary from the East Point, so as to become North-east on the northern Side, and South-east on the southern.

The Reason of which is, that, as the *equatorial* Parts are hotter than any other, both the northern and southern Air ought to have a Tendency that Way; the northern Current therefore, meeting in this Passage with the eastern, produces a North-east Wind on that Side; as the southern Current, joining with the same, on the other Side the *Equator*, forms a South-east Wind there.

These two Propositions are to be understood of open Seas, and of such Parts of them as are distant from the Land; for near the Shores, where the neighbouring Air is much rarefied, by the Reflection of the Sun's Heat from the Land, it frequently happens otherwise; particularly,

3. On the *Guinea Coast*, the Wind always sets in upon the Land, blowing westerly instead of easterly. This is because the Deserts of *Africa* lying near the *Equator*, and being a very sandy Soil, reflect a greater Degree of Heat into the Air above them. This therefore being rendered lighter, than that which is over the Sea, the Wind continually rushes in upon the Land to restore the *Æquilibrium*.

4. That Part of the Ocean, which is called the *Rains*, is attended with perpetual Calms, the Wind scarce blowing sensibly either one Way



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Way or other. (See its Situation described in Note, Page 119, Remark 7th). For this Tract being placed between the westerly Wind blowing from thence towards the Coast of *Guinea*, and the easterly Wind blowing from the same Place to the Westward thereof, the Air stands in *Æquilibrio* between both, and its Gravity is so much diminished thereby, that it is not able to support the Vapour it contains, but lets it fall in continual Rain, from whence this Part of the Ocean has its Name.

5. There is a Species of Winds, observable in some Places within the *Tropics*, called by the Sailors *Monsoons*, which, during six Months of the Year, blow one Way; and the remaining six the contrary.

The Occasion of them in general is this: When the Sun approaches the northern *Tropic*, there are several Countries, as *Arabia*, *Persia*, *India*, &c. which become hotter, and reflect more Heat than the Seas beyond the *Equator*, which the Sun has left; the Winds therefore, instead of blowing from thence to the Parts under the *Equator*, blow the contrary Way; and when the Sun leaves those Countries, and draws near the other *Tropic*, the Winds turn about, and blow on the opposite Point of the Compass.

The Regularity of these Winds making them more than ordinarily useful in Navigation, they are from thence called *Trade Winds*.

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Winds. As to other Circumstances relating to them, and the particular Times and Places of the *Monsoons*, &c. See the Historical Account in the foregoing Note; all which might easily be solved upon the same Principle, had we *Data* to go upon, and were all the several Circumstances of Situation, Heat, Cold, &c. sufficiently known \*.

From the Solution of the general *Trade Winds*, we may see the Reason, why in the *Atlantic Ocean*, a little on this Side the thirtieth Degree of North Latitude, or thereabouts, as was observed in the foregoing Dissertation, there is generally a West, or South-west Wind. For, as the inferior Air, within the Limits of those Winds, is constantly rushing towards the *Equator*, from the North-east Point, or thereabouts, the superior Air moves the contrary Way; and therefore, after it has reached these Limits, and meets with Air, that has little or no Tendency to

\* Some have thought, that the Regularity of the general *Trade Winds* is partly owing to the diurnal Motion of the *Moon* from East to West. For, as the *Sun* renders the Air specifically lighter by its Heat, so does the *Moon* by attracting it, in the same Manner, as it does the *Sea*, in raising the *Tides*: But it is to be observed, that as the *Moon* acts with the greatest Force upon the superior Parts of the Air, and as those superior Parts are unconfined, and therefore more at Liberty to rush in, in order to restore the *Equilibrium*; it will from hence follow, that the rushing in of the superior Parts of the Atmosphere will principally contribute towards restoring the *Equilibrium*; and so the Motion, produced below, will be very inconsiderable.

any

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any one Point more than to another, it will determine it to move in the same Direction with itself.

In our own Climate we frequently experience, in calm Weather, gentle Breezes blowing from the Sea to the Land, in the Heat of the Day; which *Phænomenon* is very agreeable to the Principle laid down above: For the inferior Air over the Land being rarefied by the Beams of the Sun, reflected from its Surface, more than that which impends over the Water, which reflects fewer, the latter is constantly moving on to the Shore, in order to restore the *Æquilibrium*, when not disturbed by stronger Winds from another Quarter †.

From what has been observed, nothing is more easy than to see, why the northern and southern Parts of the World, beyond the Limits of the *Trade Winds*, are subject to such

† In Confirmation of this, we have an easy, and very pertinent Experiment, related by Mr. *Clare*, in his *Motion of Fluids*. “Take, says he, a large Dish, fill it with cold Water  
“into the Middle of this put a Water-Plate, filled with warm  
“Water. The first will represent the Ocean; and the other  
“an Island, rarefying the Air above it. Blow out a Wax  
“Candle, and if the Place be still, on applying it successively  
“to every Side of the Dish, the fuliginous Particles of the  
“Smoke, being visible and very light, will be seen to move  
“towards the Plate, and rising over it, point out the Course  
“of the Air from Sea to Land. Again, if the ambient Water  
“be warmed, and the Plate filled with cold Water, let the  
“smoking Wick of the Candle be held over the Plate, and  
“the contrary will happen.”

Variety

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Variety of Winds. For the Air, upon Account of the lesser Influence of the Sun in those Parts, being undetermined to move towards any fixed Point, is continually shifting from Place to Place, in order to restore the *Æquilibrium*, wherever it is destroyed; whether by the Heat of the Sun, the rising of Vapours, or Exhalations, the melting of Snow upon the Mountains, and a great Variety of other Circumstances, more than can be easily enumerated.

We are told by Historians, of certain Caves that emit Wind; if so, it is when the included Air is rarefied by Heat, and therefore rushes out for Want of Room; or, when the Pressure of the external Air, incumbent upon the Mouth of the Cave, is diminished, and so permits the internal Air to dilate itself, and issue out.

For more on this Subject, see *Nieuwentyt's Religious Philosopher. Regnault's Philosophical Conversations. Clare's Motion of Fluids. Martin's Philosophical Grammar.* And the Authors referred to in Mr. *Johnson's Quæstiones Philosoph. Cap. IV. Quæst. 1. 2.*



## DISSERTATION VI.

*Of the Formation and Ascent of Vapours, and their Resolution into Rain, Snow, and Hail.*

**T**HAT Vapours are raised from the Surface of Water by the Action of the Sun's Heat, is agreed on by all: But the Manner in which this is done, has ever been a Controversy among Philosophers; neither is it at this Time sufficiently explained by any one.

If we consult a *Cartesian* upon this Matter, he immediately tells us, that, by the Action of the Sun upon the Water, small Particles of the Water are formed into hollow Spheres, filled with *Materia Subtilis*, and by that Means becoming lighter than an equal Bulk of Air, are easily buoyed up in it. But, as this *Materia Subtilis* is only a Fiction, this Solution is not to be regarded.

Dr. *Nieuwentyt*, and several other Philosophers, who maintain, that Fire is a particular Substance, distinct from other Matter, account for the Formation and Ascent of Vapours thus: They say, that the Rays of the Sun, or Particles of Fire separated from them, adhering to Particles of the Water, make

\* See  
† See  
Philosoph  
§ By

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up little Bodies, lighter than an equal Bulk of Air; which therefore, by the Laws of Hydrostatics, will ascend in it, till they come to an Height where the Air is of the same specific Gravity with themselves. And, that Rain is produced by the Separation of the Particles of the Fire from those of the Water; which last, being then left without Support, can no longer be sustained by the Air, but falls down in Drops of Rain\*.

This Opinion is liable to the following Difficulties; *First*, Fire has never been yet proved to be a distinct Element, or a particular Substance†; and the Change of Weight in Bodies in chemical Preparations, heretofore thought to arise from the Adhesion of Particles of Fire, is found to proceed from the Adhesion of Particles of Air§.

*Secondly*, Should the above-mentioned Supposition be allowed, the fiery Particles, which are joined to the watery ones to buoy them up, must be considerably large, or else a very great Number must fix upon a single Particle of Water; and then a Person being on the Top of an Hill in a Cloud, would be sensible of the Heat, and find the Rain produced from that Vapour, much colder than the Vapour it-

\* See *Nieuwentijt's Religious Philosopher*, Contempl. 19.

† See the Authors referred to in Mr. *Johnson's Questions Philosoph.* Cap. I. Quæst. 30.

§ By Dr. *Hales*, in his *vegetable Statics*.



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self: whereas the contrary is evident to our Senses; the Tops of Hills, though in the Clouds, being much colder than the Rain which falls below.

Besides, the Manner in which the Particles of Water should be separated from those of the Fire, so as to fall in Rain, is not easily to be conceived.

The most generally received Opinion is, That by the Action of the Sun, on the Surface of the Water, the aqueous Particles become formed into Bubbles, filled with a *Flatus*, or *warm Air*, which renders them specifically lighter than common Air, and makes them rise therein, till they meet with such as is of the same specific Gravity with themselves \*. But I ask,

*First*, How comes the Air in the Bubbles to be specifically lighter than that without, since the Sun's Rays, which act upon the Water from whence they are raised, are equally dense over all its Surface?

*Secondly*, If it could be possible for rare Air to be separated from the denser ambient Air, to form the Bubbles (as Bubbles of soaped Water are blown up by warm Air from the Lungs, whilst the ambient Air is colder and denser) what would hinder the external Air from reducing that, which is inclosed in the Bubbles, immediately to the same Degree of

\* Philosophical Transactions, No. 192.

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Coldness, and specific Gravity with itself; (Cold being readily communicated through such thin Shells of Water.) By which Means, the Bubbles would become specifically heavier than the circumambient Air, and would no longer be supported therein; but fall down, almost as soon as they were formed?

*Thirdly*, If we should grant all the rest of the Supposition, yet the following Difficulty will still remain. If Clouds are made up of Bubbles of Water filled with Air, why do not those Bubbles always expand, when the ambient Air is rarefied, and presses less upon them than it did before; and why are they not condensed, when the ambient Air is condensed by the Accumulation of the superior Air? But if this Condensation and Rarefaction should happen to them, the Clouds would always continue at the same Height, contrary to Observation; and we should never have any Rain.

The two last Opinions are more largely examined by Dr. *Desaguliers* in the Philosophical Transactions, N<sup>o</sup>. 407. After which he endeavours to establish one of his own.

He observes, with Sir *Isaac Newton*, that, when by Heat or Fermentation the Particles of a Body are separated from their Contact, their repulsive Force grows stronger, and the Particles exert that Force at greater Distances; so that the same Body shall be expanded into a very large Space, by becoming fluid; and

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may sometimes take up more than a Million of Times the Room it did in a solid and incompressible State. " Thus, says he, if the " Particles of Water are turned into Vapour, " by repelling each other strongly, and repel " Air more than they repel each other ; Ag- " gregates of such Particles, made up of Va- " pour and Vacuity, may rise in Air of different " Densities, according to their own Density " depending on their Degree of Heat." He observes farther, that Heat acts more power- fully on Water, than on common Air ; for that the same Degree of Heat, which rarefies Air two Thirds, will rarefy Water near fourteen thousand Times, changing it into Steam or Vapour, as it boils it. And in Winter, that small Degree of Heat, which in Respect of our Bodies appears cold, will raise a Steam or Va- pour from Water, at the same Time that it condenses Air. Lastly, he observes, That the Density and Rarity of this Vapour depends chiefly on its Degree of Heat, and but little on the Pressure of the circumambient Air. From all which he infers, that the Vapour being more rarefied near the Surface of the Earth, than the Air is there by the same Degree of Heat, must necessarily be buoyed up into the Atmosphere ; and since it does not expand it- self much, though the Pressure of the incum- bent Air grows less, at length it finds a Place where the Atmosphere is of the same specific Gravity

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Gravity with itself, and there floats, till, by some Accident or other, it is converted again into Drops of Water, and falls down in Rain.

And to shew that Air is not necessary for the Formation of Steam or Vapour, he gives us this Experiment.

"ABCD (*Fig. 37.*) is a pretty large Vessel of Water, which must be set upon the Fire to boil. In this Vessel must be suspended the Glass Bell E, made heavy enough to sink in Water; but put in, in such a Manner, that it be filled with Water when upright, without any Bubbles of Air at its Crown within, the Crown being all under Water. As the Water boils, the Bell will by Degrees be emptied of its Water, the Water in the Bell being pressed down by the Steam which rises from it; but, as that Steam has the Appearance of Air, in order to know whether it be Air or not, take the Vessel off the Fire, and draw up the Bell by a String fastened to its Knob at Top, till only the Mouth remains under Water; then as the Steam condenses by the cold Air on the Outside of the Bell, the Water will rise up into the Bell at F, quite to the Top, without any Bubble above it; which shews, that the Steam, which kept out the Water, was not Air."

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This Hypothesis, however plausible it may appear, is attended with Difficulties, as well as the other. For,

*First*, If the repulsive Power of the Particles of Water is sufficiently augmented by Heat as such (and that by a very small Degree of it) to make them recede from each other, and become specifically lighter than common Air; how comes it to pass, that all the Particles of Water, as soon as, or before it boils, have not their repulsive Forces thus augmented, since they are all under a much greater Degree of Heat, than is necessary to raise Vapour?

*Secondly*, Allowing that they may rise off from the Surface of the Water, and float in the circumambient Air, as being lighter than it, why do not their repulsive Forces, as they rise up into the Air, and the superincumbent Pressure becomes less, drive them to greater Distances from each other, and so cause them to continue lighter than the Air about them at all Heights?

*Thirdly*, If the Pressure of the Air about them, when near the Surface of the Earth, is not sufficient to bring them into so close Contact, as to form Drops of Water there, what Force will they find sufficient for it, when they are carried aloft into a Region of Air, where the Pressure is not near so great? The Doctor hints, that they are formed into Rain, "when  
" by the great Diminution of the specific Gra-

" vity

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“ vity of the Air about a Cloud, it has a great  
“ Way to fall, in which Case, he says, the  
“ Resistance of the Air, which increases as  
“ the Square of the Velocity of the descend-  
“ ing Cloud, causes the floating Particles of  
“ Water to come within the Power of each  
“ other’s Attraction, and form such big Drops  
“ as, being specifically heavier than any Air,  
“ must fall in Rain.” But as the inferior Air,  
from the Cloud to a considerable Depth below  
it, cannot be supposed to leave it all at once,  
the Vapours must necessarily fall all the Way  
through a resisting *Medium*; so that the little  
Velocity the Cloud can acquire, while it is in  
the Form of Vapour, will never produce a  
Resistance from the Air greater than the Pres-  
sure which the Vapours sustained below.

*Lastly*, The Experiment, brought to make  
Way for this Hypothesis, shows clearly, that  
Vapour formed without the Assistance of Air,  
is immediately condensed into Water again, as  
soon as it is suffered to cool: Which is widely  
different from what happens to Vapours buoyed  
up into the colder Regions of the Air, and  
floating there, till they fall in Rain.

These are all, or the principal *Hypotheses*,  
that have been framed for the Solution of this  
common *Pbænomenon*: Which as they seem  
inadequate to the Effects produced, and there-  
fore unsatisfactory, I thought it not amiss to



suggest to the Reader the chief Difficulties, with which I conceive them to be attended. But as it is easier to pull down, than to build up, to overturn a weak *Hypothesis*, than to raise and support a strong and *sufficient* one ; so, I must own, in this Case, I can think of no Way of accounting for the *Rise* of Vapours, according to the received Principles of Philosophy, or say *wherein* their Nature consists. Upon this Account it is impossible I should give a Philosophical Account of their Resolution into Rain. It must suffice just to mention the Causes which are observed to produce that Change.

The first is, Part of the Air beneath them being carried away by contrary Winds blowing from the same Place \*, the Remainder becomes too light to buoy them up, and so the upper ones, in all Probability, precipitate down upon the lower ones, unite with them, and form Drops of Rain †. Another is, great  
Quan-

\* Perhaps it may be thought, that when the Air, which impends over any Place, is carried away from thence by contrary Winds, the Vapours which float in it should be carried away too ; and so the few that remain should be easily supported ; or, if they fall, should not produce much Rain.

Now, in answer to this, it must be considered, that as the inferior Air is carried away from any Place by contrary Winds, the Height of the Atmosphere is diminished thereby ; wherefore, in all Probability, the superior Air rushes in by a contrary Current to fill up the Vacuity at the Top, which bringing in its Contents of Vapour affords a continual Supply for the Rain that falls.

† A remarkable Instance we have of this, in that Part of the *Atlantic Ocean*, which the Sailors call the *Rains*, (see *Dissert. V.*)  
from

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Quantities of them, being driven by the Winds against the Sides of Mountains, are thereby made to coalesce, and so constitute Rain \*.

It

from the frequent Rains that fall there : the Occasion of which is, that the Atmosphere is diminished by continual contrary Winds blowing from thence, so that it is not able to sustain the Vapour it receives.

\* Upon this depends the Difference of the Seasons of the Year at *Malabar* and *Coromandel* in the *East Indies*, and at *Jamaica* in the West. See Dr. Gordon's Discourse on the Causes of the Change of Weather, Philosophical Transactions, N<sup>o</sup>. 17. —

"The Rivers of *Indus* and *Ganges*, says he, where they enter the Ocean, contain between them a large *Chersonesus*, which is divided in the Middle by a Ridge of Hills, which they call the Gate, which run along from East to West, and quite through to Cape *Comori*. On the one Side is *Malabar*, and on the other *Coromandel*. On the *Malabar* Side, between that Ridge of Mountains and the Sea, it is after their Appellation Summer from *September* till *April* ; in which time it is always a clear Sky, without once, or very little Raining. On the other Side the Hills, on the Coast of *Coromandel*, it is at the same Time their Winter, every Day and Night yielding Abundance of Rain ; and from *April* to *September* it is on the *Malabar* Side their Winter, and on the other Side their Summer : So that in little more than twenty Leagues Journey in some Places, as where they cross the Hills to *St. Thomas*, on the one Side of the Hill you ascend with a fair Summer ; on the other you descend with a stormy Winter. The like is said to be at Cape *Rozalgate* in *Arabia*. And Dr. *Tropham* relates the same of *Jamaica*, intimating that there is a Ridge of Hills which runs from East to West, through the midst of the Island, and that the Plantations on the South Side of the Hills have, from *November* to *April*, a continual Summer, whilst those on the North Side have as constant a Winter, and the contrary from *April* to *November*.

"From these and such like Accounts it seems evident, that a bare lessening of the Atmosphere's Gravity will not occasion Rain, but that there is also needful either a sudden Change of Winds, or a Ridge of Hills to meet the Current of the Air and Vapours, whereby the Particles of the Vapours are driven together, and so fall down in Drops of Rain. And hence it is,

"that

It is generally thought, That if the Vapours in their Descent pass through a Region of Air sufficiently cold, they are there frozen into Icicles,

“ that whilst the Wind blows from the North-east, viz. from  
 “ *November* to *April* (see the Account of the Monsoons in the  
 foregoing Dissertation) “ there are continual Rains in the north-  
 “ erly Plantations of *Jamaica*, and on the Side of *Coromandel*  
 “ in the *East-Indies*, because the Winds beat against that Side  
 “ of the Hills; and so there is fair Weather on the other Side  
 “ of these Hills, in *Malabar* and in the southern Plantations of  
 “ *Jamaica*, there being no Winds to drive the Vapours toge-  
 “ ther. But in the southerly Monsoons, viz. from *April* to *Nov-*  
 “ *ember*, *Malabar* and the southerly Plantations of *Jamaica*  
 “ have Floods of Rain, the Wind beating against that Side of  
 “ the Hills, whilst in *Coromandel*, and the other Side of *Jamaica*,  
 “ there is fair and clear Weather.

“ This serves also to clear the Singularity of Seasons in *Peru*,  
 “ beyond any other Parts of the Earth, and seems to be assigned  
 “ by *Acosia* as the Cause of it. *Peru* runs along from the Line  
 “ Southwards about a thousand Leagues. It is said to be divided  
 “ into three Parts, long and narrow, which they call *Lanos*,  
 “ *Sierras*, and *Andes*; the *Lanos*, or Plains, run along the South-  
 “ Sea Coast; the *Sierras* are all Hills with some Vallies; and  
 “ the *Andes* steep and craggy Mountains. The *Lanos* have some  
 “ ten Leagues in Breadth, in some Parts less, and in some more;  
 “ the *Sierras* contain some twenty Leagues in Breadth, the *Andes*  
 “ as much, sometimes more, sometimes less; they run in Length  
 “ from North to South, and in Breadth from East to West. This  
 “ Part of the World is said to have these remarkable Things;  
 “ 1. All along the Coast, in the *Lanos*, it blows continually with  
 “ one only Wind, which is South and South-west, contrary to  
 “ that which usually blows under the torrid Zone. 2. It never  
 “ rains, thunders, snows, or hails in all this Coast, or *Lanos*,  
 “ though there falls sometimes a small Dew. 3. Upon the  
 “ *Andes* it rains almost continually, though it be sometimes  
 “ more clear than other. 4. In the *Sierras*, which lie be-  
 “ tween both Extrems, it rains from *September* to *April*, but  
 “ in the other Seasons it is more clear, which is when the Sun  
 “ is farthest off, and the contrary when it is nearest. Now  
 “ the Reason of all seems to be this; The eastern Breezes,  
 “ which blow constantly under the Line, being stopp'd in their  
 “ Course

Icicles, and form Snow. But this Opinion seems to be false ; because it frequently snows when the Barometer is high, at which Time the Vapours cannot begin to fall. It is therefore more probable, that they are first frozen into Icicles, and by that means shooting forth into several Points every Way from the Center (agreeably to the Nature of Freezing) lose their Form ; and so becoming specifically heavier than Air fall down, and in their Passage, several being congealed together, form Fleeces of Snow †.

Hail is evidently no other than Drops of Rain congealed into Ice. This happens, when in their Passage through the inferior Air, they

Course by the *Sierras* and *Andes*, and yet the same Breezes being to be found in the South-Sea beyond *Peru*, as appears by the easy Voyages from *Peru* to the *Philippines*, a Current of Wind blows from the South on the Plains of *Peru*, to supply the eastern Breeze on the South-Seas, and there being but one constant Gale on these Plains, and no contrary Winds or Hills for it to beat upon, this seems to be the Reason why the Vapours are never or very seldom driven into Rain. And the *Andes* being as high perhaps in many Places as the Vapours ascend in the highest Degree of the Atmosphere's Gravity, this may probably be the Reason, why the eastern Breeze, beating constantly against these Hills, occasions Rain upon them at all Seasons of the Year. And the *Sierras* being it seems lower than the *Andes*, therefore from *September* to *April*, when the Sun is nearest, and so the Atmosphere's Gravity less, and the Vapours lower, they are driven against the *Sierras* into Rain."

The like is to be said of other Countries. They are all, *cæteris paribus*, more or less rainy, as they are more or less mountainous. *Egypt*, which is quite without Mountains, has seldom or never any Rain ; but is plentifully watered by the *Nile*, which nearly rises above its Banks, and overflows the Country.

† See a Discourse on the Nature of Snow, Philosophical Transactions, N<sup>o</sup>. 92.

meet

meet with *nitrous* Particles, which are known to contribute greatly to Freezing. Their Magnitude is owing to a fresh Accession of Matter, as they pass along. Hence we see the Reason why Hail is so frequent in Summer, because at that Time greater Quantities of Nitre are exhaled from the Earth, and float up and down in the Air.

If the Vapours, after they are exhaled from off the Waters, do not rise very high in the Atmosphere, but hover near the Surface of the Earth, they then constitute what we call a Fog. And, if they ascend higher, they still appear in the same Form to those, who, being upon the Tops or Sides of Mountains, are at an equal Height with them; though to those, who are below, they appear as Clouds.

If they fall to the Earth, without uniting into Drops, large enough to be called Rain, they are then said to fall in Dew.

See farther on this Subject, *Derham's Physic Theolog.* Book I. Chap. 3, and Book II. Chap. 5. *Spectacle de la Nature*, Dialog. 21. and 23. *Nieuwentyt Contempl.* 19. *Clare's Motion of Fluids.* *Regnault*, Vol. III. Conversat. 7. *Musschenbroek Epitom. Phys. Mathematic.* Par. II. Cap. 24. *Melchior Verdries Physic.* Pars special. Cap. V. §. 8. And the Authors referred to in Mr. *Johnson's Questions Philosoph.* Cap. IV. Quæst. 7.

# DISSERTATION VII.

*Of the Causes of Thunder and Lightning, and of the Aurora Borealis.*

THOSE Philosophers, who maintain that Vapours are buoyed up into the Air, by Particles of Fire adhering to them (as explained in the foregoing Dissertation) account for the *Phænomena* of Thunder and Lightning, in the following Manner: They suppose, that from the Particles of Sulphur, Nitre, and other combustible Matter, which are exhaled from the Earth, and carried into the higher Regions of the Atmosphere, together with the ascending Vapours, is formed an inflammable Substance; which, when a sufficient Quantity of fiery Particles is separated from the Vapour by the Collision of two Clouds, or otherwise, takes Fire, and shoots out into a Train of Light, greater or less, according to the Strength and Quantity of the Materials.

The Opinion is false for the Reasons mentioned in the foregoing Dissertation, which plainly show, that it is impossible the Vapours should be attended with such fiery Particles, as is here supposed.

Neither



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Neither have we Occasion to fly to such an *Hypothesis* ; for, as Vapours exhale from the Surface of the Water are carried up into the Atmosphere ; in like Manner, the *Effluvia* of solid Bodies are continually ascending thither. Now we find by Experiment, that there are several inflammable Bodies, which, being mixed together in due Proportion, will kindle into Flame by Fermentation alone \*, without the Help of any fiery Particles †. When there-

\* See the Theory of Fermentation explained in the following Dissertation.

† Monsieur Lemery having covered up in the Earth about fifty Pounds of a Mixture composed of equal Parts of Sulphur, and Filings of Iron tempered with Water ; after eight or nine Hours Time, the Earth, where it was laid, vomited up Flames Hist. de l'Acad. 1700. p. 574.

From this Experiment we see the true Cause of the Fire of *Ætna* and *Vesuvius*, and other burning Mountains. They probably are Mountains of Sulphur, and some other Matter proper to ferment with it, and take Fire. From like Causes proceed the Heat of Bath-Waters, and other hot Springs.

Mix a small Quantity of Gun-Powder with Oyl of Cloves, pour gently upon this Mixture two or three times as much Spirit of Nitre, and you will observe a bright Inflammation suddenly arising from it. A Mixture of the two Fluids alone will take Fire, the Powder is added only to augment the Inflammation.

Take two Pounds of Nitre, or refined Salt-Petre, well dried and reduced to the finest Powder, with a Pound of Oyl of common Vitriol : From this Mixture may be drawn by Distillation a Spirit of Nitre capable of inflaming Oyl of Turpentine. Mem. de l'Acad. 1726, p. 97, &c. Put into a Glass an Ounce of the Spirit of Nitre, with an Ounce of Oyl of Vitriol ; pour upon it an equal Quantity of Oyl of Turpentine, and a very fine Flame will arise suddenly with a great Explosion. When the Liquors are fresh the Effect is much greater. If we mix a Dram of the Spirit of Nitre, and three of Oyl of Turpentine, with only one of the

Spirit

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fore there happens to be a proper Mixture of the *Effluvia* of such Bodies floating in the Air, they ferment, kindle, and flashing like Gun-powder, occasion those Explosions, and Streams of Fire, which we call Thunder and Lightning.

As to the particular Species of the *Effluvia*, which compose this Mixture, that cannot exactly be determined; they are thought to be chiefly sulphureous and nitrous: *sulphureous*, because of the sulphureous Smell which Lightning generally leaves behind it, and of that sultry Heat in the Air which is commonly the Fore-runner of it: *nitrous*, because we don't know of any Body so liable to a sudden and violent Explosion, as Nitre is \*.

The

Spirit of Vitriol, the Mixture will take Fire immediately. If the same Experiment be made with the Balm of Mecca, a sudden Flame will arise, with a Noise like that of the Report of a Gun.

There are two celebrated Experiments of this Kind, though they do not come up exactly to the present Purpose, because they will not succeed, unless the Ingredients be first heated, the one of *Aurum fulminans*, and the other of *Pulvis fulminans*. The first is a Mixture of Salt of Tartar, and Gold dissolved by *Aqua Regia*. A small Quantity of this, if put into a Brass Spoon, and heated with the Flame of a Candle, causes a sudden Noise resembling that of Thunder; and goes off with great Violence. The other is a Mixture of three Parts of Nitre, two of Salt of Tartar, and one of Sulphur. This Mixture when set upon the Fire, and warmed to a certain Degree, is dissipated all on a sudden with great Thundering, like the *Aurum fulminans*.

See an Account of Exhalations taking Fire of their own Accord in Coal-Pits. *Power's Experimental Philosophy*, p. 62. and 181.

\* Dr. Lister is of Opinion, That the Matter both of Thunder and Lightning, and also of Earthquakes, is the *Effluvia* of the *Pyrites*,

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The Effects of Thunder and Lightning are owing to the sudden and violent Agitation the Air is put into thereby, together with the Force of the Exploſion \*; and not to Thunderbolts falling from the Clouds, as ſuppoſed by the Vulgar †.

*Pyrites*; as he does, that the Matter of *Vulcano's* is the *Pyrites* itſelf. This is a Mineral that emits copious Exhalations, and is exceedingly apt to take Fire upon the Admiſſion of Moiſture. See the Doctör's Defence of his Notion in the Philoſophical Tranſactions, N<sup>o</sup>. 157. He thinks this may be the Reaſon why *England* is ſo little troubled with Earthquakes, and *Italy*, and almoſt all Places round the *Mediterranean* Sea, ſo very much, viz. becauſe the *Pyrites* are rarely found in *England*; and where they are, they lie very thin, in Compariſon of what they do in thoſe Countries: as the vaſt Quantity of Sulphur, emitted from the burning Mountains there, ſeems to ſhew.

\* Lightning is ſaid to have diſſolved Silver, without burning the Purſe it was in; and to have melted the Sword, without touching the Scabbard, and the like. The Occaſion of this may poſſibly be, that the Matter of the Exhalation may be ſo ſubtle and penetrating, that, as we ſee it happens with *Aqua Fortis*, or *volatile Salts*, it may paſs through ſoft Bodies without altering their Texture, while it ſpends its whole Force on hard ones, in which it finds the greater Reſiſtance.

† Some are inclined to think, that Thunderbolts are artificial, and that they were applied by the Ancients to ſome Uſe. What confirms them in their Opinion, is, that they are found more frequently where Sepulchres have been, than in other Places. And, as it was the Cuſtom of the Ancients to have their Arms buried with their Aſhes, they think they might be of ſome Uſe in War. Some are of Opinion, they were uſed in Sacrifices. See *Melchior Verdries* *Phyſic. Pars ſpecial. Cap. V. §. 9.* *Wedelius* *Exercit. Med. Philol. Cont. II. Dec. I. p. 103.* *Schminckius* *Prof. Marburg. Diſſertat. de Urnis Sepulchralibus & Armis Lapideis, A. 1714.* *Herman Nunningius* *Sepulchretum Weſtphal. Mimigard. Gentil. p. 44.* *Jo. Henr. Cöhrſen* *Offileg. Hiſtor. Phyſic. p. 44.*

The

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The Distance the Thunder is from us, may nearly be estimated by the Interval of Time between our seeing the Lightning, and hearing the Thunder. For, as the Motion of Light is so very quick, that the Time it takes up, in coming to us from the Cloud, is not perceptible; and as that of Sound is about a thousand Feet in a Second; allowing a thousand Feet for every Second, that passes between our seeing the one, and hearing the other; we have the Distance of the Cloud, pretty nearly, from whence the Thunder comes.

We sometimes see Flashes of Lightning, though the Sky be clear and free from Clouds; in this Case they proceed from Clouds, that lie immediately below our Horizon.

Of Affinity with the *Phænomena* of Lightning are those of the *Aurora Borealis*, or *Northern Lights*, which, of late Years, have very frequently appeared in our Climate \*. These Lights differ so much from each other, that to give a Description of one alone, would not be

\* *Phænomena* of this Kind are reported to have been very frequent in *Greenland*, *Iceland*, and *Norway*, Countries near the Pole. The only ones, that are upon Record, as having appeared in *England*, before that of *March* the 6th, 17<sup>th</sup> 1780, are those of *January* the 30th, 1560, *October* the 7th, 1564, *November* the 14th and 15th, 1574, and a small one seen near *London* on the 10th of *August*, 1708. On *November* the 16th, 1707, a small one appeared in *Ireland*. Since that of *March* the 6th, 1780, they have been, and still continue very common.

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sufficient to acquaint the Reader with all the Circumstances wherewith they are attended. I shall therefore collect together such *Phænomena*, as have been most generally observed, and reduce them to the ten following Propositions, adding in the Notes, by Way of *Specimen*, a full Account of that most remarkable *Aurora*, which was seen *March* the 6th, 17<sup>16</sup>, as it was laid before the *Royal Society* by Dr. *Halley*, at their Request †.

The

† “ On *Tuesday* the 6th of *March*, in the Year 1716 (the Afternoon having been very serene and calm, and somewhat warmer than ordinary) about the Time it began to grow dark (much about seven of the Clock) not only in *London*, but in all Parts of *England*, where the Beginning of this wonderful Sight was seen; out of what seemed a dusky Cloud, in the North-east Parts of the Horizon, and scarce ten Degrees high, the Edges whereof were tinged with a reddish Yellow, like as if the Moon had been hid behind it, there arose very long luminous Rays, or Streaks perpendicular to the Horizon, some of which seemed nearly to ascend to the *Zenith*. Presently after, that reddish Cloud was swiftly propagated along the northern Horizon into the North-west, and still farther westwardly; and immediately sent forth its Rays from all Parts, now here, now there, they observing no Rule or Order in their rising. Many of those Rays, seeming to concur near the *Zenith*, formed there a *Corona*, or Image, which drew the Attention of all Spectators. Some likened it to that Representation of Glory, wherewith our Painters in Churches surround the Holy Name of God. Others to those radiating Stars, wherewith the Breasts of *Knights* of the Order of the *Garter* are adorned. Many compared it to the *Concave* of the great *Cupola* of *St. Paul's Church*, distinguished with Streaks alternately light and obscure, and having in the Middle a Space less bright than the rest, resembling the *Lanthorn*. Whilst others, to express as well the Motion as Figure thereof, would have it to be like the Flame in an *Oven*, reverberated and rolling against the arch.

“ Ro

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The most general *Pbænomena* of an *Aurora Borealis* are these that follow.

1. In the northern Parts of the *Horizon*, there is commonly the Appearance of a very black

" Roof thereof : Some thought it liker to that tremulous Light,  
" which is cast against the Cieling by the Beams of the Sun, re-  
" flected from the Surface of the Water in a Bason, that's a little  
" shaken. But all agree, that this *Spectrum* lasted only a few  
" Minutes, and exhibited itself variously tinged with Colours,  
" Yellow, Red, and a dusky Green : Nor did it keep in the  
" same Place ; for when first it began, it appeared a little to the  
" Northwards of the *Zenith*, but by Degrees declining towards  
" the South, the long *Striæ* of Light, which arose from all  
" Parts of the Northern Semicircle of the *Horizon*, seemed to  
" meet together, not much above the Head of *Cassiopeia*, or the  
" northern *Twin*, and there soon disappeared.

" After the first *Impetus* of the ascending Vapour was over,  
" the *Corona* appeared no more ; but still, without any Order,  
" as to Time or Place, or Size, luminous *Radii*, like the former,  
" continued to arise perpendicularly, now oftener, and again  
" seldomer ; now here, now there ; now larger, now shorter.  
" Nor did they proceed, as at first, out of a Cloud, but oftener  
" would emerge at once out of the pure Sky, which was more  
" than ordinary serene and still. Nor were they all of the same  
" Form. Most of them seemed to end in a Point upwards,  
" like erect Cones ; others like truncate Cones, or Cylinders,  
" so much resembling the long Tails of Comets, that at first  
" Sight they might well be taken for such. Some of those  
" Rays would continue visible for several Minutes ; when others,  
" and those the much greater Part, just shewed themselves, and  
" died away. Some seemed to have little Motion, and to stand,  
" as it were, fixed among the Stars, whilst others, with a very  
" perceptible Translation, moved from East to West under the  
" Pole, contrary to the Motion of the Heavens ; by which  
" Means they would sometimes seem to run together, and at  
" other Times to fly one another.

" After this Sight had continued about an Hour and a half,  
" those Beams began to rise much fewer in Number, and not near  
" so high ; and by Degrees, that diffused Light, which had illu-



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black Cloud ; but it is evident that it is no real Cloud, because the smallest Stars are visible through it. This apparent Cloud is extended sometimes farther towards the West, than

“ strated the northern Parts of the Hemisphere, seemed to sub-  
 “ side, and settling on the Horizon, formed the Resemblance of  
 “ a very bright *Crepusculum*. That this was the State of this  
 “ *Phænomenon*, in the first Hours, is abundantly confirmed by the  
 “ unanimous Consent of several. For, by the Letters we have  
 “ received from almost all the extreme Parts of the Kingdom,  
 “ there is found very little Difference from what appeared at  
 “ London and Oxford ; unless that in the North of England,  
 “ and in Scotland, the Light seemed somewhat stronger and  
 “ brighter.

“ Hitherto I have related the Observations of others ; as to  
 “ myself, I had no Notice of this Matter, till about nine of the  
 “ Clock : I immediately perceived toward the South and South-  
 “ west Quarter, that though the Sky was clear, yet it was tinged  
 “ with a strange Sort of Light ; so that the smaller Stars were  
 “ scarce to be seen, and much as it is when the Moon of four  
 “ Days old appears after Twilight. I perceived at the same  
 “ Time a very thin Vapour to pass before us, which arose from  
 “ the precise East Part of the Horizon, ascending obliquely, so  
 “ as to leave the *Zenith* about fifteen or twenty Degrees to the  
 “ Northward. But the Swiftnes, wherewith it proceeded, was  
 “ scarce to be believed, seeming not inferior to that of Light-  
 “ ning ; and exhibiting, as it passed on, a Sort of momentaneous  
 “ *Nubecula*, which discover’d itself by a very diluted and feint  
 “ Whiteness ; and was no sooner formed, but, before the Eye  
 “ could well take it, it was gone, and left no Signs behind it.  
 “ Nor was this a single Appearance ; but for several Minutes,  
 “ about six or seven Times in a Minute, the same was again and  
 “ again repeated ; these Waves of Vapour regularly succeeding  
 “ one another, and at Intervals very nearly equal ; all of them  
 “ in their Ascent producing a like transient *Nubecula*.

“ By this Particular we were first assured, that the Vapour we  
 “ saw, became conspicuous by its own proper Light, without the  
 “ Help of the Sun’s Beams ; for these *Nubeculae* did not discover  
 “ themselves in any other Part of their Passage, but only between  
 “ the South-east and South, where being opposite to the Sun,  
 “ they

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than to the East; sometimes farther towards the East, than to the West; and generally takes up a Quarter of the Horizon, more or less.

2. The

" they were deepest immersed in the Cone of the Earth's Shadow; nor were they visible before or after. Whereas the contrary must have happened, had they borrowed their Light from the Sun:

" On the western Side of the northern Horizon, *viz.* between West and North-west, not much past ten of the Clock, I observed the Representation of a very bright Twilight, contiguous to the Horizon, out of which arose very long Beams of Light, not exactly erect towards the *Vertex*, but something declining towards the South; which, ascending by a quick and undulating Motion to a considerable Height, vanished in a little Time; whilst others, though at uncertain Intervals, supplied their Place. But at the same Time, through all the rest of the northern Horizon, *viz.* from the North-west to the true East, there did not appear any Sign of Light to arise from, or join to, the *Horizon*; but what appeared to be an exceeding black and dismal Cloud, seemed to hang over all that Part of it. Yet was it no Cloud, but only the serene Sky, more than ordinary pure and limpid, so that the bright Stars shone clearly in it, and particularly *Canuda Cygni*, then very low in the North; the great Blackness manifestly proceeding from the Neighbourhood of the Light, which was collected above it. For the Light had now put on a Form quite different from all that we have been describing, and had fashioned itself into the Shape of two *Laminae*, or Streaks, lying in a Position parallel to the *Horizon*, whose Edges were but ill terminated. They extended themselves from the North by East to the North-east, and were each about a Degree broad; the undermost about eight or nine Degrees high, and the other about four or five Degrees over it; these kept their Places for a long Time, and made the Sky so light, that I believe a Man might easily have read an ordinary Print by the Help thereof.

" Whilst I was viewing this surprising Light, and expecting what was farther to come, the northern End of the upper *Lamina* by Degrees went downwards, and at length closed with the End of the other that was under it, so as to shut up on the North Side an intermediate Space, which still continued open

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2. The upper Edge of this Cloud (which is sometimes within less than six Degrees of the Horizon, and sometimes forty or fifty above it)

“ to the East. Not long after this, in the said included Space, I  
“ saw a great Number of small Columns, or whitish Streaks, to  
“ appear suddenly erect to the Horizon, and reaching from the  
“ one *Lamina* to the other; which instantly disappearing, were  
“ too quick for the Eye, so that I could not judge, whether they  
“ arose from the under, or fell from the upper; by their sudden  
“ Alterations, they made such an Appearance, as might well  
“ enough be taken to resemble the Conflict of Men in Battle.

“ And much about the same Time, there began on a sudden to  
“ appear, low under the Pole, and very near due North, three or  
“ four lucid *Areas*, like Clouds, discovering themselves in the  
“ pure but very black Sky, by their yellowish Light. These, as  
“ they broke out at once, so after they had continued a few  
“ Minutes, disappeared as quick, as if a Curtain had been drawn  
“ over them: Nor were they of any determined Figure, but both  
“ in Shape and Size might properly be compared to small  
“ Clouds illuminated by the full Moon, but brighter.

“ Not long after this, from above the foresaid two *Laminae*,  
“ there arose a very great *Pyramidal* Figure, like a Spear, sharp  
“ at the Top, whose Sides were inclined to each other, with an  
“ Angle of about four or five Degrees, and which seemed to  
“ reach up to the *Zenith*, or beyond it. This was carried with  
“ an equable, and not very slow Motion, from the North-east,  
“ where it arose, into the North-west, where it disappeared, still  
“ keeping in a perpendicular Situation, or very near it; and  
“ passing successively over all the Stars of the *little Bear*, did  
“ not efface the smaller ones in the Tail, which are of the fifth  
“ Magnitude; such was the extream Rariety, and Perspicuity  
“ of the Matter whereof it consisted.

“ This single Beam was very remarkable for its Height above  
“ all those, that, for a great while before, had preceded it, or  
“ that followed it.

“ It being now past eleven of the Clock, and nothing new of-  
“ fering itself to our View, but repeated *Phases* of the same  
“ Spectacle; I observed, that the two *Laminae*, or Streaks,  
“ parallel to the *Horizon*, had now wholly disappeared; and  
“ the whole Spectacle reduced itself to the Resemblance of a very  
“ bright

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it) is generally terminated with a very lucid Arch, from one to four or five Degrees broad, whose Center is below the Horizon. Some times

" bright *Crepusculum* settling on the Northern *Horizon*, so as to  
 " be brightest and highest under the Pole itself; from whence it  
 " spread both Ways into the North-east and North-west. Un-  
 " der this, in the Middle thereof, there appeared a very black  
 " Space, as it were the Segment of a lesser Circle of the Sphere  
 " cut off by the *Horizon*. It seemed to the Bye like a dark  
 " Cloud, but was not so; for by the Telescope the small Stars  
 " appeared through it more clearly than usual, considering  
 " how low they were: And upon this as a *Basis*, our *Lumen*  
 " *Auroriforme* rested, which was no other than a Segment of a  
 " Ring, or *Zone* of the Sphere, intercepted between two pa-  
 " rallel lesser Circles, cut off likewise by the *Horizon*; or the  
 " Segment of a very broad *Iris*, but of one uniform Colour,  
 " viz. a Flame-Colour inclining to Yellow, the Center thereof  
 " being about forty Degrees below the *Horizon*. And above  
 " this there were seen some Rudiments of a much larger Seg-  
 " ment, with an Interval of Sky between, but this was so ex-  
 " ceeding feint and uncertain, that I could make no proper  
 " Estimate thereof.

" I attended this *Phænomenon* till near three in the Morning,  
 " and the Rising of the Moon: But for above two Hours toge-  
 " ther it had no Manner of Change in its Appearance, nor Di-  
 " minution, nor Increase of Light; only sometimes, for very  
 " short Intervals, as if new Fuel had been cast on a Fire, the  
 " Light seemed to undulate and sparkle not unlike the rising of  
 " a vaporous Smoak out of a great Blaze when agitated. But  
 " one Thing I assured myself of, that the *Iris*-like Figure did by  
 " no means owe its Origin to the Sun's Beams: For that about  
 " three in the Morning, the Sun being in the Middle between  
 " the North and East, our *Aurora* had not followed him, but  
 " ended in that very Point where he then was: Whereas in the  
 " true North, which the Sun had long passed, the Light re-  
 " mained unchanged, and in its full Lustre."

Appearances of this Kind have been taken Notice of both by  
*Pliny*, *Seneca*, and *Aristotle*; the last of which calls the vibrating  
 Light near the *Zenith*, *Δαλοί*; the more steady perpendicular  
 Streams, *Δοκός*; and the *Aurora* itself, from the apparent dark  
 Cloud just below it, *Χάσμα*. That *Aurora* which was observed  
 by

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times there are two or more of these Arches, one above another. In some, the Cloud is not terminated by an Arch, but by a long bright Streak

by Monsieur Gassendi in *Provence*, on the 21st of *September*, in the Year 1621, was very remarkable, at that Time.

He tells us, that about the End of Twilight in the Evening, when the Sky was very clear, and there was no Moon, there appeared in the North a Sort of a rising Morn, which ascending by Degrees, became intermingled with certain Streaks, as it were, or Rays perpendicular to the Horizon: And that at the same Time there appeared some small *passing* whitish Clouds between the South and the Place of the Sun's setting in Winter; and that in the Place where the Sun sets in Summer, a bright Redness seemed to arise in the Form of a *Pyramid*, which moved towards the setting of the Sun at the *Equinox*; and which was distinguished into three several *Pyramids*, which in a little Time were confounded one with another, and at last disappeared. When this Redness ceased, the northern Whiteness was risen forty Degrees and more, that is, about the Altitude of the Pole-Star, forming itself into an Arch, and taking up near sixty Degrees of the *Horizon*. After this, certain *Chevrons*, or Columns of Rays, some whiter, and some a little darker, began more plainly to be distinguished, of about two Degrees in Breadth, and perpendicularly posited; so that all that Part appeared as it were fluted. The whole Circumference immediately appeared scalloped; and then some of those Columns which were in the Middle, and that were the whitest, began as it were to leave their Places with great Impetuosity, and in less than a Quarter of a Minute, raised themselves almost to the Top, putting on the Form of *Pyramids*, which they would retain four or five Minutes. It was about nine of the Clock, when the Arch of Whiteness began to decrease or sink; at which Time certain very white Streams of Smoke began to issue out from the Columns which were remaining under the *Pyramids*, and darting upwards with very great Rapidity through the *Pyramids*, like Javelins, vanished immediately when they came to the Tops of them. This Spectacle lasted about an Hour; after which the Whiteness sunk down to about six Degrees of the *Horizon*. *Vide Abregé de Gassendi, Tom. V. P. 245.*

This

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Streak of Light, lying parallel to the *Horizon*. The Limb of this luminous Arch, or parallel Streak, is not always even and regular, but sinks lower in some Parts, than in others.

3. Out of this Arch proceed Streams of Light, generally perpendicular to the *Horizon*, but sometimes a little inclined to it. Most of them seem to end in a Point, like *Pyramids* or *Cones*; and often very much resemble the Tails of *Comets*. Sometimes there is no luminous Arch, nor Streak of Light; and then the Streams seem to issue out from behind the dark Cloud, being distinct from each other at their *Bases*.

4. The upper Ends of the Streams incessantly appear and vanish again, as quick as if a Curtain were drawn backwards and forwards before them; which sometimes causes such a seeming Trembling in the Air, that you would

This *Phænomenon* appeared not only to *Gassendi* in *Provence*, but was seen at Places very distant from thence, as *Tolose*, *Montauban*, *Bordeaux*, *Grenoble*, *Dijon*, *Paris*, and *Roan*, &c. and at most other Places in *France*, and the neighbouring Countries, that lie to the Northwards of *Provence*, unless where it was intercepted by Clouds; but no where in such as lie at any great Distance to the Southwards of it.

Monsieur *Gassendi* is thought to have given the Name of *Aurora Borealis* to this *Phænomenon*; but this is observed by Monsieur *Mairan*, to be a Mistake. See Mr. *Mairan*'s Physical and Historical Treatise of the *Aurora Borealis*, in the *Memoires de l'Academie Royale des Sciences*, Année 1731. or an Abstract of it in *Philosoph. Transact.* N<sup>o</sup>. 431.

imagine



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imagine the upper Part of the Heavens to be, as it were, in Convulsions \*.

5. They sometimes seem to meet in the Zenith, or more commonly to the Southward of it, about ten Degrees, more or less (sometimes they deviate a little to the South-east of the Meridian; and sometimes to the South-west); and there curling round, in some Measure, imitate Flame confined under an Arch; and being frequently tinged with various Orders of Colours, exhibit a most beautiful Appearance, resembling a Canopy finely painted †, of about ten or twenty Degrees in Breadth.

In many *Aurora*'s, there are Streams ascending from those Parts of the Heavens, which lie several Degrees to the Southwards of the Canopy; and in some, they appear to arise, though very rarely, almost as large and numerous from the southern, as from the northern Parts of the Horizon.

6. The Height of the *Aurora Borealis* is very great; for that of *March* the sixth, 17<sup>th</sup>, was visible from the West Side of *Ireland*, to the Confines of *Russia* and *Poland* on the East, and perhaps farther; extending at least over thirty Degrees of Longitude, and in La-

\* See their Motions well described in the Account we have of an *Aurora* in the Philosoph. Transact. N<sup>o</sup>. 395. Art. 2.

† See the various Colours of the Canopy, as well as those of an *Aurora* itself, accurately described by *Greenwood*, in Philosoph. Transact. N<sup>o</sup>. 418. Art. 1.

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titude from about the fiftieth Degree over almost all the North of *Europe*, and at all Places exhibiting, nearly at the same Time, the same Appearances.

7. These Appearances have always been very frequent (as far as the Accounts we have of them inform us) in Countries that lie in, or near the *frigid Zone*, but very rare in those of our Latitude; they are now become very frequent with us, but always seem to proceed from the North; and are as yet unknown to the Inhabitants of the more southern Parts of our *Hemisphere*. Whether they are seen to those, who inhabit in, or near the other *frigid Zone*, is to us unknown.

8. In some, there are Trains of Light running horizontally, sometimes from the Middle to the Extremes, and sometimes from one Extreme to the other. And from these Trains often arise Streams perpendicular to the *Horizon*, and accompanying them as they pass along.

9. When all the Streaming is over, the *Aurora Borealis* commonly degenerates into a bright Twilight in the North, and there gradually dies away.

10. It is probable, that these *Phænomena* often happen in cloudy Nights, though we are not sensible of them; for 'tis observable, that in such Nights there is frequently more Light than

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than what usually proceeds from the Stars alone.

The most obvious Solution of the *Aurora Borealis*, or at least what would appear so, to such as have only attended to the Circumstances of some particular ones, and which has accordingly been affixed by several \* to their Accounts of the *Aurora's* they have seen, is, that it is a thin Nitro-sulphureous Vapour, raised in our Atmosphere considerably higher than the Clouds; that this Vapour by Fermentation takes Fire, and the Explosion of one Portion of it kindling the next, the Flashes succeed one another, till the whole Quantity of Vapour within their Reach is set on Fire.

\* Professor Cotes, at the End of his Description of a *Phenomenon* of this Kind, inserted in the Philosophical Transactions, No. 365, observes, that supposing a Fund of Vapours or Exhalations at a considerable Height above us to be diffused every Way into a large and spacious Plane, parallel to the *Horizon*; that Fund of mixed Matter by Fermentation will emit Streams; and that, if the Wind be still, they will ascend perpendicularly upwards; otherwise they will be inclined towards that Point of the *Horizon* which is opposite to that from which the Wind blows; and that they will appear, by the Rules of Perspective, in the first Case, to converge to the Spectator's *Zenith*, in the other, to some Point not far from it; and that, if this Fund of Vapours be situated more to the North than the South, it will produce Streams of Light, attended with such Circumstances as then appeared: But he does not say, why the Vapours should be situated rather to the North than to the South, or proceed to account for all the *Phænomena* of the *Aurora Borealis* in general from thence.

Some

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Some have thought, that Vapours rarefied exceedingly by subterraneous Fire, and tinged with sulphureous Steams, might from thence be disposed to shine in the Night, and rising up to the Top of the Atmosphere, or even beyond its Limits (as we find the Vapours of Gun-powder, when heated *in Vacuo*, will shine in the Dark, and ascend to the Top of the Receiver, though exhausted) might produce those Undulations in the Air, which constitute this *Phænomena*.

But these *Hypotheses* have been rejected, as insufficient; it having been thought impossible to account for all the Circumstances of the *Aurora* by them \*.

\* In the Commentaries of the Academy of Sciences at *Petersburgh*, I find a late Solution of the *Aurora Borealis* from Exhalations fermenting and taking Fire in the Atmosphere, which, the Author *Chr. Maier* says, occasion the Appearance of the lucid Arch in the North, and thinks that the Streams, which seem to issue from thence, are no other than the Light of that Arch reflected to us from the under Side of some thin Clouds, that lie above it. As to its appearing in the North rather than in the South, he supposes that may be owing to the Cold of those Regions condensing the Exhalations, and thereby rendering them more liable to ferment than they are in the southern; but acknowledges ingenuously, that he has no Reason to suppose this, but its being necessary to his Solution. At the End he tells us, That it was known in ancient Times as well as lately: But omits taking Notice, that it appears much oftner of late Years in our Climate than it used to do; and so avoids accounting for that Particular. *Vide Commentar. Academ. Scientiar. Imperial. Petropolitanae. Tom. I. p. 351.*

Dr.

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Dr. *Halley* therefore has Recourse to the *magnetic Effluvia* of the Earth, which he supposes to perform the same Kind of Circulation with Regard to it, as the *Effluvia* of any particular *Terrella* \* do with respect to that, viz. that they enter the Earth near the South Pole and pervading its Pores, pass out again at the same Distance from the northern : And thinks they may sometimes, by the Concurrence of several Causes very rarely coincident, and to us as yet unknown, be capable of producing a small Degree of Light, either from the greater Density of the Matter, or perhaps from the greater Velocity of its Motion ; after the same Manner, as we see the *Effluvia* of *Electric* Bodies emit Light in the Dark.

Monsieur de *Mairan* has given us a Physical and Historical Treatise of the *Aurora Borealis*, and endeavours to prove that it is owing to the *Zodiacal Light*, or the Atmosphere of the Sun, spread on each Side of it along the *Zodiac* in the Form of a *Pyramid*. This, he says, is sometimes extended to such a Length as to reach beyond the Orbit of our Earth, and then mixing itself with our Atmosphere, and being of an *Heterogeneous* Nature, produces

\* A round Magnet, so called from the Resemblance it bears to the Earth.

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the several Appearances, which are observed in the *Aurora Borealis* \*.

I have just mentioned these two Solutions, because they come from two very ingenious Philosophers; though I doubt not but the Reader will agree with me, that they are much too fine spun to hold, and that they are no other than the ingenious *Reveries* of Persons determined to frame an Hypothesis at any Rate. I question not, but we may find Materials for the *Aurora Borealis*, without going so far for them, as these Gentlemen have done; and in particular that we have no Occasion to have Recourse either to the *magnetic Effluvia* of the Earth, or the *Zodiacal Light*, the Nature of both which we are wholly unacquainted with. The Materials employed in the first Solution (I mean such *Effluvia* as are continually exhaled from the Surface and Bowels of the Earth) if rightly considered, will afford a very easy and natural one, as I shall endeavour to shew in the following Manner.

First, we are assured by Experiment, that there are some Steams (as inflammable sulphureous ones) which are capable of so great a Degree of Expansion, that they will render themselves lighter than the Air they float in, though

\* See his Account at large, referred to at the End of Note, Page 155.



it be as rare, as it can be made by Art; for they will rise to the Top of the Receiver, when exhausted \*, that is, when as much Air, as is possible, is drawn out †: Such Steams therefore or Exhalations, rising out of the Earth from Mines, Vulcano's, &c. must necessarily be buoyed up towards the Top of the Atmosphere, at least, till they come to a Region where the Air is as rare and expanded, as it can be made by the *Air Pump*, here below. This Height, according to Dr. *Halley's* Computation §, (which he founded upon the Air's Elasticity) is about forty or fifty Miles: but probably it is much greater; for the Air, which is higher than Vapours, and other *Heterogeneous Matter*, that is not elastic, rise to being much purer than any we can make Experiments upon, may be indued with a much greater Degree of Elasticity, and so the Atmosphere may be considerably higher, than what he, upon that Principle, computes it to be.

Secondly, These *Effluvia* being raised to the Top of the Atmosphere, or near it, and floating there, will necessarily be carried towards the polar Parts thereof, for the following Reasons. 1. Because the superior Current

\* See Philosoph. Transact. N°. 347. and 360.

† It is impossible to extract all the Air out of a Vessel, because it is by the Spring of the Air remaining in the Vessel, that the Valves of the Pump are opened at each Stroke.

§ Philosoph. Transact. N°. 181.

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of the Air, to a great Distance from the *Equator*, is that Way \*. 2. We know from Experiment, that whatever swims upon a Fluid which revolves upon an *Axis*, is thereby carried towards that *Axis*. This is exactly the Case of these *Effluvia*, for they swim near the Top of the Atmosphere which continually revolves about the *Axis* of the Earth; they must therefore necessarily be carried towards the polar Parts thereof.

Thirdly, These *Effluvia* being collected together at, or near the Poles, and of an inflammable Nature, may easily be supposed to ferment, when they meet with other *beterogeneous* ones proper to produce such an Effect, and emit Streams of Fire; which Streams will naturally rise up into such Parts of the Atmosphere as are still lighter than that wherein the *Effluvia* rest, that is, directly upwards from the Center of the Earth. But, according to the Rules of *Perspective*, those Streams, though they really diverge, as *Radii* from a Center, will appear to a Spectator on the Surface of the Earth to converge towards a Point: which Point will seem to be directly over his Head, if the Streams ascend in right Lines from the Center of the Earth: but if they deviate all one Way from that Direction, the Point will be on

\* As explained in Dissertation V.

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that

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that Side the *Zenith* towards which they incline \*.

To illustrate this; suppose several Strings hung down from the Ceiling of a Room, and a Candle placed upon a Table below them, the Shadows of them all will converge towards the Point, that is over the Candle. And, if they are made to incline, suppose all one Way, the Point of Convergency will remove from over the Candle, towards that Side of the Room to which the upper Ends of the Strings incline: Now if a Person had viewed them from the Place where the Candle was, and referred their Places to the Ceiling, they would have seemed to him to have converged towards the Point, where the Shadows did.

And if the Streams spread themselves as they arise, but not too much, they will nevertheless appear tapering towards the upper Ends, like *Cones* or *Pyramids*; just as the Sides of a long Walk seem to a Person that views them

\* This may be made to appear in the following Manner; Let ADB (*Fig. 38.*) represent the *Concave* of the Heavens, AB the *Horizon*, C the Place of the Spectator, TV a luminous Substance sending forth the parallel Streams EG, LM, NO, &c. These Streams will all seem to converge towards the Point D, if that Point be taken such, that the Line DC drawn from thence to the Spectator's Eye, be parallel to the Streams. For Instance, the Stream EG will seem to rise from *e* to *g*, LM from *l* to *m*, and FH from *f* to *h*, and so of the rest. And NO will appear wholly in D, the Place to which the rest seem to converge. And if the Streams are as large, or somewhat larger at the upper Ends, than at the lower, they will still appear leis there, those Ends being farthest from the Spectator's Eye.

from

Diff. VII. *Of the Aurora Borealis.* † 65

from one End of it, or from a distant Place, to approach each other at that which is farthest from him.

This being premised, we may now account for the several *Phænomena* of the *Aurora Borealis* before laid down. As,

1. The Blackness of the Sky, which generally appears in the northern Parts of the *Horizon*, like a dark Cloud, is occasioned by the Brightness of the luminous Matter of the *Aurora* just above it. That the Sky is clear here, is evident (as was observed before) because the smallest Stars are seen through it.

2. The lucid Arch immediately above, is the luminous Matter of the *Aurora* itself, which sometimes exhibits the Appearance of a Curve, sometimes of a straight Line, according to its Form and Situation in the Atmosphere, though generally that of a Curve: For, by the Rules of *Perspective*, when a straight Line is distended horizontally, and above the Spectator's Eye, it ought to appear bent into a Curve, whose Center is below the *Horizon* \*. Sometimes it appears on one Side the North Point, more

\* Thus, when a Person stands fronting a Row of Houses, and looks over the Tops of them, if they are all of an equal Height, that House which is nearest him, will seem to cut the Heavens in a Point that will be higher than where it is cut by any of the rest; and the Points where the Heavens will seem to be cut by the Tops of those, which are on the Right and Left Hand of the Spectator, will descend lower and lower, as the Houses are farther off; so that the Points, taken all together, will represent a Curve.

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than on the other ; sometimes regular, sometimes irregular, as the various Circumstances of the Air's Motion at the Top of the Atmosphere, and of the Situation of the flaming Matter, may be.

3. The Streams of Light, issuing out of the lucid Arch, are Streams of Fire emitted upwards from the Matter of the *Aurora*, and seem, for the Reasons already laid down, to converge towards the *Zenith* of the Spectator. Why they incline a little sometimes from the Perpendicular, will be explained in the fifth Remark, where we account for the Situation of the Canopy. When no luminous Arch appears, it is probable, that it is intercepted by the *Horizon*, or by the Vapours which float in great Quantities therein.

4. The trembling observed in the upper Part of the Heavens, is owing to the Quickness wherewith the Flashes succeed one another, and also to the irregular Motions and Agitations of the superior Parts of the Atmosphere.

5. So long as the luminous Matter of the *Aurora* is all of it towards the North of us, the Streams cannot seem to meet in a Point at the Top, as will appear to any one that considers the Figure referred to in the Note (p. 164); but after it has advanced forwards, or become kindled over our Heads, then they appear to meet, and form the Canopy already described ; and when it has passed further still, they seem

to

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to arise from all Parts; though they are much fainter on the southern than on the northern Side, so long as the main Body of the *Aurora* remains on the northern Side of the Canopy, which it rarely, if ever, passes. The Reason why the Center of the Canopy is generally a few Degrees to the South of the Spectator's *Zenith* \* is, because the luminous Streams, which issue forth from the extreme Parts of the Substance of the *Aurora*, will naturally diverge a little from the middle ones; and, as those which appear to us, proceed chiefly from the southern Side (that being nearest to us) the Point of Convergency will necessarily be placed to the South of our *Zenith*, according to what was said above about the Inclination of the Strings hanging from the Ceiling of a Room. If the Center of the Canopy is sometimes to the Eastward, and sometimes to the Westward of the Meridian, that depends upon the Motion of that Part of the Air, which is above the Substance of the *Aurora*, and through which the Streams pass, as they rise. This also it is that makes the Streams seem to arise sometimes a little obliquely.

\* According to this Theory, the Center of the Canopy will always be near the Spectator's *Zenith*, where-ever he is; which I believe is the Case, for I have met with no Account where it is otherwise; and so every Spectator sees a different Canopy, just as, when several Persons are viewing a Rainbow, no two Persons see the same Rainbow at the same Time. See Part III. p. 208.



## 168 *Of the Aurora Borealis. Part II.*

6. The great Height of the *Aurora* is owing to the exceeding Lightness of the *Effluvia*, which compose the Substance of it (as explained above) and the darting of the Streams upwards, into Regions perhaps quite above the Atmosphere, occasions it to exhibit at very distant Places the same Appearances at the same Time.

7. That the *Aurora* appears near the *Pole*, and never at or near the *Equator*, is because of the Tendency the Matter of it has towards the Poles, as explained above. And that it appears in Places more distant from the *Pole*, than it formerly did, is because the *Effluvia*, which are now raised from the Earth, are prevented from approaching so near the polar Parts of the Atmosphere, as they used to do; those Parts being already stocked with others, which were formerly raised, and are now grown *effete* by frequent Fermentations and Explosions.

8. The *horizontal* Trains of Light are the Substance of the *Aurora* just taking Fire, which runs from one Part to another, as in a Train of Gun-powder kindled in any one Part; and sends up Streams perpendicularly from Places, where it meets with a greater Quantity of Matter than ordinary.

9. When the Matter of the *Aurora* is so far spent, as to emit no more Streams, it appears only as a bright steady Light in the North, which

Diff. VII. *Of the Aurora Borealis.* 169

which gradually dies away, for Want of fresh Fewel to support it.

10. As the Vapours, of which Clouds are formed, never rise so high, as where the Matter of the *Aurora Borealis* floats; it is not at all inconsistent with the foregoing Theory, if it is sometimes intercepted from our Sight, by the Interposition of Clouds below.

See farther on this Subject, *Aristotel.* Meteor. Lib. I. Cap. 4, 5. *Plinii* Histor. Natural. Cap. 26, 27. *Senec.* Quæst. Natural. Lib. I. *Lycost.* Prodigiorum ac Ostentorum Chronicon, passim. *Julius Obsequens* de Prodigis, Cap. 13, 43, 88. *Gassendi* Animadvers. in *Diog. Laert.* Lib. X. p. 1157. *Cornelius Gemma* de divinis Naturæ Characterismis. *Nicephori* Histor. Ecclesiast. Lib. XII. Cap. 37. *Isid.* *Hispal.* Histor. Goth. Tom. I. p. 65. Bibliothec. Orientalis Clementino-Vaticana, Tom. I. p. 407. *Gregor.* *Tur.* passim. Mem. de Lit. de l'Acad. des Inscriptions & Belles Lettres, Tom. IV. p. 431. Miscellan. *Berolin.* Tom. I. p. 137. *Theatr.* Comet. *Stanisf. Lubienietz*, p. 264, 348. Mem. pour servir à l'Histor. de France, Tom. I. p. 168. Mem. de l'Acad. Royal de Sciences, for almost each Year since 1716. *Philosoph. Transf.* N°. 305, 310, 320, 347, 348, 349, 351, 352, 363, 365, 368, 376, 385, 395, 398, 399, 402, 410, 418, 431; and the Authors referred to by Mr. *John-son*, in his Quæst. *Philosoph.* Cap. IV. §. 3.

D I S-

## DISSERTATION VIII.

## Of Fermentation.

**H**AVING had Occasion to mention some of the Effects of Fermentation, it may not be amiss, before I put an End to these Dissertations, to add a short Account of the Nature of it, and to shew how those Effects are produced by it.

Fermentation is a mutual Commotion of the constituent Particles of Bodies, one among another; and arises from an Inequality in their Attractions of Cohesion. Authors distinguish it into two Kinds; the one is that which happens when a Solid is dissolved by a Fluid; the other is, when two Fluids, being mixed together, ferment with each other.

Those Authors, who have treated of the first of these, tell us, That to cause a Fermentation between a Solid and a Fluid, several Circumstances are necessary. Particularly Dr. Friend\*, and Keil†, are of Opinion,

1. That the Particles of the Solid must attract those of the Fluid with a greater Force, than the Particles of the Fluid attract one another.

\* See his Chymical Lectures.

† See his Letter to Dr. Cockburn, *De Legibus Attractionis*.

2. That

II. Diff. VIII. *Of Fermentation.* 171

2. That the Pores of the Solid must not be too small to admit the Particles of the Fluid into them.

3. That the Body be of so loose a Texture, that the Force of Impact, with which the Particles of the Fluid rush into its Pores, may be sufficient to disunite its Parts.

4. That the Elasticity of the Particles tends very much to promote, and augment the Fermentation.

Dr. *Boerhaave* makes also four Conditions requisite \*.

1. That there be a due Proportion between the Size of the Particles of the Fluid, and the Pores of the Body to be dissolved.

2. That the Figure of the Particles of the Fluid have a determinate Relation to that of the Pores of the Solid.

3. That the Particles of the Fluid be sufficiently solid, that their Moment, or Force of Action may not be too weak.

4. The last Qualification, he mentions, is a fit Disposition of the Particles of the Fluid, when received into the Pores of the Solid, to make some Stay there, and not immediately to pass through; but to act every Way upon the Solid, as they move towards the external Surface thereof.

\* Dr. *Boerhaave's* Chemistry, by *Shaw*, p. 344.

But

But we have no Occasion to have Recourse to so many Suppositions: If the Particles of the Solid attract those of the Fluid with a greater Degree of Force than either those of the Fluid, or those of the Solid attract one another \*, it is sufficient; and there will follow a Dissolution of the Body, as may clearly be demonstrated from the Laws of *Mechanics*, whatever the other Circumstances, relating to the Figure or Magnitude of Pores, &c. may be †.

When

\* This may be thought an impossible Supposition, for the Force of Attraction of Cohesion being as the Surfaces of the attracting Particles, whatever Size or Form the Particles of the Solid and Fluid are of, there cannot be a greater Quantity of Surface between every two Particles, one of which is a Particle of the Solid, and the other a Particle of the Fluid, than there is between every two Particles, which are either both of the Solid, or both of the Fluid; and therefore the Particles of the Solid cannot attract those of the Fluid with greater Force than either those of the Solid or those of the Fluid attract one another. But it is to be considered, that we are not so well acquainted with the Nature of the Attraction of Cohesion, as to determine *exactly* in what Manner, and by what Laws it acts. The Experiments made Use of for this Purpose, only shew that so long as we try them with the same Kind of Bodies, the Attraction is larger where the Contact is so. See Part I. Chap. III. But we have no Method of determining, whether the Difference of Attraction, which *various* Bodies exert upon one another, arises solely from a Difference in their Surfaces; or not.

† *Dem.* Thus, let *f, f, f, &c.* (Fig. 39.) represent a Series of the Particles of a Fluid, and *s, s, s, &c.* a Series of those of a solid Body, contiguous to one another: and let the pricked Lines *ff, ff, &c.* represent the Forces of Attraction between the Fluid Particles one among another, and *ss, ss, &c.* those of the solid ones among themselves; and let the black Lines *fs, fs, &c.* express those which are between the fluid and solid Particles. Now, the latter Forces being by the Supposition stronger than the former, the fluid Particles will recede

from

# Diff. VIII. Of Fermentation. 173

When a Solid is put into a Fluid, if their Particles have the above-mentioned Relation to each other, those of the Solid being attracted with greater Force towards the Fluid, than they are the contrary Way, they will fall off from the Solid, and enter in between the Particles of the Fluid; and for the like Reason, those of the Fluid will open to themselves a Way in between those of the Solid, and will separate them from each other. Neither will their respective Motions cease, unless their

from each other, and suffer those of the Solid to enter in between them; and for the same Reason the solid Particles will give Way to those of the Fluid. By which means, the Distances represented by the pricked Lines becoming greater, the Attractions, which they express, will be diminished; so that the fluid Particles will enter quite in between the solid ones, and the solid ones between the fluid ones; and both of them together will constitute such a Series, as is represented in *Figure 40*, in the middle Row *s, f, s, f, &c.* where the solid and fluid Particles lie mixed interchangeably one with another in a right Line. Now let it be supposed, that this Series is contiguous to one which consists wholly of Fluid above it, as is expressed in the Figure, and to another below, consisting of solid Particles only. Every solid Particle in this Series will be attracted upwards with greater Force, than it is downwards; and every fluid one with greater Force downwards than it is upwards, as appears by bare Inspection of the Figure, where the black Lines, as in the former, express the stronger Attractions, and the pricked ones the weaker. And, if we suppose the Number of Particles in the Solid and in the Fluid to be nearly equal, those of the Fluid will not stop, till they have quite passed through the Solid; for they will always find a Series wholly consisting of solid Particles before them, whilst that which they leave behind, will be a Mixture of both. In like manner, the solid ones will pass quite through the fluid ones; for they will always meet with more fluid ones before them, than they leave behind within the Sphere of their own Attraction.

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Quantities be very unequal, till they are diffused uniformly one among another, as we may very easily conceive; for till then there will always be some Particles attracted with greater Degree of Force one Way than they are another.

And if more of the Solid be added to this Fluid, the Particles of the Fluid will also enter into that Solid, till each is surrounded on all Sides with solid Particles, as far as its attractive Force reaches. After which the Fluid will (as they say) be *saturated*, and will dissolve no more.

Again, if more of the Fluid be poured upon that Solid, the solid Particles will diffuse themselves farther into the Fluid, till each of them is encompassed with Particles of the Fluid, as far as its attractive Force extends; and then they will spread themselves no farther.

But in either Case, if another Solid, or Fluid, the attractive Force of whose Particles differ from those of the former, be added, a fresh Fermentation will begin, provided the attractive Forces between the Particles of the former Mixture, and of those which are now added, have such a Relation to each other, as is necessary to produce it.

Upon this Principle it should seem, that a Fluid should always be capable of dissolving more than an equal Quantity of a Solid; and that a Solid should be capable of entering in  
and

Diff. VIII. *Of Fermentation.* 175

and diffusing itself through more than an equal Quantity of Fluid. The Reason why it is frequently not so is, because it commonly happens, that the Fluid and the Solid are not of equal specific Gravities. When the Solid is heaviest, so many of its Particles will not ascend and enter into the Fluid, as would otherwise have done; and on the contrary, when the Fluid is heaviest, the Weight of its Particles will be an Impediment to their rising into, and dissolving so much of the Solid as it otherwise might have done.

We have no Occasion to distinguish Fermentation into two Kinds, with regard to its Causes; for, according to the foregoing Theory, whenever two Fluids, or a Solid and a Fluid, are put together, if the Particles of the one attract those of the other, with greater Force than either those of the one, or those of the other attract themselves, a Fermentation will *equally* ensue, the Cause being the same in both Cases.

When two Fluids, or a Solid and a Fluid, ferment with each other, if the Agitation and *intestine* Motion of their Particles be very great, or continues a long Time, and if the Substance of them be of the inflammable Kind, they will, by continually rubbing one against another, be sufficiently heated to take Fire, and burst out into Flame; as was said of the several Compositions mentioned in the foregoing Dissertations.

See

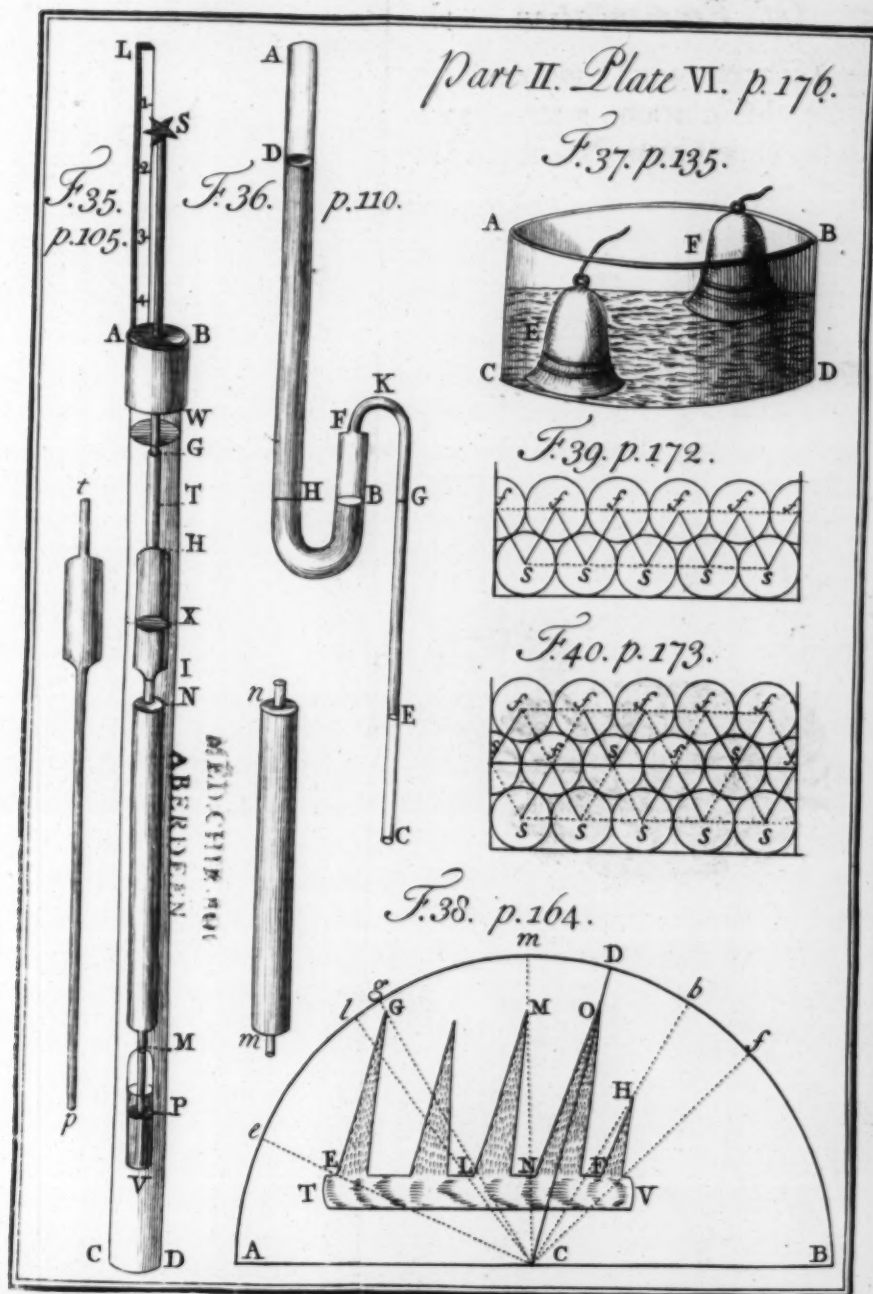
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See the Authors, who have explained and defended the old Solution, referred to by Mr. *Johnson* in his *Quæstiones Philosoph.* Cap. III. Quæst. 5, 6, 7.

*The End of the Second Part.*



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PART. III.  
OPTICS.

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CHAP. I.

*Of the Nature and Propagation of Light.*

**I**N treating of the Nature of Fluids, I have explained such *Phænomena* as result from small Particles of Matter collected together, and acted upon according to the Laws of *Mechanism*; the Order of my Design now brings me to shew how according to the same Laws such *Phænomena*, as result from the Emission of infinitely small Particles from luminous Bodies, are produced; which *Phænomena*, being the Means, whereby the Images of external Objects are represented to our Minds, by the Intervention of our Organs of  
A Sight,



*Sight*, are for that Reason called *Optical*, and the Doctrine, by which they are explained, the Science of *Optics* \*.

Every visible Body emits or reflects inconceivably small Particles of Matter from each Point of its Surface, which issue from it continually, (not unlike Sparks from a Coal) in straight Lines and in all Directions. These Particles entering the Eye, and striking upon the *Retina* (a Nerve expanded on the back Part of the Eye to receive their Impulses) excite in our Minds the Idea of Light. And as they differ in Substance, Density, Velocity, or Magnitude †, they produce in us the Ideas of different Colours; as will be explained in its proper Place.

That

\* Optics is generally divided into two Parts, *viz.* *Dioptrics*, under which is comprehended every Thing that relates to the Appearances of Bodies seen through transparent Substances; and *Catoptrics*, or what relates to the seeing of Bodies by reflected Light. To these we may add a third, which properly comes under neither of the former Distinctions, and that is, *the Doctrine of Colours*, which explains every Thing that relates to the Causes of the Diversity of Colours observable in natural Bodies.

† It is more probable, that they differ either in Magnitude, or Density, than in Velocity or Substance. For, if the Difference of Colours arise from the different Velocity of the Rays of Light, then the Colours of Objects would appear changed to an Eye placed under Water, or within any *Medium* differing from the Air in Density: For when a Ray of Light passes out of a *Medium* into another of different Density, it undergoes an Alteration in its Velocity, as will be explained hereafter. And to suppose them to differ in Substance, is contrary to that Uniformity of Things, which is observable in the Universe; as well as repug-

nant

## Chap. I. *Propagation of Light.*

5

That the Particles, which constitute Light, are exceedingly small, appears from hence, *viz.* that if a Hole be made through a Piece of Paper with a Needle, Rays of Light from every Object on the farther Side of it, are capable of passing through it at once without the least Confusion; for any one of those Objects may as clearly be seen through it as if no Rays passed through it from any of the rest. Further, if a Candle is lighted, and there be no Obstacle in the Way to obstruct the Progress of its Rays, it will fill all the Space within two Miles of it every Way with luminous Particles, before it has lost the least sensible Part of its Substance thereby.

That these Particles proceed from every Point of the Surface of a visible Body, and in all Directions, is clear from hence, *viz.* be-

nant to that *Homogeneity* in the 'primogeneal Parts of Matter, which from the Experiments hitherto made, is thought to exist every where. Whereas, if we suppose them to differ either in Magnitude or Density, nothing is more easy than to see how those of the same Kind should, however refracted, produce the same Colours; and also how those which produce different Colours, should suffer different Degrees of Refraction in passing through the same *Medium*. As to the first, it is self-evident, because Refraction cannot alter their Magnitudes or Densities; as to the second, it is probable, that the more intense and stronger Colours, the Rays of which suffer the least Refraction, are produced by the larger, or more dense Particles of Light: For, that such Particles should be less refracted than others, is quite consonant to the Laws of Attraction of Cohesion, which, as it acts in Proportion to the Surfaces of Bodies only, must necessarily affect the larger, or the more dense Particles, less than it does the rest; because such have larger *Momenta* or Forces in Proportion to their Surfaces, than others have.

6 *Of the Nature and* Part III.

cause where-ever a Spectator is placed with Regard to the Body, every Point of that Part of the Surface which is turned towards him, is visible to him. That they proceed from the Body in right Lines, we are assured, because just so many and no more will be intercepted in their Passage to any Place, by an interposed Object, as that Object ought to intercept, supposing them to come in such Lines.

The Velocity, with which they proceed from the Surface of the visible Body is no less surprizing, than their Minuteness: The Method whereby Philosophers estimate their Swift-ness, is by Observations made on the Eclipses of *Jupiter's* Satellites, which Eclipses to us appear about seven Minutes sooner than they ought to do by Calculation, when the Earth is placed between the Sun and him; that is, when we are nearest him, and as much later, when the Sun is between him and us, at which Time we are farthest from him; from whence it is concluded, that they require about seven Minutes to pass over a Space equal to the Distance between the Sun and us, which is about eighty one Millions of Miles \*.

A

\* This affords us another Proof of the surprizing Fineness of the Particles of Light; for the above-mentioned Velocity of the Rays is considerably more than a Million of Times greater than that of a Canon Ball. Were they not therefore inconceivably small, the Eye would be rather wounded than delighted with them;

# III. Chap. I. *Propagation of Light.*

7

A Stream of these Particles issuing from the Surface of a visible Body in one and the same Direction, is called a *Ray of Light*.

As Rays proceed from a visible Body in all Directions, they necessarily become thinner and thinner, continually spreading themselves, as they pass along, into a larger Space, and that in Proportion to the Squares of their Distances from the Body \*; that is, at the Distance of two Spaces, they are four Times thinner, than they are at one; at the Distance of three Spaces, nine Times thinner, and so on: the Reason of which is, because they spread themselves in a twofold Manner, *viz.* upwards and downwards, as well as Side-Ways.

them; and the tender Flowers of Plants would be so far from being cherished by them, that they would be torn in Pieces, and not able to stand before them.

\* This Proposition is demonstrated mathematically thus; let us conceive two concentric Surfaces ABD, and EFG (*Fig. 1.*) and in these, two similar Portions ELFI, and AHBK; let the Rays CE and CF, with the rest proceeding from the Center C, fall upon the Portion ELFI and cover it; it is evident from Inspection of the Figure, that the same Rays at the Distance CH will cover the Portion AHBK only; now these Rays being the same in Number at each Place, will be thinner in the former, than they are in the latter, in Proportion as that is larger than this; but these Spaces being similar Portions of the Surfaces of Spheres, bear the same Proportion to each other, that the Surfaces themselves do, that is, they are to each other as the Squares of their Radii CL, CH; the Rays therefore are more diffused, or thinner in Proportion to the Squares of the same Radii, or of their Distances from the luminous Point C.  
Q. E. D.

The

8 *Of the Nature, &c.* Part III

The Particles of Light are subject to the Laws of Attraction of Cohesion like other small Bodies, for if a Ray of Light be made to pass by the Edge of a Knife, it will be diverted from its natural Course, and be inflected towards the Edge of the Knife. The like Inflection happens to a Ray when it enters obliquely into a denser or rarer Substance than that in which it was before, in which Case it is said to be *refracted*; the Laws of which Refraction are the Subject of the following Chapter \*.

\* The *Cartesian* Notion of Light, was not, that it is propagated from luminous Bodies by the Emission of small Particles, but that it was communicated to the Organ of Sight by their Pressure upon the *Materia subtilis*, with which they supposed the Universe to be full. But according to this *Hypothesis*, it could never be dark; because when a Fluid sustains any Pressure, if that Fluid fills all the Space it takes up, absolutely, without leaving any Pores, which is the Case of the supposed *Materia subtilis*; then that Pressure must necessarily be communicated *equally* and *instantaneously* to every Part: And therefore, whether the Sun were above or below the Horizon, the Pressure communicated, and consequently the Light, would be the same. And farther, as the Pressure would be instantaneous, so would the Light, which is contrary to what is collected, as we observed above, from the Eclipses of *Jupiter's* Satellites.

Chap. II. *The Cause of Refraction, &c.* 9

C H A P. II.

*Of the Cause of Refraction, and the Law by which it is performed.*

**W**HATEVER Substance a Ray of Light passes through, or if it pass through a Space void of all Substance, it is said by Philosophers to pass through a *Medium*; and therefore if it passes out of any Substance, as Air or Glass, into a *Vacuum*, or the contrary, it is said to pass out of one *Medium* into another.

All Bodies being endued with an attractive Force, which is extended to some Distance beyond their Surfaces; when a Ray of Light passes out of a rarer into a denser *Medium* (if this latter has a greater attractive Force than the former, as is commonly the Case \*, and what we shall hereafter always suppose, unless it be mentioned to the contrary) the Ray just before its Entrance, will begin to be attracted towards the denser *Medium*, and this Attraction will continue to act upon it, till some Time after it has entered the *Medium*, as we shall shew by and by; and therefore if a Ray approaches a denser *Medium* in a Direction perpendicular to its Surface, its Velocity will

\* In oily and inflammable Bodies it happens otherwise; for they are observed to attract more strongly than others of greater Density.



10 *The Cause of Refraction, &c.* Part III.

be continually accelerated during its Passage through the Space in which that Attraction exerts itself; and therefore, after it has passed that Space, it will move on, till it arrives at the opposite Side of the *Medium*, with a greater Degree of Velocity than it had before it entered. So that in this Case its Velocity only will be altered. Whereas, if a Ray enters a denser *Medium* obliquely, it will not only have its Velocity augmented thereby, but its Direction will become less oblique to the Surface. Just as when a Stone is thrown downwards obliquely from a Precipice, it falls to the Surface of the Ground in a Direction nearer to a perpendicular one, than that with which it was thrown from the Hand. From hence we say a Ray of Light in passing out of a rarer into a denser *Medium*, is refracted *towards the Perpendicular*; that is, supposing a Line drawn perpendicularly to the Surface of the *Medium*, through the Point where the Ray enters, and extended both Ways, the Ray in passing through the Surface is refracted or bent towards the perpendicular Line; or, which is the same Thing, the Line which it describes by its Motion after it has passed through the Surface, makes a less Angle with the Perpendicular, than the Line it described before. All which may be illustrated in the following Manner.

Let

## Chap. 2. *The Cause of Refraction; &c.* 11

Let us suppose first, that the Ray passes out of a *Vacuum* into the denser *Medium* *A* *BCD*, (*Fig. 2.*) and that the attractive Force of each Particle in the *Medium* is extended from its respective Center to a Distance equal to that which is between the Lines *AB* and *EF*, or *AB* and *GH*; and let *KL* be the Path described by a Ray of Light in its Progress towards the denser *Medium*. This Ray when it arrives at *L* will enter the attractive Forces of those Particles which lie in *AB* the Surface of the denser *Medium*, and will therefore cease to proceed any longer in the right Line *KLM*, but will be diverted from its Course by being attracted towards the Line *AB*, and will begin to describe the Curve *LN*, passing through the Surface *AB* in some new Direction as *OQ*, thereby making a less Angle with a Line as *PR* drawn perpendicularly through the Point *N*, than it would have done, had it proceeded in its first Direction *KLM*.

Farther, whereas we have supposed the attractive Force of each Particle to be extended through a Space equal to the Distance between *AB* and *EF*, it is evident, the Ray after it has entered the Surface, will still be attracted downwards, till it has arrived at the Line *EF*; for till that Time, there will not be so many Particles above it which will attract it upwards, as below, that will attract it downwards. So that after it has entered

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12 *The Cause of Refraction, &c.* Part III.

tered the Surface at N, in the Direction OQ, it will not proceed in that Direction, but will continue to describe a Curve, as NS, after which it will proceed strait on towards the opposite Side of the *Medium*, being attracted equally every Way; and therefore will at last proceed in the Direction XST still nearer the Perpendicular PR than before.

Now if we suppose the Space ABYZ not to be a *Vacuum*, but a rarer *Medium* than the other, the Case will still be the same; but the Ray will not be so much refracted from its rectilineal Course, because the Attraction of the Particles of the upper *Medium* being in a contrary Direction to that of the Attraction of those in the lower one, the Attraction of the denser *Medium* will in some Measure be destroyed by that of the rarer.

On the contrary, when a Ray passes out of a denser into a rarer *Medium*, if its Direction be perpendicular to the Surface of the *Medium*, it will only lose somewhat of its Velocity, in passing through the Space of Attraction of that *Medium* (that is, the Space wherein it is attracted more one Way than it is another.) If its Direction be oblique, it will continually recede from the Perpendicular during its Passage, and by that Means have its Obliquity encreased, just as a Stone thrown up obliquely from the Surface of the Earth increases its Obliquity all the time it rises. Thus, supposing the Ray TS passing out of the denser

III. Chap. 2. *The Cause of Refraction, &c.* 13

denser *Medium* ABCD into the rarer ABYZ, when it arrives at S it will begin to be attracted downwards, and so will describe the Curve SNL, and then proceed in the right Line LK, making a larger Angle with the Perpendicular PR, than the Line TSX in which it proceeded during its Passage through the other *Medium*.

The Space through which the Attraction of Cohesion of the Particles of Matter is extended is so very small, that in considering the Progress of a Ray of Light out of one *Medium* into another, the Curvature it describes in passing through the Space of Attraction is generally neglected; and its Path is supposed to be bent, or in the usual Terms, the Ray is supposed to be refracted only in the Point where it enters the denser *Medium*.

Now the Line, which a Ray describes before it enters a denser or a rarer *Medium* is called the *Incident Ray*; that which it describes after it has entered, is the *Refracted Ray*.

The Angle comprehended between the Incident Ray and the Perpendicular, is the *Angle of Incidence*; and that between the refracted Ray and the Perpendicular, is the *Angle of Refraction*.

There is a certain and immutable Law or Rule, by which Refraction is always performed; and that is this: Whatever Inclination a Ray of Light has to the Surface of any *Medi-*

# 14 The Cause of Refraction, &c. Part III.

um before it enters it, the Degree of Refraction will always be such, that the Proportion between the Sine of the Angle of its Incidence, and that of the Angle of its Refraction, will always be the same in that Medium \*.

To

\* *Lemma.* If from a Point as M (Fig. 4.) taken any where without the Circle PNQ, a Line as MP be drawn passing through L the Center of the Circle, and terminated in the Circumference at P, the Product of MQ multiplied by MP is equal to the Difference between the Squares of ML and PL.

*Demonstration of the Lemma.* Call MQ,  $a$ ; and the Radius of the Circle LQ or LP,  $b$ ; then will the Diameter QP be expressible by  $2b$ , and the whole Line MP, by  $a+2b$ ; then multiplying MQ by MP, that is,  $a$  by  $a+2b$ , we have for the Product of this,  $aa+2ab$ . Now the Square of the Line ML, which is expressible by  $a+b$ , is  $aa+2ab+bb$ , and the Square of PL is  $bb$ ; but the Difference between these Squares, viz.  $aa+2ab+bb$  and  $bb$  is evidently  $aa+2ab$ ; and therefore the Product of MQ multiplied by MP is equal to the Difference between the Squares of ML and PL. Q. E. D.

*Demonstration of the Proposition.* When a Ray of Light passes through the Space of Attraction of any Medium, it is evident that its Motion will be subject to the like Laws with that of *Projectiles*, provided we suppose it to be acted upon with an equal Degree of Force during its whole Passage through that Space, as is commonly supposed to be the Case in *Projectiles* to whatever Height they are thrown from the Earth. We will therefore put a Case as nearly parallel as may be to that which was demonstrated of *Projectiles* in the seventh Chapter of the first Part; and suppose first, that the Force of Attraction of the denser Medium is at all Distances the same as far as it reaches, and that the Ray proceeds out of a denser into a rarer Medium; in which Case it will be attracted back towards the denser Medium, during its Passage through the Space of Attraction, in like Manner as a *Projectile* thrown upwards is while it rises from the Earth. Let then ABCD (Fig. 4.) represent the denser Medium, and ABCE the Space of Attraction; and let GH be a Ray about to enter the Force of Attraction at H, and let GH be produced to M. Now it is evident, that in this Supposition, the Ray when at H, is in the same Circumstances with a *Projectile*

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To illustrate this, Let us suppose ABCD (Fig. 3.) to represent a rarer, and AB EF a denser *Medium*; let GH be a Ray of Light passing

about to be thrown upwards from H towards M, it will therefore describe a Portion of a *Parabola* as HI; to which the Line HM will be a Tangent at H; and the Line IK, in which it would proceed after it has passed the Space of Attraction, a Tangent to it at I, for after having left the attractive Force at I, it goes straight on in its last Direction. Let the Perpendicular IR be drawn meeting GH produced in M, and let KI be produced to L. On the Center L with the Radius LI, describe the Circle PNQ, let fall the Perpendicular LO upon MR, and join the Points L and N. Now it is demonstrated in the Case of Projectiles, that the *Parameter* of the Point H is equal

to  $\frac{HM^2}{MI}$ , and therefore the *Parameter* multiplied by MI is equal to  $HM^2$ . And it is there farther demonstrated, that the said *Parameter* is equal to four times the Height which a Body must fall from, to acquire the Velocity the Projectile has at H; this *Parameter* therefore is a Quantity not at all depending on the Direction of the Projectile, but on its Velocity only; and consequently in the present Supposition it is a given Quantity, the Ray GH being supposed to have the same Velocity, whatever is its Inclination to the Surface AB. Now the Tangent KI being produced to L, will by the Property of the *Parabola*, bisect the other Tangent HM, wherefore the Line LO being parallel to HR, MR will also be bisected in O; and adding the equal Lines OI and ON to each Part, MN will be equal to IR; but the Line IR is also a Line independent of the Inclination of the Ray GH, its Length being determined by the Breadth of the Space of Attraction AB EF only, and therefore MN is a given Quantity. Now, whereas MI, when multiplied by the *Parameter* of the Point H, which before was shewn to be a given Line, is equal to the Square of HM, therefore the same Line MI when multiplied by any other given Line (*viz.* MN) if it is not equal to, will nevertheless bear a given Proportion to the Square of HM: But since MI multiplied by MN bears a given Proportion (*viz.* a Proportion that does not depend on the Inclination of the Ray GH) to the Square of MH, its equal, *viz.* the Product of MQ multiplied by MP (37. El. 3.) or what is equal to this, the Difference between the Squares of

ML



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passing through the first and entering the second at H, and let HI be the refracted Ray; then supposing the Perpendicular PR drawn through the Point H, on the Center H, and with any

ML and PL (by the foregoing *Lemma*) or, which is the same Thing, of ML and LI, (because PL and LI are *Radii* of the same Circle) does so too. Now the Square of ML bears also a given Proportion to the Square of MH (ML being equal to half MH) consequently there is a given Proportion between the Square of ML and the Difference of the Squares of ML and LI; and therefore there is a certain Proportion between the Lines themselves, *viz.* between ML and LI. But in every Triangle the Sides are proportionable to the Sines of their opposite Angles, therefore in the Triangle MLI, the Sine of the Angle LMI has a given Proportion to the Sine of the Angle LIM, or of its Complement to two right ones MIK (for they have the same Sine.) But LMI being an Angle made by the incident Ray GH produced, with the Perpendicular RM, is the Angle of Incidence, and MIK being made by the refracted Ray IK, and the same Perpendicular, is the Angle of Refraction, therefore in this Case there is a constant *Ratio* between the Sine of the Angle of Incidence, and that of the Angle of Refraction.

We have here supposed that the Force of Attraction is every where uniform, but if it be otherwise, provided it be the same every where at the same Distances from the Surface AB, the Proportion between the forementioned Sines will still be a given one. For, let us imagine the Space of Attraction divided into parallel Planes, and the Attraction to be the same through the whole Breadth of each Plane though different in different Planes, the Sine of the Angle of Incidence out of each will, by what has been demonstrated above, be to the Sine of the Angle of Refraction into the next in a given *Ratio*; and therefore, since the Sine of the Angle of Refraction out of one will be the Sine of the Angle of Incidence into the next, it is evident that the Sine of the Angle of Incidence into the first will be in a given *Ratio* to the Sine of the Angle of Refraction out of the last. Now let us suppose the Thickness of these Planes diminished in *infinitum*, and their Number proportionably increased, the Law of Refraction will still continue the same; and therefore whether the Attraction be uniform or not, there will be a constant *Ratio* between the Sine of the Angle of Incidence and of Refraction. Q. E. D.

*Radius*

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*Radius* describe the Circle APBR, and from G and I where the incident and refracted Rays cut the Circle, let fall the Lines GK and IL perpendicularly upon the Line PR, the former of these will be the Sine of the Angle of Incidence, the latter of Refraction. Now if in this Case, the Ray GH is so refracted at H, that GK is double or treble, &c. of IL, then whatever other Inclination the Ray GH might have had, the Sine of its Angle of Incidence would have been double, or treble, &c. to that of its Angle of Refraction. For Instance, had the Ray passed in the Line MH before Refraction, it would have passed in some Line, as HN afterwards, so situated that MO should have been double or treble, &c. of NQ.

When a Ray passes out of a *Vacuum* into Air, the Sine of the Angle of Incidence is found to be to that of Refraction, as 100036 to 100000.

When it passes out of Air into Water, as about 4 to 3.

When out of Air into Glass, as about 17 to 11.

When out of Air into a Diamond, as about 5 to 2.

C H A P.

C H A P. III.

*Of the Refraction of Light in passing thro' plain and spherical Surfaces.*

**A**S Rays of Light are capable of having their Progress altered by Refraction or Reflection, it is possible they may acquire various Inclinations and Directions different from those which they had at their Emission from the Surfaces of visible Bodies.

When they recede from each other as they pass along, they are said to *diverge*; and the Point they proceed from, is called the *Radiant Point*.

When they proceed towards any Point approaching nearer together in their Progress, they are then said to *converge*; and the Point towards which they tend, is called the *Focus*.

This *Focus* may be either *real* or *Imaginary*; it is said to be real, when the Rays actually proceed to it; but if they are intercepted in their Progress, or turned another Way before they reach it, it is called their *imaginary Focus*.

Sometimes it happens, that Rays are so refracted or reflected, that they proceed afterwards, as from some Point, which is not their true Radiant, then also that Point is called their *imaginary Focus*.

When

*187*

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When they proceed in parallel Lines, they are then called *parallel Rays*; and both their Focus and radiant Point is supposed to be at an infinite Distance.

When Rays pass out of one *Medium* into another, they suffer various Alterations in their Motion. All which are expressed in the eighteen following Propositions.

I. When *parallel Rays* fall obliquely on a plain Surface of a *Medium* of different Density, they are *parallel* also after Refraction. For having all the same Inclination to the Surface, they suffer an equal Degree of Refraction.

II. When diverging Rays pass out of a rarer into a denser *Medium* through a plain Surface, they are made thereby to diverge less.

For being all refracted towards their respective Perpendiculars, (but those the most that are the most oblique to the Surface,) they are brought nearer to a Parallelism among themselves; that is, they are made to diverge less than before.

See this and the following Cases expressed more determinately, and demonstrated in the Note below \*.

III. When

\* I. When Rays pass out of one *Medium* into another of different Density through a plain Surface; if they diverge, the focal Distance will be to that of the radiant Point; if they converge, it will be to that of the imaginary Focus of the incident

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III. When they proceed out of a denser into a rarer *Medium*, the contrary happens; for then being refracted from their respective Perpendiculars, and those the most that are the most oblique, they are made to diverge more.

IV. So

dent Rays, as the Sine of the Angle of Incidence is to that of the Angle of Refraction.

This Proposition admits of four Cases.

*Case 1.* Of diverging Rays passing out of a rarer into a denser *Medium*.

*Dem.* Let *X* (Fig. 10.) represent a rarer, and *Z* a denser *Medium*, separated from each other by the plain Surface *AB*; suppose *CE* and *CD* to be two diverging Rays proceeding from the Point *C*, the one perpendicular to the Surface, the other oblique; through *E* draw the Perpendicular *PK*. The Ray *CD* being perpendicular to the Surface will proceed on in the right Line *CQ*, but the other falling on it obliquely at *E*, and there entering a denser *Medium*, will suffer a Refraction towards the Perpendicular *EK*. Let then *EG* be the refracted Ray, and produce it back till it intersects *DC* produced also, in *F*; this will be the focal Point. On the Center *E* and with the Radius *EF*, describe the Circle *AFBQ*, and produce *EC* to *H*; draw *HI* the Sine of the Angle of Incidence and *GK* that of Refraction; equal to this is *FP* or *CM*, which let be drawn. Now if we suppose the Points *D* and *E* contiguous, or nearly so, then will the Line *HE* be almost coincident with *FD*, and therefore *FD* will be to *CD* as *HE* to *CE*; but *HE* is to *CE* as *HI* to *CM*, because the Triangles *HIE* and *CME* are similar; that is, the focal Distance of the Ray *CE* is to the Distance of the Radiant Point, as the Sine of the Angle of Incidence is to that of the Angle of Refraction. *Q. E. D.*

*Obs. 1.* Whereas the Ratio of *IE* to *ME*, or which is the same Thing, that of *nD* to *CD* bears the exact Proportion of *HI* to *CM*, and because this, (being the Ratio of the Sine of the Angle of Incidence to that of the Angle of Refraction) is always the same, the Line *In* is in all Inclinations of the Ray *CE*, at the same Distance from *CM*; consequently had *CE* been coincident with *CD*, the Point *H* had fallen upon *n*; and because the Circle passes through both *H* and *F*, *F* would also have fallen upon *n*.

### Chap. 3. *The Cause of Refraction, &c.* 21

IV. So when converging Rays pass out of a rarer into a denser Medium, through a plain Surface, they are made thereby to converge less.

For

upon which Account the Focus of the Ray CE would have been there. But the Ray CE being oblique to the Surface DB, the Point H is at some Distance from n; and therefore the Point F is necessarily so too, and the more so by how much the greater that Distance is: from whence it is clear, that no two Rays flowing from the radiant Point C and falling with different Obliquities on the Surface BD, will after Refraction there, proceed as from the same Point; therefore strictly speaking, there is no one Point in the Line D produced, that can more properly be called the Focus of Rays flowing from C, than another: for those which enter the refracting Surface near D, will after Refraction proceed, as has been observed, from the Parts about n; those which enter near E, will flow as from the Parts about F; those which enter about T, as from some Points in the Line DF produced, &c. And it is farther to be observed, that when the Angle DCE becomes large, the Line nF increases apace; wherefore those Rays which fall near T, proceed after Refraction, as from a more diffused Space, than those which fall at the same Distance from each other near the Point D. Upon which Account it is usual with Optical Writers to suppose the Distance between the Points where the Rays enter the plain Surface of a refracting Medium, to be inconsiderable with Regard to the Distance of the radiant Point, if they diverge; or to that of their imaginary Focus, if they converge: and unless there be some particular Reason to the contrary, they consider them, as entering the refracting Medium in a Direction as nearly perpendicular to its Surfaces as may be.

Case 2. Of diverging Rays proceeding out of a denser into a rarer Medium.

Dem. Let X be the denser, Z the rarer Medium, FD and FE two diverging Rays proceeding from the Point F; and supposing the Perpendicular PK drawn as before, FP will be the Sine of the Angle of Incidence of the oblique Ray FE, which in this Case being refracted from the Perpendicular, will pass on in some Line as ER, which being produced back to the Circumference of the Circle will cut the Ray FD somewhere, suppose



## 22 *The Cause of Refraction, &c.* Part III.

For being all refracted towards their respective Perpendiculars, and those the most that are the most oblique, they themselves are brought nearer to a Parallelism, and so converge less.

V. On

pose in C, this therefore will be the imaginary Focus of the refracted Ray ER; draw RS the Sine of the Angle of Refraction, to which HI will be equal: but here also FP or its equal CM, is to HI, as EC to EH, or (if the Point D and E be considered as contiguous) as DC to DF; that is, the Sine of the Angle of Incidence is to the Sine of the Angle of Refraction, as the focal Distance to that of the radiant Point. *Q. E. D.*

*Case 3.* Of converging Rays passing out a denser Medium into a rarer.

*Dem.* Let Z be the denser, X the rarer Medium, and GE the incident Ray; this will be refracted from the Perpendicular into a Line as EH; then all Things remaining as before, GK, or its equal FP, or CM will be the Sine of the Angle of Incidence, and HI that of Refraction: but these Lines, as before, are to each other, as DC to DF; that is, the focal Distance is to the Distance of the imaginary Focus, as the Sine of the Angle of Incidence to that of the Angle of Refraction. *Q. E. D.*

*Case 4.* Of converging Rays passing out of a rarer into a denser Medium.

*Dem.* Let Z be the rarer, X the denser Medium, and RE the incident Ray; this will be refracted towards the Perpendicular into a Line, as EF; C will be the imaginary Focus, and F the real one, HI which is equal to RS, the Sine of the Angle of Incidence, and FP that of the Angle of Refraction: but these are to each other, as DF to DC; and therefore the focal Distance is to that of the imaginary Focus, as the Sine of the Angle of Incidence is to that of the Angle of Refraction. *Q. E. D.*

II. When parallel Rays fall upon a spherical Surface of different Density, the focal Distance will be to the Distance of the Center of Convexity, as the Sine of the Angle of Incidence is to the Difference between that Sine and the Sine of the Angle of Refraction.

This

### Chap. 3. *The Cause of Refraction, &c.* 23

V. On the contrary, when they proceed out of a denser into a rarer Medium, they are refracted the contrary Way, and so made to converge more.

All

This Proposition admits of four Cases.

*Case 1.* Of parallel Rays passing out of a rarer into a denser Medium through a convex Surface of the denser.

*Dem.* Let AB (Fig. 11.) represent a convex Surface, C its Center of Convexity; HA and DB two parallel Rays, passing out of the rarer Medium X into the denser Z, the one perpendicular to the refracting Surface, the other oblique: draw CB, this being a Radius, will be perpendicular to the Surface at the Point B; and the oblique Ray DB being in this Case refracted towards the Perpendicular, will proceed in some Line, as BF, meeting the other Ray in F, which will therefore be the Focal Point: produce CB to N, then will DBN, or its equal BCA be the Angle of Incidence, and FBC that of Refraction. Now, *whereas any Angle has the same Sine with its Complement to two right ones*, the Angle FCB being the Complement of ACB, which is equal to the Angle of Incidence, may here be taken for that Angle; and therefore, *as the Sides of a Triangle have the same Relation to each other, that the Sines of their opposite Angles have*, FB being opposite to this Angle, and FC being opposite to the Angle of Refraction, they may here be considered as the Sines of the Angles of Incidence and of Refraction; and for the same Reason CB may be considered as the Sine of the Angle CFB, which Angle being together with the Angle FBC, equal to the external one ACB (32. El. 1.) is itself equal to the Difference between those two last Angles; and therefore the Line FB is to CB as the Sine of the Angle of Incidence is to the Sine of an Angle which is equal to the Difference between the Angle of Incidence and of Refraction. Now, because in very small Angles as these are, *for we suppose in this Case also the Distance AB to vanish, the Reason of which will be shown by and by*, their Sines bear nearly the same Proportion to each other that they themselves do, the Distance FB will be to CB as the Sine of the Angle of Incidence is to the Difference between that Sine and the Sine of the Angle of Refraction; but *because BA vanishes*, FB and FA are equal, and therefore FA is to CA in that Proportion. Q. E. D.

Obf. 2.

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All which may be illustrated in the following Manner. 1. Let AB, CD, (Fig. 5.) be two parallel Rays falling on the plain Surface EF

Obs. 2. It appears from the foregoing Demonstration, that the focal Distance of the Oblique Ray DB, is such, that the Line BF shall be to the Line CB or CA as the Sine of the Angle of Incidence to the Sine of an Angle, which Angle is equal to the Difference between the Angle of Incidence and Refraction; therefore so long as the Angles BCA, &c. are small, so long the Line BF is pretty much of the same Length, because small Angles have nearly the same Relation to each other that their Sines have. But when the Point B is removed far from A, so that the Ray DB enters the Surface, suppose about O, the Angles BCA, &c. becoming large, the Sine of the Angle of Incidence begins to bear a considerably less Proportion to the Sine of an Angle which is equal to the Difference between the Angle of Incidence and Refraction than before, and therefore the Line BF begins to bear a much less Proportion to BC; wherefore its Length decreases apace: Upon which Account those Rays which enter the Surface about O, not only meet nearer the Center of Convexity than those which enter at A, but are collected into a more diffused Space. From hence it is, that the Point where those only which enter near A, are collected, is reckoned the true Focus; and the Distance AB in all Demonstrations relating to the Foci of parallel Rays entering a spherical Surface whether convex or concave, is supposed to vanish.

Case 2. Of parallel Rays passing out of a denser into a rarer Medium through a concave Surface of the denser.

Dem. Let X be the denser, Z the rarer Medium. AB the Surface by which they are separated, C the Center of Convexity, and HA and DB two parallel Rays, as before. Through B the Point where the oblique Ray DB, enters the rarer Medium draw the Perpendicular CN; and let the Ray DB, being in this Case refracted from the Perpendicular, proceed in the Direction BM; produce BM back to H; this will be the imaginary Focus, and DBN, or its equal ACB will be the Angle of Incidence, and CBM, or its equal HBN (for they are vertical) that of Refraction; produce DB to L and draw BF such, that the Angle LBF may be equal to DBH: then because NBD and DBH together are equal to NBH the Angle of Refraction, therefore BCA which is equal to the first, and LBF which

### Chap. 3. *The Cause of Refraction, &c.* 25

EF of a *Medium* of a different Density : Now because they both make equal Angles of Incidence with their respective Perpendiculars GH, IK,

is equal to the second, are together equal to the Angle of Refraction ; but LBF is equal to BFA (as being alternate to it) consequently BFA and BCA together are equal to the Angle of Refraction ; and therefore since one of them, *viz.* BCA is equal to the Angle of Incidence, the other is the Difference between that Angle, and the Angle of Refraction. Now FB the Sine of the Angle FCB, or which is the same Thing, of its Complement to two right ones BCA, the Angle of Incidence, is to CB the Sine of the Angle BFC, as FB to CB, that is as HB to CB ; for the Angles DBH and LBF being equal, the Lines BF and BH are so too ; but the Distance BA vanishing, HB is to CB, as HA to CA : that is, the Sine of the Angle of Incidence is to the Sine of an Angle which is the Difference between the Angle of Incidence and Refraction, or because the Angles are small, to the Difference between the Sine of the Angle of Incidence and that of Refraction, as the Distance of the Focus from the Surface is to that of the Center from the same. *Q. E. D.*

*Case 3.* Of parallel Rays passing out of a rarer into a denser *Medium* through a concave Surface.

*Dem.* Let X be the denser *Medium* having the concave Surface AB, and let LB and FA be the incident Rays. Now whereas, when DB was the incident Ray, and passed out of a rarer into a denser *Medium*, as in Case the first, it was refracted into the Line BF, this Ray LB having the same Inclination to the Perpendicular, will also suffer the same Degree of Refraction, and will therefore pass on afterwards in the Line FB produced, *v. g.* towards P. So that, whereas in that Case the Point F was the real Focus of the incident Ray DB, the same Point will in this be the imaginary Focus of the incident Ray LB : But it was there demonstrated, that the Distance FA is to CA, as the Sine of the Angle of Incidence is to the Difference between that and the Sine of the Angle of Refraction, therefore the focal Distance of the refracted Ray BP is to the Distance of the Center of Convexity in that Proportion. *Q. E. D.*

*Case 4.* Of Parallel Rays passing out of a denser into a rarer *Medium* through a convex Surface of the denser.

*Dem.* Let Z be the denser *Medium*, having the convex Surface AB, and let LB and FA be the incident Rays, as before. Now

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IK, before Refraction, they will make equal Angles of Refraction with them afterwards, and so proceed on in the parallel Lines BL DM.

Now whereas when DB was the incident Ray passing out of a denser into a rarer Medium, it was refracted into BM, as in Case the second, having a Point as H in the Line MB produced for its imaginary Focus; therefore LB, for the like Reason as was given in the last Case, will in this be refracted into BH, having the same Point H for its real Focus. So that here also the Focal Distance will be to that of the Center of Convexity, as the Sine of the Angle of Incidence is to the Difference between that and the Sine of the Angle of Refraction. *Q. E. D.*

III. When diverging or converging Rays enter into a Medium of different Density through a spherical Surface, the Ratio compounded of that which the focal Distance bears to the Distance of the Radiant Point (or of the imaginary Focus of the incident Rays, if they converge;) and of that, which the Distance between the same radiant Point (or imaginary Focus) and the Center, bears to the Distance between the Center and the Focus, is equal to the Ratio, which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction.

This Proposition admits of sixteen Cases.

Case 1. Of diverging Rays passing out of a rarer into a denser Medium, through a convex Surface of the denser, with such a degree of Divergency, that they shall converge after Refraction.

*Dem.* Let BD (Fig. 12.) represent a spherical Surface, C its Center of convexity, and let there be two diverging Rays AB and AD, proceeding from the radiant Point A, the one perpendicular to the Surface, the other oblique. Though the Center C produce the perpendicular Ray AD to F, and draw the Radius CB and produce it to K, and let BF be the refracted Ray; then will F be the focal Point; produce AB to H, and through the Point F draw the Line FG parallel to CB. AB being the incident Ray, and CK perpendicular to the Surface at the Point B, the Angle ABK, or which is equal to it, because of the parallel Lines CB and FG, FGH is the Angle of Incidence. Now whereas the Complement of any Angle to two right ones has the same Sine with the Angle itself, the Sine of the Angle FGB, that being the Complement of FGH to two right ones, may be considered as the Sine of

### Chap. 3. The Refraction of Light, &c. 27

DM. 2. Let the diverging Rays AB, AE, AF, (Fig. 6.) pass out of a rarer into a denser Medium, through the plain Surface GH, and

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the Angle of Incidence ; which Sine the Line FB, as the Sides of a Triangle have the same Relation to each other, that the Sines of their opposite Angles have, may be taken for. Again, the Angle FBC is the Angle of Refraction, or its equal, because alternate to it, BFG, to which BG being an opposite Side, may be looked upon as the Sine. But FB is to BG in a Ratio compounded of FB to BA, and of BA to BG, for the Ratio that any two Quantities bear to each other, is compounded of the Ratio, which the first bears to any other, and of the Ratio which that other bears to the second. Now FB is to BA, supposing BD to vanish, as FD to DA ; and BA is to BG, because of the parallel Lines CB and FG, as AC to CF. That is, the Ratio compounded of FD, the focal Distance, to DA, the Distance of the radiant Point, and of AC, the Distance between the radiant Point and the Center, to CF, the Distance between the Center and the Focus, is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. Q. E. D.

Obs. 3. Whereas the focal Distance of the oblique Ray AB is such, that the compound Ratio of FB to BA and of AC to CF shall be the same, whatever be the Distance between B and D ; it is evident, that since AC is always of the same Length, the more the Line AB lengthens, the more FB must lengthen too, or else FC must shorten ; but it appears by Inspection of the Figure, that if BF lengthens, CF will do so too, and in a greater Proportion with respect to its own Length than BF will, therefore the Lengthening of BF will conduce nothing towards preserving the Equality of the Proportion : but as AB lengthens, BF and CF must both shorten, which is the only possible Way wherein the Proportion may be continued the same. And it is also apparent, that the farther B moves from D towards O, the faster AB lengthens, and therefore the farther the Rays enter from D, the nearer to the refracting Surface is the Place where they meet, but the Space they are collected in, is the more diffused : and therefore in this Case, as well as those taken Notice of in the two foregoing Observations, different Rays, though flowing from the same Point, shall constitute different Focus's ; and are so effectual as those which enter at or very near the

D

Points



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and let the Ray AB be perpendicular to that Surface; the rest being refracted towards their respective Perpendiculars EK, FM, and those the most that fall the farthest from B, they will

pro-

*Point D.* And since the same is observable of converging as well as of diverging Rays, none except those which enter very near that Point, are usually taken into Consideration; upon which Account it is, that the Distance DB, in determining the focal Distances of diverging or converging Rays entering a convex or concave Surface, is supposed to vanish.

Those who would see a Method of determining the precise Point which the Ray AB, whether it be parallel, converging, or diverging to the Ray AF, converges to or diverges from after Refraction at B or any other given Point in the Surface DO, may find it in the Appendix to Molineux's Optics, which for the Sake of those who have not that Book, I shall subjoin at the End of this Note.

*Case 2.* Of converging Rays passing out of a rarer into a denser Medium through a concave Surface of the denser with such a Degree of Convergency, that they shall diverge after Refraction.

*Dem.* Let the incident Rays be HB and FD passing out of a rarer into a denser Medium through the concave Surface BD, and tending towards the Point A, from whence the diverging Rays flowed in the other Case; then the oblique Ray HB having its Angle of Incidence HBC equal to ABK the Angle of Incidence in the former Case, will be refracted into the Line BL such, that its refracted Angle KBL will be equal to FBC the Angle of Refraction in the former Case; that is, it will proceed after Refraction in the Line FB produced, having the same focal Distance FD with the diverging Rays AB, AD, in the other Case. But, by what has been already demonstrated, the Ratio compounded of FD, the focal Distance, to DA, in this Case, the Distance of the imaginary Focus of the incident Rays, and of AC, the Distance between the same imaginary Focus and the Center, to CF, the Distance between the Center and the Focus, is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. Q. E. D.

*Case 3.* Of diverging Rays passing out of a rarer into a denser Medium through a convex Surface of the denser, with such a Degree of Divergency as to continue diverging.

*Dem.*

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proceed in the Directions EN and FO, diverging in a less Degree from the Ray AP, than they did before Refraction. 3. Had they proceeded

*Dem.* Let AB, AD (*Fig. 13.*) be the diverging Rays, and let their Divergency be so great, that the refracted Ray BL shall also diverge from the other; produce LB back to F which will be the focal Point; draw the Radius CB and produce it to K, produce BA likewise towards G, and draw FG parallel to BC. Then will ABK be the Angle of Incidence, whose Sine BF may be taken for, as being opposite to the Angle BGF, which is the Complement of the other to two right ones. And LBC is the Angle of Refraction, or its equal KBF, or which is equal to this, BFG, as being alternate; therefore BG the opposite Side to this may be taken for the Sine of the Angle of Refraction. But BF is to BG, for the like Reason as was given in Case the first, in a *Ratio* compounded of BF to BA, and of BA to BG. Now BF is to BA, (DB vanishing) as DF to DA, and because of the parallel Lines FG and BC, the Triangles CBA and AGF are similar; therefore BA is to AG as CA to AF, consequently BA is to BA together with AG, that is, to BG, as CA is to CA together with AF, that is, CF. Therefore the *Ratio* compounded of DF the focal Distance to DA the Distance of the radiant Point, and of CA the Distance between the radiant Point and the Center, to CF the Distance between the Center and the Focus, is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

*Case 4.* Of converging Rays passing out of a rarer into a denser Medium through a concave Surface of the denser in such Manner that they shall continue converging.

*Dem.* Let HB and CD be the incident Rays passing out of the rarer into the denser Medium through the concave Surface BD, and tending towards A the same Point from whence the diverging Rays flowed in the last Case. Then because the Ray HB has the same Inclination to the Perpendicular CK that AB had before, it will suffer the same Degree of Refraction, and pass on in the Line LB produced, having its Focus F at the same Distance from the refracting Surface as that of the diverging Ray AB in the other Case. Therefore, *&c. Q. E. D.*

*Case 5.* Of diverging Rays passing out of a denser into a rarer Medium through a concave Surface of the denser.

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ceeded out of a denser into a rarer *Medium*, they would have been refracted from their Perpendiculars EK, FM, and those the most which were the most oblique, and therefore would have

*Dem.* Let AB, AD (*Fig. 14.*) be the incident Rays passing out of a denser into a rarer *Medium* through the concave Surface BD, whose Center is C; and let BL be the refracted Ray, which produce back to F, and draw FG parallel to CB. Here ABK is the Angle of Incidence, to which its alternate one FGB being equal, FB the opposite Side may be considered as the Sine of it. The Angle of Refraction is LBC or FBK, of which BFG being the Complement to right ones, BG the opposite Side may be looked upon as its Sine. But BF is to BG, in the compound *Ratio* of BF to BA and of BA to BG for the Reason given above. Now (BD vanishing) BF is to BA as DF to DA, and BA is to BG as CA to CF. That is, the *Ratio* compounded of the focal Distance to the Distance of the radiant Point, &c. *Q. E. D.*

*Case 6.* Of converging Rays passing out of a denser into a rarer *Medium* through a convex Surface of the denser.

*Dem.* Let HB, CD, be the incident Rays tending towards the Point A which was the Radiant in the last Case. Then, for the Reason already given, the oblique Ray will suffer such a Degree of Refraction, as to have its Focus F at the same Distance from the Surface, as the diverging Rays AB, AD had in that Case. Therefore, &c. *Q. E. D.*

When the *Mediums* through which Rays pass, and the refracting Surfaces are such, that Rays flowing from A (*Fig. 12.*) are collected in F, then Rays flowing from F through the same *Mediums* the contrary Way, will be collected in A. For when Rays pass out of one *Medium* into another, the Sine of the Angle of Incidence bears the same Proportion to the Sine of the Angle of Refraction, as the Sine of the Angle of Refraction does to the Sine of the Angle of Incidence, when they pass the contrary Way. This is applicable to each of the six following Cases compared respectively with the six foregoing; therefore they may be considered as the Converse of them; or they may be demonstrated independently of them, as follows.

*Case 7.* Of diverging Rays passing out of a denser into a rarer *Medium* through a convex Surface of the denser, so as to converge afterwards.

*Dem.*

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have diverged more than before. 4. Let the converging Rays AB, CD, EF (*Fig. 7.*) pass out of a rarer into a denser *Medium*, through the

*Dem.* Let AB, AD (*Fig. 15*) be two diverging Rays passing through the convex Surface BD into a rarer *Medium*. Let C be the Center of Convexity, and BF the refracted Ray. Draw CB and produce it to K, and draw FG parallel to it meeting AB produced in G. Then will ABC be the Angle of Incidence, of which FB being opposite to its alternate and equal Angle FGB, may be considered as the Sine. The Angle of Refraction is FBK, of which GB being opposite to its Complement to two right ones GFB, may be taken for the Sine. Now FB is to BG, in a *Ratio* compounded of FB to BA, and of BA to BG. But (BD vanishing) FB is to BA as FD to DA, and because of the parallel Lines CB and FG, BA is to BG as CA to CF. Therefore the focal Distance, &c. *Q. E. D.*

*Case 8.* Of converging Rays passing out of a denser into a rarer *Medium* through a concave Surface of the denser so as to diverge afterwards.

*Dem.* Let GB and FD be the incident Rays tending towards A, and produce FB to L. Then as AB in the last Case was refracted into BF, GB will in this be refracted into BL, for the Reasons already given, having F for its focal Point. Therefore, &c. *Q. E. D.*

*Case 9.* Of diverging Rays passing out of a denser into a rarer *Medium* through a convex Surface of the denser, in such Manner as to continue diverging.

*Dem.* Let AB, AD (*Fig. 16.*) be two Rays passing out of a denser into a rarer *Medium*, through the convex Surface DB, whose Center of Convexity is C. Draw CB, produce it to K, and let BL be the refracted Ray, produce BL back to F, and draw FG parallel to CB meeting BA produced in G. Then will ABC be the Angle of Incidence, of which FB being opposite to its alternate and equal Angle FGB, may be considered as the Sine. The refracted Angle is LBK, or its equal CBF, of which BG being opposite to its Complement to two right ones BFG, is the Sine. Now BF is to BG in the compound *Ratio* of BF to BA and of BA to BG: but BF is to BA as DF to DA; and because of the parallel Lines CB and FG, the Triangles BCA, AGF are similar, therefore BA is to AG as CA to AF, and consequently BA is to BG as CA to CF. Therefore, &c. *Q. E. D.*

*Case*

*Dem.*

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the plain Surface GH, and let the Ray AB be perpendicular to that Surface, then the other Rays being refracted towards their respective

*Case 10.* Of converging Rays passing out of a denser into a rarer *Medium* through a concave Surface of the denser, in such Manner as to continue converging.

*Dem.* Let HB, MD be the incident Rays tending towards the Point A. Then will the oblique Ray HB for the Reasons already given be refracted into BF. Therefore, &c. *Q. E. D.*

*Case 11.* Of diverging Rays passing out of a rarer into a denser *Medium* through a concave Surface of the denser.

*Dem.* Let AB, AD (*Fig. 17.*) be the incident Rays passing out of a rarer into a denser *Medium*, through the concave Surface BD, whose Center of Convexity is C, and supposing the Line CB drawn and produced to K, the refracted Ray BL drawn and produced back to F, and also FG drawn parallel to CB, ABC will be the Angle of Incidence, of which FB being opposite to its Complement to two right ones BGF, is the Sine. The Angle of Refraction will be LBK or its equal FBC, of which BG being opposite to its equal and alternate one BFG, is the Sine. Now FB is to BG in the compound *Ratio* of FB to BA and of BA to BG. But (BD vanishing) FB is to BA as FD to DA, and because of the parallel Lines FG and CB, BA is to BG as CA to CF. Therefore, &c. *Q. E. D.*

*Case 12.* Of converging Rays passing out of a rarer into a denser *Medium* through a convex Surface of the denser.

*Dem.* Let HB, MD be the incident Rays tending towards A the radiant Point in the last Case; then, as was explained above, BF will be the refracted Ray. Therefore, &c. *Q. E. D.*

*Case 13.* Of Rays passing out of a rarer into a denser *Medium* from a Point between the Center of Convexity and the Surface.

*Dem.* Let AB, AD (*Fig. 18.*) be two Rays passing out of a rarer into a denser *Medium* from the Point A, which let be posited between C the Center of Convexity and the refracting Surface BD; through B draw CK, and let BL be the refracted Ray; produce BL back to F and draw FG parallel to BC. Then will ABC be the Angle of Incidence, of which BF being opposite to its Complement to two right ones BGF, is the Sine. LBK will be the Angle of Refraction, or its equal FBC, of which BG being opposite to its alternate and equal one BFG,



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ive Perpendiculars DK, FM, and EF for Instance more than CD, they will proceed in the Directions DN, FN converging in a less Degree towards the Ray AN, than they did

is the Sine. But, as before, BF is to BG in a compound Ratio of BF to BA and of BA to BG ; and (BD vanishing) BF is to BA as DF to DA, and because the Lines CB and FG are parallel, BA is to BG as CA to CF. Therefore, &c. *Q. E. D.*

*Case 14.* Of Rays passing out of a rarer into a denser Medium towards a Point between the Center of Convexity and the Surface.

*Dem.* Let the incident Rays be MD, HB, tending towards A from whence the other proceeded in the last Case. Then as in that Case the refracted Ray BL being produced back passed through F, in this the refracted Ray itself, for the like Reasons as were given in the foregoing Cases, will pass through that Point. Therefore, &c. *Q. E. D.*

*Case 15.* Of Rays passing out of a denser into a rarer Medium from a Point between the Center of Convexity and the Surface.

*Dem.* Let AB, AD (Fig. 19.) be two diverging Rays passing out of a denser into a rarer Medium through the refracting Surface BD, whose Center of Convexity is C, a Point beyond that from whence the Rays flow. Through B draw CK, and let BL be the refracted Ray, produce it back to F, and draw FG parallel to BC meeting BA produced in G. ABC will be the Angle of Incidence, of which BF being opposite to its alternate and equal Angle BGF, is the Sine. The Angle of Refraction is LBK or its equal FBC, of which BG being opposite to its Complement to two right ones BFG, is the Sine. But BF is to BG in the compound Ratio of BF to BA and of BA to BG ; and (BD vanishing) BF is to BA as DF to DA, and because of the parallel Lines CB and GF, the Triangles AFG and ABC are similar. BA therefore is to AG as CA to AF, consequently BA is to BA and AG together, that is, to BG, as CA is to CA and AF together, that is, to CF ; and therefore the focal Distance, &c. *Q. E. D.*

*Case 16.* Of Rays passing out of a denser into a rarer Medium towards a Point between the Center of Convexity and the Surface.

*Dem.*



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did before. 5. Lastly, had the first Medium been the denser, they would have been refracted the other Way, and therefore have converged more.

#### VI. When

*Dem.* Let HB, MD be the incident Rays, having for their imaginary Focus the Point A which was the Radiant in the last Case, and let C the Center of Convexity of the refracting Surface be posited beyond this Point. Then will HB, for the Reasons already given, be refracted into BF, having the Point F for its real Focus which was the imaginary one of the diverging Rays AB, AD in the former Case. Therefore as before the Ratio compounded of that which the focal Distance bears to the Distance of the imaginary Focus of the incident Rays, and of that which the Distance between the same imaginary Focus and the Center, bears to the Distance between the Center and the Focus, is equal to the Ratio which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

The first Term in the foregoing Proportion (*viz.* that in Proposition the 3d of this Note) being always an unknown Quantity, those who are not well versed in the Use of such Propositions, may think it impossible to investigate the focal Distance of any refracting Surface by it: I shall therefore exemplify it in the following Instance, by which the Manner of doing it in all others will clearly be understood, *v. g.* Let it be required to determine the focal Distance of diverging Rays passing out of Air into Glass through a convex Surface, and let the Distance of the radiant Point be 20, and the Radius of Convexity be 5: Now because we must make Use of the focal Distance before we know it, let that be expressed by some Symbol or Character as  $x$ : Then, because by the aforesaid Proposition the Ratio compounded of that which the focal Distance bears to the Distance of the radiant Point (that is in this Supposition, of  $x$  to 20); and of the Ratio which the Distance of the same radiant Point from the Center bears to the Distance between the Center and the Focus (in this Case, of 25 to  $x-5$ ) is equal to the Ratio which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction (that is, of 17 to 11), we shall have in the Instance before us, the following Proportion, *viz.*

$$x : 20$$

$$25 : x-5$$

$$\left. \begin{array}{l} x : 20 \\ 25 : x-5 \end{array} \right\} :: 17 : 11, \text{ and compounding them into one,}$$

which

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VI. When Rays proceed out of a rarer into a denser *Medium*, through a convex Surface of the denser, if they are parallel before Refraction, they become converging afterwards.

For

which is done by multiplying the two first Parts together, we have  $25x : 20x - 100 :: 17 : 11$ , and multiplying the extrem Terms and middle Terms together,  $340x - 1700 = 275x$ , which Equation after due Reduction gives  $x = 1\frac{2}{5}$ .

In some Cases which might have been put, the Quantity 65 would have been negative, and then the Quotient arising from 1700 divided by that, would have been so too; that is  $x$  the focal Distance would have been Negative, in which Case the *Focus* must have been taken on the contrary Side the Surface to that on which it was supposed to fall in stating the Problem; that is, it must have been taken on the same Side with the radiant Point, for in calling the Distance between the Center and the *Focus*  $x - 5$  it was supposed the *Focus* would fall on the same Side with the Center or on that which is opposite to the radiant Point, because otherwise that Distance must have been expressed by  $x + 5$ , as any one may see by Inspection of the 13th or 14th Figure, in which the *Focus* of diverging Rays entering a convex Surface, is supposed to fall on the same Side with the radiant Point.

In like Manner as this Problem was performed a general Theorem may be raised to solve it in all Cases whatsoever, by using Characters instead of Figures; as every one who is not unacquainted with algebraic Operations very well knows.

See this done, and applied to the Passage of Rays through the Surface of *Lenfes* in the second Note to the following Chapter.

A Method of determining the Point which a Ray, entering a spherical Surface at any given Distance from the Vertex of it, converges to or diverges from after Refraction at the same.  
*From the Appendix to Molineux's Dioptrics.*

" *Prop.* To find the *Focus* of any Parcel of Rays diverging from, or converging to a given Point in the *Axis* of a spherical Lens [Surface] and inclined thereto under the same Angle; the Ratio of the Sines in Refraction being known.

" Let GL (Fig. 20.) be the *Lens*, P any Point in its Surface, V the Pole [Vertex] thereof, C the Center of the Sphere whereof it is a Segment, O the Object or Point in the *Axis* to

E

" or

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For in this Case, the Perpendiculars at the Points where the Rays enter the Surface, are all drawn from the Center of Convexity on the

“ or from which the Rays do proceed, OP a given Ray; and  
 “ let the *Ratio* of Refraction be as  $r$  to  $s$ ; make CR to CO as  
 “  $s$  to  $r$  for the Immersion of a Ray, or as  $r$  to  $s$  for the Emer-  
 “ sion, (that is, as the Sines of the Angles in the *Medium* which  
 “ the Ray enters, to their corresponding Sines in the *Medium*  
 “ out of which it comes) and laying CR from C towards O,  
 “ the Point R shall be the same for all the Rays of the Point O.  
 “ Then draw the Radius PC (if Need be) continued, and with  
 “ the Center R and Distance OP sweep a Touch of an Arch in-  
 “ tersecting PC in Q; the Line QR being drawn shall be pa-  
 “ rallel to the refracted Ray, and PF being made parallel  
 “ thereto, shall intersect the Axis in the Point F, which is the  
 “ *Focus* sought. Or make it as CQ : CP :: CR : CF, and CF  
 “ shall be the Distance of the *Focus* from the Center of the  
 “ Sphere.

“ *Dem.* Let fall the Perpendiculars PX on the Axis, CY on the  
 “ given Ray, and CZ on the refracted Ray. By the Construction  
 “ PF and QR are parallel, whence the Triangles QRC and PFC  
 “ are similar, and CR to QR, as CF to PF, that is CR to OP as  
 “ CF to PF. Now CF : PF :: CZ : PX *ob similia Triang.*  
 “ whence CR : OP :: CZ : PX, and CR : CZ :: OP : PX. Again,  
 “ CR is to CO as the Sines of Refraction by Construction, that  
 “ is, as  $s$  to  $r$ , or  $r$  to  $s$ ; and as CR to CZ, so  $(CO =) \frac{r}{s}$  or  
 “  $\frac{s}{r}$  CR to  $\frac{r}{s}$  or  $\frac{s}{r}$  CZ, and so is PO to PX: But as PO to PX,

“ so CO to CY. Ergo  $CY = \frac{r}{s}$  or  $\frac{s}{r}$  CZ, that is CY to CZ is as

“ the Sines of Refraction, but CY is the Sine of the Angle of  
 “ Incidence, and CZ of the refracted Angle. Ergo *constat*  
 “ *Propositio.* —

“ Hitherto we have considered only *oblique Rays*; it now re-  
 “ mains to add something concerning *Rays parallel to the Axis*:  
 “ In this Case the Point O must be considered as infinitely di-  
 “ stant, and consequently OP, OC, and CR are all infinite;  
 “ and OP and OC are in this Case to be accounted as always  
 “ equal, (since they differ but by a Part of the *Radius* of the  
 “ Sphere

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the other Side; and therefore, as the Rays are refracted towards those Perpendiculars, they are necessarily refracted towards each other, and thereby made to converge.

VII. If they enter diverging, then for the same Reason, they are made to diverge less,

" Sphere GPVL, which is no Part of either of them) wherefore the *Ratio* of CR to OP will be always the same, viz. as  $s$  to  $r$  for immerging Rays, and as  $r$  to  $s$  for those that emerge. And by this Proposition CF is to PF in the same *Ratio*. It remains therefore to shew on the Base CP how to find all the Triangles CPF, wherein CF is to PF in the *Ratio* given by the Degree of Refraction. This Problem has been very fully considered by the celebrated Dr. Wallis in his late Treatise of Algebra, p. 258, to which I refer; but I must here repeat the Construction thereof. See Fig. 21 and 22.

" Let GPVL be a *Lens*, VC or PC the *Radius* of its Sphere, and let it be required to find all the Points  $f, f$ , such, as Cf may be to Pf in the given *Ratio* of  $s$  to  $r$  for immerging Rays, or as  $r$  to  $s$  for the emerging. Divide CV in K, and continue CV to F, that CK may be to VK, and CF to VF in the proposed *Ratio*. Then divide KF equally in the Point  $a$ , and with that Center sweep the Circle FKF; this Circle being drawn gives readily all the *Foci* of the parallel Rays OP, OP. For having continued CP till it intersect the Circle in F, PF shall be always equal to Vf the Distance of the *Focus* of each respective Parcel of Rays OP from the Vertex or Pole of the *Lens*.

" To demonstrate this, draw the pricked Line VF, and by what is delivered by Dr. Wallis in the above-cited Place, VF and CF will be always in the same proposed *Ratio*. Again, Vf being made equal to PF, CF and Cf will be likewise equal, as are CP, VC; and the Angles PCf, VCF being adjacent are also equal: Wherefore Pf will be equal to VF, and consequently Cf to Pf in the same *Ratio* as CF to VF, whence, and by what foregoes, the Points  $f, f$ , are the several respective *Foci* of the several Parcels of Rays, OP, OP." Q. E. D.—

If any one would see how this is to be applied in all other Instances, he may consult the Place.

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to be parallel, or to converge, according to the Degree of Divergency they have before they enter.

For if they diverge very much, their being bent towards their respective Perpendiculars in passing through the Surface, may only diminish their Divergency; whereas, if they diverge in a small Degree it may make them parallel, or even to converge. What Degree of Divergency or Convergency before Refraction in this and the following Cases, is necessary to make Rays become parallel, will be shewn at Section the 17th of this Chapter.

VIII. If they converge in such Manner as to tend directly towards the Center of Convexity before they enter the Surface, they fall in with their respective Perpendiculars, and so pass on to the Center without suffering any Refraction.

IX. If they converge less than their Perpendiculars, that is, if they tend to a Point beyond the Center of Convexity, they are made by Refraction to converge more; and if they converge more than their Perpendiculars, that is, if they tend towards a Point between the Center and the Surface, then by being refracted towards them, they are made to converge less.

This and the three foregoing Propositions may be illustrated in the following Manner.  
1. Let AB, CD (*Fig. 8.*) be two parallel Rays entring a denser Medium through the convex



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convex Surface DB, whose Center of Convexity is E; and let one of these, viz. AB be perpendicular to the Surface. This will pass on through the Center without suffering any Refraction, but the other being oblique to the Surface, will be refracted towards the Perpendicular ED, and will therefore be made to proceed in some Line, as DG, converging towards the other Ray, and meeting it in G, which Point for that Reason is called the *Focus*. 2. Had the Ray CD diverged from the other, suppose in the Line AD, it would, by being refracted towards its Perpendicular ED, have been made either to diverge less, be parallel, or to converge. 3. Let the Line ED be produced to F, and if the Ray had converged, so as to have described the Line FD, it would then have been coincident with its Perpendicular, and have suffered no Refraction at all. 4. If it had proceeded from any Point between C and F, as from H, or which is the same Thing, towards any Point beyond E in the Line BE produced, it would have been made to converge more, by being refracted towards the Perpendicular DE, which converges more than it; and had it proceeded from some Point, as I, on the other Side F, that is, towards any Point between B and E, it would then have converged more than its Perpendicular, and so, being refracted towards it, would have been made to have converged less.

X. When



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X. When Rays proceed out of a denser into a rarer *Medium*, through a concave Surface of the denser, the contrary happens in each Case.

For being now refracted from their respective Perpendiculars, as they were before towards them, if they are parallel before Refraction, they diverge afterwards; if they diverge, their Divergency is increased; if they converge in the Direction of their Perpendiculars, they suffer no Refraction; if they converge less than their respective Perpendiculars, they are made to converge still less, to be parallel, or to diverge; if they converge more, their Convergence is increased. All which may clearly be seen by the Figure, without any farther Illustration, imagining the Rays AD, CD, &c. bent the contrary Way in their Refractions to what they were in the former Cases.

*not at all* XI. When Rays proceed out of a rarer into a denser *Medium*, through a concave Surface of the denser, if they are parallel before Refraction, they are made to diverge.

For in this Case, the Perpendiculars at the Points where the Rays enter the Surface, being drawn from a Point on that Side of the Surface from which the Rays tend, if we conceive them to pass through the Surface, they will be so many diverging Lines on the other Side, and therefore the Rays after they have passed through the same Points, must necessarily be rendered diverging in being refracted towards them.

XII. If

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XII. If they diverge before Refraction, then for the same Reason, they are made to diverge more.

XIII. Unless they proceed directly from the Center, in which Case they fall in with their Perpendiculars, and suffer no Refraction: or from some Point between the Center of Convexity and the Surface, for then they diverge more than their respective Perpendiculars, and therefore being by Refraction brought towards them, they become less diverging.

XIV. If they converge, then being refracted towards their Perpendiculars, they are either made less converging, parallel, or diverging, according to the Degree they converged in before Refraction.

To illustrate this, and the three foregoing Cases. 1. Let AB, CD (*Fig. 9.*) be two parallel Rays entering the concave and denser Medium X, the Center of whose Convexity is E, and the Perpendicular to the refracting Surface at the Point D, is EF; the Ray AB if we suppose it perpendicular to the Surface at B will proceed on directly to G; but the oblique one CD being refracted towards the Perpendicular DF, will recede from the other Ray AG in some Line as DH. 2. If the Ray CD had proceeded from A diverging in the Direction AD it would have been bent nearer to the Perpendicular, and therefore have diverged more. 3. But if it had diverged from the Center

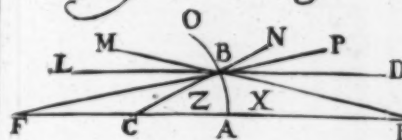
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ter E, it would have fallen in with the Perpendicular EF, and not have been refracted at all: and had it proceeded from I, a Point on the other Side the Center E, it would by being refracted towards the Perpendicular DF have proceeded in some Line nearer it than it otherwise would have done, and so would diverge less than before Refraction. 4. If it had converged in the Line LD, it would have been rendered less converging, parallel, or diverging, according to the Degree of Convergency, which it had before it entered into the refracting Surface.

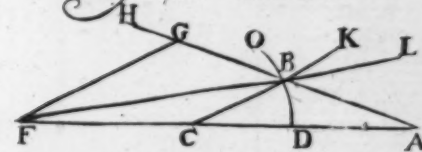
XV. If the same Rays proceed out of a denser into a rarer *Medium* through a convex Surface of the denser, the contrary happens in each Supposition: The parallel are made to converge; those which diverge less than their respective Perpendiculars, that is, those which proceed from a Point beyond the Center, are made less diverging, parallel, or converging, according to the Degree in which they diverge before Refraction; those which diverge more than their respective Perpendiculars, that is, those which proceed from a Point between the Center and the refracting Surface, are made to diverge still more. And those which converge, are made to converge more. All which may easily be seen by considering the Situation of the Rays AD, CD, &c. with Respect to the Perpendicular EF; and therefore requires no farther Illustration.

XVI. When

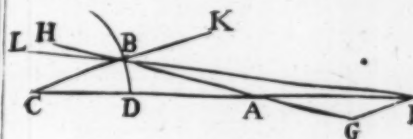
*Fig. 11. P. 23.*



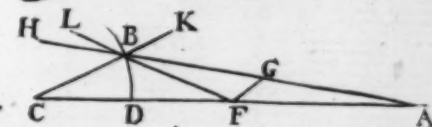
*Fig. 12. P. 26.*



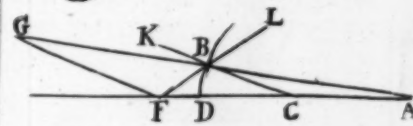
*Fig. 13. P. 28.*



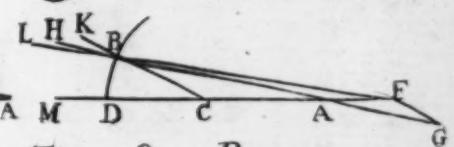
*Fig. 14. P. 29.*



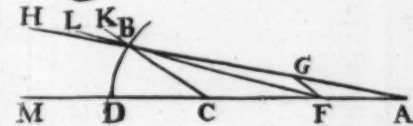
*Fig. 15. P. 30.*



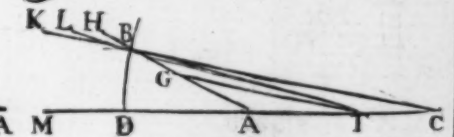
*Fig. 16. P. 31.*



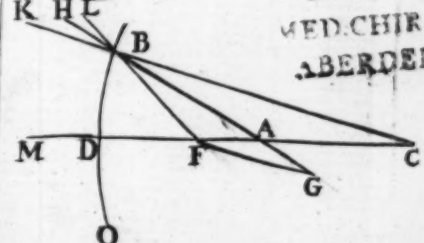
*Fig. 17. P. 32.*



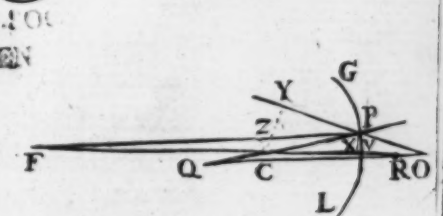
*Fig. 18. P. 32.*



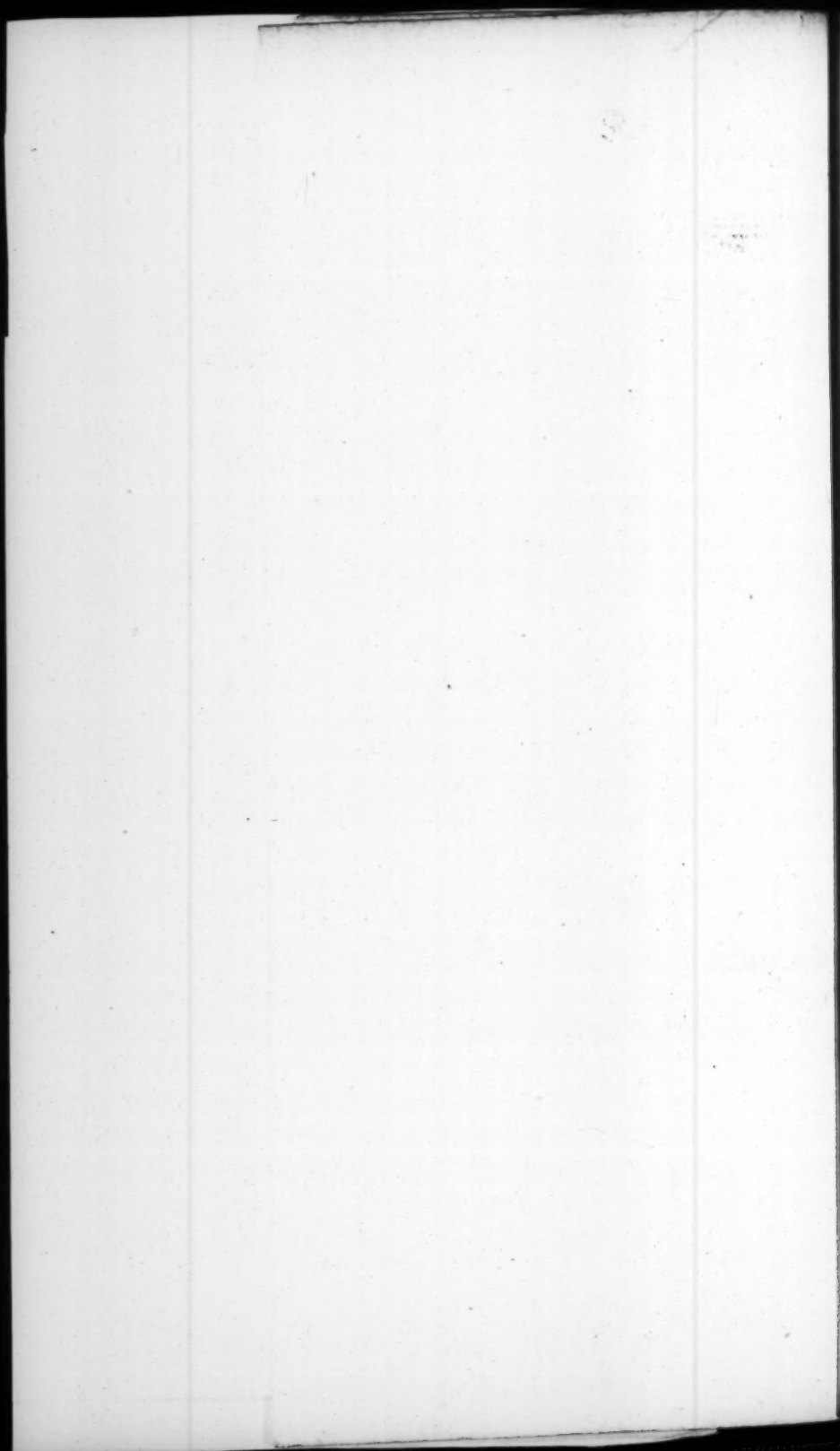
*Fig. 19. P. 33.*



*Fig. 20. P. 35.*



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Chap. 3. *The Cause of Refraction, &c.* 43

XVI. When diverging Rays are by Refraction made to converge, the nearer their radiant Point is to the refracting Surface, the farther is their *Focus* from it on the other Side, and *vice versâ*.

For the nearer the Radiant Point is to the refracting Surface, the more the Rays which fall upon the same Points of it, diverge before Refraction, upon which Account they converge the less afterwards.

XVII. When the radiant Point is at that Distance from the Surface, at which parallel Rays coming through it from the other Side would by Refraction be collected, then Rays flowing from that Point become parallel on the other Side, and are said to have their *Focus* at an infinite Distance. For the Power of Refraction in the *Medium* is the same, whether the Ray passes one Way or the other. For Instance, if the parallel Rays AB, CD (*Fig. 8.*) in passing through the refracting Surface BD are brought to a *Focus* in G, then Rays flowing from G as a radiant Point will afterwards proceed in the parallel Lines BA and DC. And the Point G, where the parallel Rays AB and CD meet after Refraction, is called the *Focus of parallel Rays*.

XVIII. When Rays proceed from a Point nearer the refracting Surface than the *Focus* of parallel Rays, they continue to diverge after Refraction, and their *Focus* is then an imaginary

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nary one, and situated on the same Side the Surface with the Radiant.

For in this Case, their Divergency being greater then that which they would have, if they had proceeded from the *Focus* of parallel Rays, they cannot be brought to a Parallelism with one another, much less be made to converge, and therefore they continue to diverge, though in a less Degree than before they passed through the refracting Surface; upon which Account, they proceed after Refraction, as if they came from some Point farther distant from the refracting Surface than their Radiant.

#### C H A P. IV.

*Of Lenfes, and the Manner in which Rays are affected in passing through them.*

**A** *Lens*, is a *Medium* terminated on one Side by a spherical Surface, on the other by a Surface either plain or spherical. And of these there are five Sorts. The first, as A, (*Fig. 23.*) is plain on one Side and convex on the other; the second, B, convex on both Sides; the third, plain on one Side and concave on the other, as C; the fourth, D, concave on both Sides; the fifth, convex on one Side and concave on the other, as E, which is by some called a *Meniscus*.

The

The *Axis* of a *Lens* is a Line passing perpendicularly through both its Surfaces: Thus, the Line FG is an *Axis* common to all the five.

*Lenses* are distinguished into two general Kinds, convex and concave; the first and second *Lenses* are considered, as convex; the third and fourth, as concave: the last, if its Convexity is greater than its Concavity, is looked upon, as convex; if on the contrary, it is considered as concave.

A *Lens* is always supposed to consist of a *Medium* denser than the circumambient one, unless where the contrary is expressed.

When parallel Rays fall upon the Surface of a convex *Lens*, they are refracted towards each other in passing through it, and thereby collected to a *Focus* on the other Side.

To explain this, let us trace the Progress of a Ray as AB (*Fig. 24.*) through the convex *Lens* CDEH, whose *Axis* is IK. Let L be the Center of the first Convexity CDE, and M that of the other CHE; and let the Ray AB be parallel to the *Axis*; through B draw the Line LN which will be perpendicular to the Surface CDE at that Point. The Ray AB in entering the denser Substance of the *Lens* will be refracted towards the Perpendicular, and therefore proceed after it has entered the Surface at B in some Direction inclined towards the *Axis*, as BP. Through M the Center of Convexity of this Surface and the Point P draw

the Line MR, which passing through the Center will be perpendicular to the Surface at P, and the Ray now entring a rarer *Medium* will be refracted from the Perpendicular into some Direction as PF. In like Manner, and for the same Reasons, the parallel Ray ST on the other Side the Axis, and also all the intermediate ones as XZ, &c. will meet it in the same Point, unless the Rays AB and ST enter the Surface of the *Lens* at too great a Distance from the Axis IF, the Reason of which has already been fully explained \*.

The Point F where the parallel Rays AB, SF, &c. are supposed to be collected by passing through the *Lens* CE, is called the *Focus of parallel Rays of that Lens*.

If the Rays converge before they enter the *Lens*, they are then collected at a Point nearer to the *Lens* than the *Focus* of parallel Rays. If they diverge before they enter the *Lens*, they are then collected in a Point beyond F; unless they proceed from a Point on the other Side at the same Distance with the *Focus* of parallel Rays, in which Case they are rendered parallel. If they proceed from a Point nearer than that, they diverge afterwards, but in a less Degree than before they enter the *Lens*.

If the *Lens* is plain on one Side and convex on the other, the Rays are refracted the same Way, but in a less Degree.

\* See *Observation* 3, in the foregoing Note.

If the Rays AB, ST, be supposed to proceed from a radiant Point on one Side of the *Lens*, and be refracted into a *Focus*, as at F, on the other ; then Rays proceeding from that focal Point, F, as from a Radiant, and supposed to pass through the *Lens* the contrary Way, will be collected in that Point which was the Radiant in the other Case : and the nearer the Radiant Point is to the *Lens*, the farther is the *Focus* from it on the other Side, and *vice versa*.

If the Rays AB, CD, EF, &c. (*Fig. 25.*) be parallel to each other, but oblique to GH the Axis of the *Lens* IK, or if the diverging Rays CB, CF, proceed as from some Point C which is not situated in the Axis of the *Lens*, they will be collected into some Point as L, not directly opposite to the Radiant C, but nearly so : for the Ray CD which passes through M the Middle of the *Lens* and falls upon the Surface of it with some Obliquity, will itself suffer a Refraction at D and N ; but then it will be refracted the contrary Way in one Place to what it is in the other, and these Refractions will be equal in Degree if the *Lens* has an equal Convexity on each Side, as we may easily perceive if we imagine ND to be a Ray passing out of the *Lens* both at N and D, for it is evident the Line ND has an equal Inclination to each Surface at both its Extremities. Upon which Account the Difference between the Situation of the Point L and one directly opposite

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site to C, is so small, that it is generally neglected; and the *Focus* is supposed to be in that Line, which a Ray, that would pass through the middle Point of the *Lens*, were it to suffer no Refraction, would proceed in.

All which is sufficiently clear, from what has been said concerning the Laws of Refraction explained in the foregoing Chapter.

When parallel Rays fall upon a concave *Lens*, they are refracted from each other in passing through it, and thereby made to diverge, proceeding as from an imaginary *Focus* on the first Side of the *Lens*.

In order to comprehend this, let ABCD (*Fig. 26.*) represent a concave *Lens*, EO its Axis, GH the Radius of the first Concavity, IK that of the second; produce HG to L, and let MG be a Ray of Light entering the *Lens* at the Point G. This Ray being refracted towards the Perpendicular GL, will pass on to some Point as K in the other Surface more distant from the Axis than G, and being there refracted from the Perpendicular IK, will be diverted farther still from the Axis, and proceed in the Direction KN as from some Point, O, on the first Side of the *Lens*. In like Manner other Rays as PQ parallel to the former, will proceed after Refraction at both Surfaces as from the same Point O; which upon that Account will be the imaginary *Focus* of parallel Rays of this *Lens*.

If

If the Rays diverge before they enter the *Lens*, their imaginary *Focus* is then nearer the *Lens* than that of the parallel Rays. If they converge before they enter the *Lens* proceeding towards some distant Point in the Axis as E, they are then rendered less converging: if they converge to a Point at the same Distance from the *Lens* with the *Focus* of parallel Rays, they then go out parallel; if to a Point at a less Distance they remain converging, but in a less Degree than before they entered the *Lens*.

When the Rays enter the *Lens* diverging, the nearer their radiant Point is to it, the nearer also is their imaginary *Focus* after Refraction, and *vice versa*.

If the *Lens* is plain on one Side and concave on the other, the Rays suffer a like Refraction in each Case, but in a less Degree.

The Truth of what has been said concerning the Passage of Rays through a concave *Lens*, is easily to be deduced from the Laws of Refraction delivered in the foregoing Chapter.

But the Method of determining the exact focal Distances of *Lenses* is to be had from the Propositions laid down and demonstrated in the Note in the foregoing Chapter. Thus, the Progress of the Rays after their Refraction at the first Surface where they enter a *Lens*, is had by one of those which determines the focal Distance of Rays entring a denser Medium of such Form: And their Progress after their

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Refraction at the other Surface where they go out, is had by computing what Progress Rays, moving in the Direction they are found to have after their Entrance at the first Surface, will acquire by being refracted at the other; which is to be effected by one which determines the focal Distance of Rays passing out of a denser Medium of like Form with that of the Lens\*.

When

\* Or a general Theorem may be made after the following Manner, to determine the Progress of Rays after Refraction at both Sides of the Lens, whatever be the Matter of it, or the Form wherein it is made.

Thus suppose GH (Fig. 28.) to be a given Lens, and E a Point in its Axis from whence the diverging Rays EL, &c. fall upon the Lens, AL, the Radius of the first Convexity, and CK that of the second; let LKf be the Direction of the diverging Ray EL after its Refraction at the first Surface, and KF its Direction after Refraction at both. Then will f be the Focus of the Rays after their first Refraction, and F the Point they will meet in after both. Let BD be the Thickness of the Lens, and let the Proportion which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction be expressed by the Ratio of I to R. Call EB, d; BD, t; AB, r; CD, s; Bf, x; DF, y: Now, to find f their Focus after Refraction at L where they enter the first Surface of the Lens, comes under the third Proposition in the forementioned Note: According to which the Ratio compounded of x, the focal Distance sought, to d, the Distance of the radiant Point; and of d+r, the Distance between the same Point and the Center, to x-r, the Distance between the Center and the Focus, is as I to R; compounding these two Ratios therefore (that is multiplying them together) we have  $dx + rx : dx - dr :: I : R$ ; which Proportion being converted into an Equation, and duly reduced, gives  $x =$

$$\frac{Idr}{Id - Rd - Rr}.$$

Thus having found the Distance Bf, and consequently the Point f, to which the Rays converge from L, we must proceed to find F, that to which they will converge after having passed through K where

When a Ray passes through a *Medium* terminated by two plain and parallel Surfaces, it is

where they suffer a second Refraction: This comes under the same Proposition; but, if we would use the same Letters as before, to express the Proportion which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, they must be put one for the other; because, when Rays pass out of a denser into a rarer *Medium*, the Sine of the Angle of Incidence bears the same Proportion to the Sine of the Angle of Refraction, that the Sine of the Angle of Refraction does to the Sine of the Angle of Incidence, when they pass out of a rarer into a denser. This being observed, by the aforesaid Proposition, we shall have the *Ratio* compounded of  $y$ ,

the focal Distance, to  $\frac{Idr}{Id-Rd-Rr} - t$ , the imaginary Focus of the incident Rays, and of  $\frac{Idr}{Id-Kd-Rr} - t + s$ , the Distance between the imaginary Focus and the Center, to  $y + s$ , the Distance between the Center and the Focus, as R to I. Which Equation, if we reduce the mixed Quantities  $\frac{Idr}{Id-Rd-Kr} - t$ , and  $\frac{Idr}{Id-Rd-Rr} - t + s$  into improper Fractions, will stand thus:

$$y : \frac{Idr - Idt + Rdt + Rrt}{Id - Rd - Rr} \text{ and } \frac{Idr - Idt + Rdt + Rrt + Ids - Rds - Rrs}{Id - Kd - Kr} : y + s \left. \vphantom{\frac{Idr - Idt + Rdt + Rrt}{Id - Rd - Rr}} \right\} :: R : I.$$

And, compounding these Ratio's, we have

$$\frac{Idy - Idty + Rdy + Rty + Idsy - Rdsy - Rrsy}{Id - Rd - Rr} : \frac{Idy - Idty + Rdy}{Id - Kd} + \frac{Rty + Ids - Ids + Rds + Rrts}{-Rr} :: R : I. \text{ And, throwing out}$$

the two equal Denominators  $Id - Rd - Rr$  and  $Id - Rd - Rr$ , and multiplying Extreams together and Means together, we have  $Idry - Idty + IRdy + IRty + Idsy - IRdsy - IRrsy = IRdy - IRdy + RRdy + RKty + IRds - IRds + RRds + RRrts$ ;

which Equation, being reduced, gives  $y = \frac{IRds - IRds + RRds + RRrts}{Idr - Idt + 2IRdt +$

$$\frac{+RRds + RRrts}{IRr + Ids - IRds - IRrs - IRdr - RRds - RRrs}$$

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is refracted one Way ingoing out of the ſecond, as much as it was the other in entering the firſt; and therefore proceeds afterwards not in the ſame Direction, but in one that is parallel to that which it had before. Thus if the Ray AB (*Fig. 27.*) enters the denſer Medium CDEF terminated by the parallel Surfaces CD and EF, it is refracted at B towards the Perpendicular BI, proceeding to a Point as G, where it is as much refracted from the Perpendicular GK in going out, and proceeds in the Direction GH. not the ſame, but parallel to the former ABL.

This Theorem may be applied to all Caſes whatever, even to plain Surfaces *mutatis mutandis*; v. g. the Radius of a concave Surface being negative (as lying the contrary Way) with Reſpect to that of a convex, and the Radius of a plain Surface being an infinite Line; if we would apply this Theorem to a concave Surface, we muſt change all the Sines of thoſe Members wherein the Symbol expreſſing the Radius of that Surface occurs; and, if to a plain Surface, all the Members which involve the Radius muſt be conſidered as infinite Quantities; that is, all, except them, muſt be ſtruck out of the Equation as nothing. So, likewise if we would have it extend to other Rays beſides diverging ones, the Point where converging Rays would meet lying on the contrary Side to that from whence the diverging ones were ſuppoſed to flow, its Diſtance muſt be made negative; and, the Diſtance where parallel Rays meet being infinite, it is only changing the Sines of all thoſe Members in which *d* is found, if the Rays are ſuppoſed converging, or making thoſe Members infinite, in Caſe the Rays are ſuppoſed parallel; which is done by ſtriking out all the reſt, as bearing no Proportion to them.

See the Method of reducing this Equation to fewer Terms, where it is alſo illuſtrated with divers Inſtances, in Dr. Brown's Appendix to Gregory's Optics, or in Dr. Halley's Method of finding the principal Focus of Optic Glaſſes univerſally, Philoſoph. Tranſact. No. 205.

C H A P. V.

*Of the Eye.*

THE Form of the Eye is such as is represented in Figure 29, and would be a perfect Sphere, were not the fore Part AA somewhat more protuberant than the rest.

The Description of it, so far as is necessary to explain the Nature of Vision, is as follows:

It is inclosed in three distinct Coats or Teguments; the outermost of which, viz. *aa*, is called *Tunica Sclerotica*; the next, *cc*, *Choroides*, or *Uvea*; the third and innermost, *dd*, derives its Name from that of its Discoverer, and is called *Tunica Ruyschiana*.

These Coats are contiguous to each other every where, except on the fore Part of the Eye.

That Portion of the *Sclerotica* which lies between A and A is more protuberant than the rest, is transparent, and has the Name of *Tunica Cornea*.

That Portion of the *Choroides* which is situated between *b* and *b* is called the *Iris*, and is that which by its Colour denominates an Eye black, grey, &c. In the Middle of this there is a round Hole as *pp*, called the *Pupil*.

The *Iris* consists of two Kinds of muscular Fibres; the first lie extended from its Extre-



mity like the *Radii* of a Circle, and point towards the Middle of the Pupil as towards a Center : the other are circular ones and surround the Pupil, having the Middle of it for their common Center. These are connected to the former where they cross them : and therefore when these contract, the Pupil is diminished ; when the other, it is enlarged.

Within the Cavity of the Eye, and not far behind the Pupil, there is a soft transparent Substance CC, not unlike a double convex *Lens*, one of whose Surfaces as S, is more convex than the other. This is called the *CrySTALLINE Humour*, and is suspended within the Eye by certain Ligaments, as Cl, Cl, called *Ligamenta Ciliaria*, or *Processus Ciliares*: these are convex towards the Pupil, as expressed in the Figure, and concave on the other Side, and are muscular, and therefore capable of Contraction and Dilatation. The convex Sides of these Ligaments are lined with a very black Substance, as is also that Side of the *Iris* which is next them.

The *Tunica Ruyschiana* leaves the *Choroides* at l, and, passing behind the *Ligamenta Ciliaria* and the *CrySTALLINE Humour*, is contiguous to them, and joins the *Choroides* again at l, on the other Side the Pupil.

By means of the forementioned Parts, the Cavity of the Eye is divided into two Portions; the one of which VV is filled with a Fluid nearly of the same Density with Water, and

Chap. 5. *Of the Eye.* 55

is therefore called *Humor Aqueus*; the other TT contains a Fluid whose Consistency is greater than that of the former, and is called *Humor Vitreus*: And both these Humours are rarer than the Substance of the Crystalline.

At the back Part of each Eye, but not directly opposite to the Pupil, there enters a Nerve as NN, which is called the *Optic Nerve*.

The Fibres of this Nerve, after their Entrance into the Eye at N, spread themselves over the innermost Coat of it as far as the *Ligamenta Ciliaria*, and form a very thin Membrane, called *Tunica Retina*.

The innermost Coat of the Eye is every where covered (except that Part of it which is contiguous to the back Part of the Crystalline Humour) with a very black Substance, not unlike that with which the back Part of the *Iris* and fore Part of the *Ligamenta Ciliaria* were observed to be covered. This is to hinder any Light from being reflected from those Parts to the *Retina*; for that would render the Images of Objects indistinct; as we shall see when we have explained the Nature of Vision, which is the Subject of the next Chapter.

## C H A P. VI.

*Of the Nature of Vision.*

**S**UCH is the Substance and Form of the Humours of the Eye, when lodged in their proper Receptacles, that Rays of Light in passing through them are affected in the like Manner as in passing through a convex *Lens*, as we shall see immediately: and therefore, to understand the Nature of Vision, which depends on the Passage of Rays flowing from the *several* Points of a distant Object through those Humours, little more is required than to know how the same Rays would be affected, were they supposed to pass through a convex *Lens*: Which may easily be done by applying to this Case what has been delivered in the fourth Chapter concerning the Manner in which Rays flowing from a *single* Point are affected in passing through *Lenses* of that Kind.

We have already seen, in the abovementioned Chapter, that Rays flowing from a *single* radiant Point, and afterwards falling on a convex *Lens*, are collected to a *Focus* in some Point opposite, or nearly so, to the radiant. Let us now suppose an Object placed before a *Lens*, but farther from it than the focal Distance of its parallel Rays; and let it send forth

Rays

## Chap. 6. *Of the Nature of Vision.* 57

Rays from each Point of its Surface in every Direction, as from so many radiant Points. Some of the Rays which flow from each Point of that Surface of the Object which is turned towards the *Lens* will necessarily fall upon it, and, passing through it, will be collected in so many distinct focal Points on the opposite Side, as there are distinct radiant Points in the Surface of the Object from whence they came. Now, as the radiant Points are contiguous to each other in the Surface of the Object on one Side of the *Lens*, the focal Points will also be contiguous on the other; and as each focal Point is opposite to its respective radiant, their Places will have the same Relation to each other, that those of the radiant have; and, consequently these Points, taken together, will be a true Representation and perfect Image of that Object; for each Point will exhibit the same Colour that its correspondent Point in the Object is of: But, because each Point in the Representation is opposite to its respective one in the Object, the Image will be inverted. The Truth of this may easily be experienced, if we hold a clean white Paper facing the *Lens* in the Place where the focal Points are, and take Care to prevent all other Light from falling upon the Paper, except that which passes through the *Lens* \*.

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\* On this depends the Structure of the *Obscura Camera*, which is a Contrivance to exhibit the Representation of such Objects

## 58 *Of the Nature of Vision. Part III.*

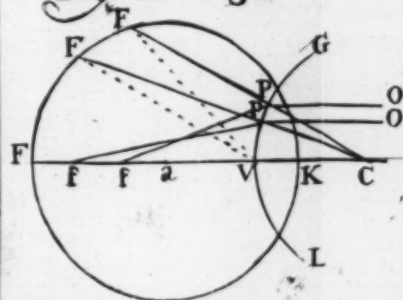
To illustrate this, let PQR (*Fig. 30.*) represent an Object placed before the *Lens* AB, and sending forth Rays from each Point in its Surface; and let  $q$  be the focal Distance of Rays proceeding from Q and passing through the said *Lens*. Then will all the Rays that proceed from the Point Q, between the Lines QA and QB, be collected in  $q$ ; in like Manner all that flow from P, between the Lines PA and PB, will meet in the opposite Point  $p$ ; and so many as proceed from R, and pass through the *Lens*, will be collected in  $r$ ; and all the Rays that flow from the remaining Points between P and R and fall upon the *Lens*, will be collected in as many Points between  $p$  and  $r$ ; and, if the Rays are received there upon a white Surface, there will be exhibited an Image of the Object PR, but inverted; because the Rays PL $p$  and RL $r$

Objects as may be seen from a Window upon some plain white Surface held before the Window within the Room. In order to do this, a common Spectacle Glass or Burning Glass (both which are convex *Len(es)*) must be fixed in an Hole in the Windowshutter; for then, if no Light be suffered to enter into the Room, but what passes through the Hole, and a Sheet of white Paper be held opposite to the Hole at that Distance where the Rays proceeding from the Objects abroad, and passing through the Glass, are collected into their respective *Foci*, we shall have the Images of all the Objects which lie before the Hole represented upon the Paper, inverted; but in a much more lively and exact Manner than can be done by the Pencil; and not only the Objects and their respective Situations, but, what is peculiar to this Sort of Painting, their Motions will also be expressed.

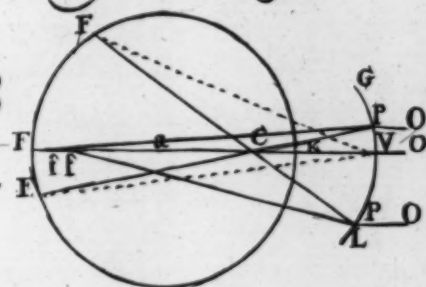
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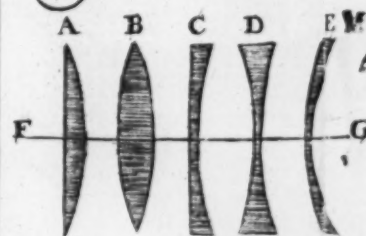
*Fig. 21. P.36.*



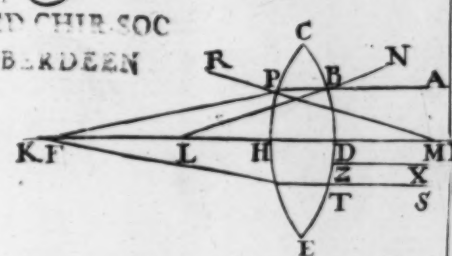
*Fig. 22. P<sub>36</sub>.*



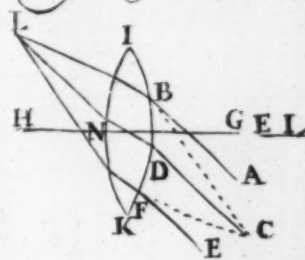
*Fig. 23. P. 44.*



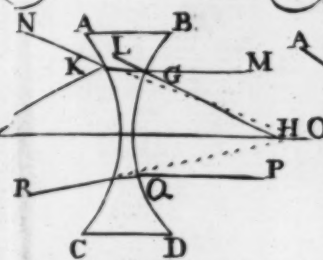
*Fig. 24. P.45.*



*Fig. 25. P. 47.*



*Fig. 26. P. 48.*



*Fig. 27. P<sub>52</sub>*



*Fig. 28. P. 50.*





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cross each other at *L* in passing through the *Lens* \*.

Those Rays which flow from the same Point of an Object, when considered together, are called a *Cone* or *Pencil* of Rays. Thus the Rays *QA*, *QL*, &c. constitute a Pencil flowing from the Point *Q*; so the Rays *PA*, *PL*, &c. a Pencil from the Point *P*; and the middle Ray of each Pencil, as *PL*, *QL*, &c. is called the *Axis* of that Pencil, to which it belongs.

Now in like Manner as the several Pencils of Rays flowing from the distinct Points in the Surface of an Object placed before a *Lens*, are collected in so many Points at a certain Distance on the other Side of the *Lens*, and form an Image there when received upon a white Paper; so Pencils proceeding from an Object placed before the Eye at a proper Distance from it, and being refracted in passing through the Humours of it, are collected into their respective *Foci* upon the *Retina*, where they form a Representation of that Object; and by their Impulses upon the tender Nerves of the *Retina*, an *Idea* of the Object is excited in the Mind.

\* A burning Glass is no other than a Piece of Glass ground into the Form of a convex *Lens*; for if the Rays of the Sun are permitted to pass through such an one, they will burn very strongly in the Place where they are collected into their respective *Foci*; upon which Account it is, that the Point where Rays in general are collected, is called their *Focus*, that is, their Place of Burning.

## 60 *Of the Nature of Vision. Part III.*

The Progress of the Rays through the Humours of the Eye, are expressed in the 31<sup>st</sup> Figure: where *FGt* is the Eye, *FG* the *Tunica Cornea*, *pp* the Pupil, *AA* the aqueous Humour, *HH* the cristalline, and *VV* the vitreous. And *RS* represents an Object placed before it, emitting Pencils of Rays from its several Points *R*, *S*, *T*, &c. The Rays which constitute the Pencil *GTF*, in entering the aqueous Humour, pass out of a rarer into a denser Medium through a convex Surface, in which Case diverging Rays are made to diverge less, to become parallel or to converge (Chap. 3. Prop. 7.); in entering the cristalline they do the like; and in passing out of that, they proceed out of a denser into a rarer Medium, through a convex Surface of the denser, which also has the same Effect (Chap. 3. Prop. 15.) By which Means they are made to converge, as described in the Figure, and to meet together in a *Focus* at *t*, a Point in the *Retina*. In like Manner the Rays flowing from *R*, and constituting the Pencil *GRF*, will proceed as described in the Figure, and after Refraction meet in *r*; and the Rays proceeding from *S* will be collected in *s*, &c. by which Means an Image of the Object will be formed in *rts* upon the *Retina*, but because the Pencils cross each other in passing through the Pupil, it will be inverted \*.

The

\* Of this we have experimental Proof: For if we cut away the back Part of an Eye, and apply a Paper there, we shall see the Images

## Chap. 6. Of the Nature of Vision. 61

The less the Distance between the Object and the Eye is, the more the Rays which come from the Object, are said to diverge, and *è contra*: Not that the Situation of the Eye makes any Alteration in the Progress of those Rays, but that, when the Eye is placed nearer the Object, it receives into its Pupil Rays which diverge in a greater Degree than those which it can receive when placed farther off. The following Illustration will make this clear: Let *AB* (*Fig. 32.*) represent an Object emitting Rays from each Point of its Surface, and let *Cd*, *Cr*, &c. express those which flow from the Point *C*: let *m* be a Pupil of an Eye placed at the Distance *Cm* from it; 'tis plain this Pupil will receive into it the diverging Rays *Cr*, *Cs*; whereas the Rays *Co*, *Ct* will diverge the most of any that can enter the same Pupil, when placed at the Distance *Cn*; but these diverge less than the former, the Angle *oCt* being included in the Angle *rCs*.

Images of external Objects placed thereon, as accurately as in the *Obscura Camera*, provided no Light is permitted to fall upon the Paper, except that which passes through the Humours of the Eye.

Optic Writers have made it Matter of great Difficulty to determine the Point where the *Axis* of the Pencils which enter the Eye, cross each other; some placing it in the Center of the Eye, others in the Vertex of the cristalline Humour, others in that of the *Tunica Cornea*: But as the Rays of each Pencil fill the Pupil, or as the Pupil itself is a common Base to each Pencil, it is inconceivable how the *Axis* of those Pencils should cross each other in any other Place than the Center of the Pupil. See Figure 31, or any other where several Pencils are represented, as passing through the Pupil of an Eye.

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Vision is distinguished into *bright* and *obscure*; *distinct* and *confused*.

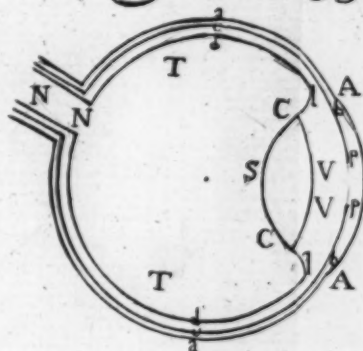
It is said to be *bright*, when a sufficient Number of Rays enter the Pupil at the same Time; *obscure*, when too few. It is *distinct*, when each Pencil of Rays is collected into a *Focus* exactly upon the *Retina*; *confused*, when they meet before they come at it, or when they would pass it before they meet; for in either of these last Cases, the Rays flowing from different Points of the Object, will fall upon the same Part of the *Retina*, which must necessarily render the Image confused and indistinct.

Now that Objects may appear with a due *Brightness*, whether more or fewer Rays proceed from them, we have a Power of contracting, or dilating the Pupil by means of the muscular Fibres of the *Iris* (as explained in the foregoing Chapter), in order to take in more or fewer Rays as Occasion requires. But this Power has its Limits. \*

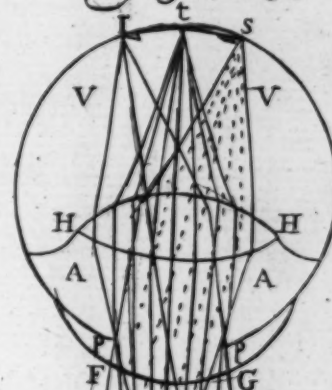
And that the Rays may be collected into Points exactly upon the *Retina*, that is, that Objects may appear *distinct*, whether they be nearer or farther off, that is, whether the Rays proceeding from them diverge more or less, we have a Power of contracting or relaxing the *Ligamenta Ciliaria*, and thereby altering the

\* In some Animals, this Power is much greater than in others; particularly in such as are obliged to make Use of their Eyes by Night, as well as by Day, as in Cats, &c.

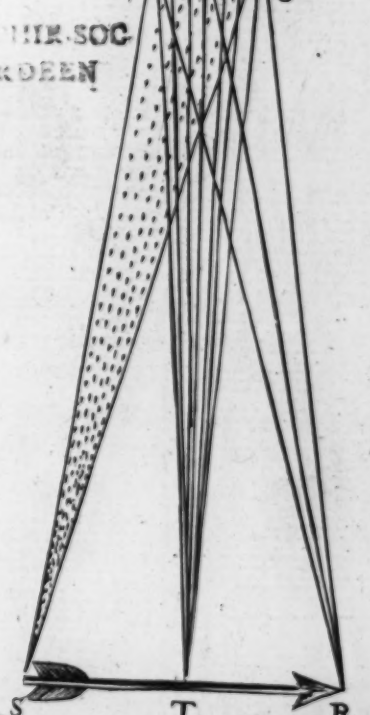
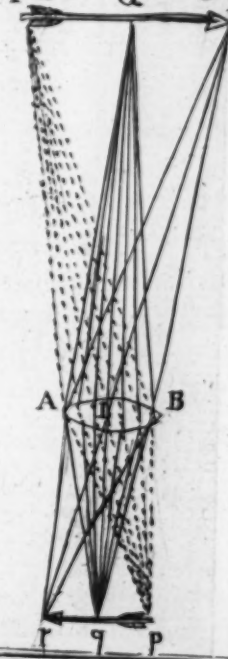
*Fig. 29. P. 53.*



*Fig. 31. P. 60*



*Fig. 30. P. 58*





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## Chap. 6. *Of the Nature of Vision.* 63

Form of the chriftalline Humour, and with that the focal Distance of the Rays. Thus, when the Object we view is far off, and the Rays fall upon the Pupil with a very small Degree of Divergency, we contract the *Ligamenta Ciliaria*, which being concave towards the vitreous Humour, do thereby compress it more than otherwise they would do; by this means it is made to press harder upon the back Side of the chriftalline Humour, which is thereby rendered flatter; and so the Rays proceed farther before they meet in a *Focus* than otherwise they would have done. Add to this, that we dilate the Pupils of our Eyes (unless in Cases where the Light is so strong that it offends the Eye) and thereby admit Rays into them, that are more diverging than those which would otherwise enter. And when the Rays come from an Object that is very near, and therefore diverge too much to be collected into their respective *Foci* upon the *Retina*, by relaxing the *Ligamenta Ciliaria* we give the Chrifalline a more convex Form, by which means the Rays are made to suffer a proportionably greater Degree of Refraction in passing through it \*.

\* Some Philosophers are of Opinion, that we do this by a Power of altering the Form of the Eye; and others, by removing the Chrifalline forwards or backwards as Occasion requires; but neither of these Opinions is probable; for the Coats of the Eye are too hard, especially in some Animals, for the first; and as to moving the Chrifalline out of its Place, the Cavities of the Eye seem to be too well filled with the other Humours to admit of such Removal.

And

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And besides this, by contracting the Pupils of our Eyes, we exclude the more diverging Rays, and admit only such as are more easily refracted into their respective *Foci* \*. But Vision is not distinct at all Distances, for our Power of contracting and relaxing the *Ligamenta Ciliaria* is also circumscribed within certain Limits.

The nearer an Object is placed to the Eye, the greater is the Image of it upon the *Retina*. Because the Pencils flowing from the extreme Parts of the Object when near, make a larger Angle with each other in the Pupil where they cross, than the same Pencils do when the Object is placed farther off. Thus AB (*Fig. 33.*) the Image of the Object CD, far exceeds EF that of the same Object GH, placed at a greater Distance from the Eye, as is evident from Inspection of the Figure.

In those Eyes where the *Tunica Cornea* is very protuberant and convex, the Rays of Light suffer a very considerable Refraction at their Entrance into the aqueous Humour, and are therefore collected to a *Focus* before they fall upon the *Retina*, unless the Object be placed very near, so that the Rays which enter the Eye, may have a considerable Degree of Divergency. People that have such Eyes, are

\* Accordingly it is observable, that if we make a small Hole with the Point of a Needle through a Piece of Paper, and apply that Hole close to the Eye, making Use of it, as it were, instead of a Pupil, we shall be able to see an Object distinctly through it, though the Object be placed within half an Inch of the Eye.

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said to be *purblind*. Now the nearer an Object is placed to the Eye, the greater is the Image of it therein, as explained above; these People therefore can see much smaller Objects than others, as seeing much nearer ones with the same Distinctness. And their Sight continues good longer than that of other People, because the *Tunica Cornea* of their Eyes, as they grow old, becomes plainer, for Want of that Redundancy of Humours with which they were filled before.

On the contrary, old Men having the *Cornea* of their Eyes too flat for want of a sufficient Quantity of the aqueous Humour to fill them out, if the Rays diverge too much before they enter the Eye, they cannot be brought to a *Focus* before they reach the *Retina*; on which Account those People cannot see distinctly, unless the Object be situated at a greater Distance from the Eye, than is required for those whose Eyes are of a due Form.

Since the Images of the Objects we look at are inverted in the Eye, it may be thought the Objects themselves ought to appear so; but it must be considered, that there is no *natural* Connection between the *Idea* in our Mind, and the Image upon the *Retina*; we find by Experience, that when such an *Idea* is excited in our Mind, such an Object stands before us in such a Position and of such a Form; whenever therefore the like *Idea* is excited again, we

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## 66 *Of the Nature of Vision.* Part III.

conclude there is a like Cause of it. For it is found by Observation, that People who have been born blind, and have afterwards received their Sight, have had no Information from their Eyes at first, concerning the particular Situation or Form of Bodies; but have been obliged to stay till Experience has taught them what Figures and Situation of Bodies correspond to such and such Sensations in the Mind \*.

In like Manner it is from Experience that an Object appears single, though there be an Image of it in each Eye; for after we find, that its Place, according to the Representation of it in each Eye, is the same, we necessarily conceive it to be but one. The Manner how we come to find this, seems to be as follows: There is one Part of the *Retina* upon which when the Image falls, the Object appears brighter and more distinct, than when it falls upon any other, as is evident, because we always see one Part of an Object with greater Distinctness than any of the rest. This Point I shall hereafter call *the Point of distinct Vision*. This naturally leads us to turn our Eyes so, that the Object may be situated directly opposite to this Point. And this Action of ours is that which has given Rise to those imaginary Lines, which are supposed to pass di-

\* See Mr. *Cheffelden's* Observations on a young Gentleman couched by him at the Age of 13 Years. *Philosoph. Transact.* No. 402.

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rectly through the Eye and to terminate in the Object we view, and are commonly called the *optic Axes*. We therefore turn our Eyes so that the Object may appear in those Lines. Therefore, since these Lines concur at the Object, when we endeavour to view it with Distinctness, each Eye affords us an *Idea* of the Object in the same Place, from whence it necessarily appears but one \*.

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\* There are other Methods of accounting for these two last *Phænomena*, some of which, perhaps, the Reader may think more plausible; for the Connection between the Image on the *Retina* and the Idea in the Mind being purely *metaphysical*, we can never hope to arrive at Certainty in this Matter.

Some are of Opinion, that we judge those Rays which paint the uppermost Part of the Image in the Eye to proceed from the lowermost Part of the Object, because they strike upon the *Retina*, as coming from that Part; and that we conclude from hence that the Object is erect, though the Image be inverted; as if the Direction, wherewith the Rays strike the *Retina*, informed the Mind which Way they came. This Solution serves also to explicate the *Phænomenon* of seeing but one Object with both Eyes; for as the Mind is informed, by the Direction with which the Rays strike the *Retina*, of the Place from whence they come; therefore when it appears that they enter each Eye as from the same Place, the Object necessarily seems to be but one; because we can't suppose two to exist in the same Place at the same Time.

Some have been so absurd as to embrace an Opinion, the first Author of which was *Gassendus*, that we see one and the same Point of an Object only with one Eye at a Time, (*otiante alio*, as they express it) while the other does nothing. Vid. *Gassendi* Epistol. de Magnitud. Solis; or *Tacquet*. Optic. Lib. I. Prop. 2.

Some imagining that the *optic Nerves* consist of a Bundle of small ones wrapped up in one common Tegument, are of Opinion, that such as lie upon the *Retina* at equal Distances from the Point of distinct Vision, and on the same Side of it in each Eye, are connected together in one, before they terminate in the Brain; and so, whether one or both are affected, only one *Idea*



### 68 *Of the Nature of Vision. Part III.*

Whenever the Eyes are so situated with respect to an Object, that the same Part of the *Retina* in each Eye is affected by the Rays that flow from it, which is are wont to be affected when two Objects are placed before the Eyes, the Mind, receiving no Information from without, but by the Impulses of the Rays upon the *Retina*, judges that there are two Objects. Thus, let A, B (*Fig. 34.*) represent two Eyes whose *optic Axes* are directed to the Point C, and let E be an Object on one Side the Point C; and F an Object on the other. Now, Objects thus situated must appear separate; otherwise, every time we viewed an Object we must imagine all the different Points in its Surface to be but one, which is contrary to Experience. In this Case, the Point *d* in each Eye will be affected by the Rays which flow from those Objects; but so it will, if a single Object be placed at D; and therefore, for the Reason given above, an Object in that Situation shall appear as the two separate ones E and F, that is, double. A-

is excited in the Mind, *Gravesand* confirms this Opinion, by asserting that, in all Animals which look at the same Object with both Eyes, the optic Nerves concur, before they enter the Brain; and that, in such as look at one Object with one Eye, and at a different one with the others they are separate all the Way.

Others, with *Briggius* (see his *Optalmog.* Chap. 11.) do not contend, that the forementioned corresponding Parts of the *optic Nerves* are connected before they terminate in the Brain; but that they are of an equal Tension, and therefore excite the same Sensation in the Mind.

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gain, let there be an Object placed without the *optic Axes*, as at G, Rays flowing from this will affect the same Part in each Eye, as if there were two distinct Objects, *viz.* one at E, and the other at H; this therefore will also appear double. Farther, as the Objects D and G are situated in this Figure, if both are attended to at the same Time (the *optic Axes* being still directed to the same Point C) they will appear as three, being situated opposite to the three Points F, E, and H. And what has been said of the Appearance of the Objects D and G, as they are situated in this Figure, may be applied to their Appearance as they are placed in the next, where they are represented as being beyond the Line HF. So that wherever an Object is placed, provided it be nearer to the Eyes than the Point where the *optic Axes* concur, or farther from them, it appears double.

There is one Part of the *Retina* of each Eye, upon which when the Image falls, the Object cannot be seen at all with that Eye; the Proof of this we have from the following Experiment. Fix two Objects upon a Wall, of such Bigness that each may hide a square Inch of it, or thereabouts, and at the Distance of about a Foot or two from each other, and go back from the Wall about three times that Distance; then shutting the left Eye, look at the left Object with the right one, and while the right Eye is in that Position, the right Object will not be

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seen. So, if in that Station we look at the right Object with only the left Eye, the left Object will disappear. The Reason of this is supposed to be, that the Image of that Object which disappears falls upon the Blood-vessels of the *optic* Nerve, from which no Sensation is conveyed to the Brain.

The Angle comprehended between the Rays which flow from the extreme Parts of the Object, and cross in the Pupil, is called the *optic Angle*.

'Tis by means of this Angle that we are able to form some Judgment of the Magnitude of an Object; because, the larger this is, the larger is the Image upon the *Retina*, that is, a greater Portion of it is affected by the Rays which flow from that Object. But this is not sufficient alone, because different Objects at different Distances from the Eye, may subtend equal Angles at the Pupil. We ought therefore to know also the Distance of the Object.

This, if the Object be very near, we are able to form a tolerable Judgment of, by the Degree of Divergency, wherein the Rays which flow from the same Point of the Object enter the Eye; because we find it necessary to adapt the Eye accordingly, in order to bring them to a *Focus* upon the *Retina*.

But, when the Object is at a greater Distance from us, a considerable Variation in the Distance of it makes but a very small one in the Diver-

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Divergency of those Rays, and therefore this Rule of judging ceases to be of Use. The only Expedient then is the Angle comprehended between the *optic Axes* at the Object to which they are directed, or, which is the same Thing, the Position of the Eyes with respect to each other when they view the Object \*. But in very large Distances this Position varies so little, that it is also of no Use; in which Case, we make the best Judgment we can from the Brightness, Distinctness, and apparent Magnitude of the Object, and likewise from its Situation with respect to others which are interposed †.

When we are unable to judge rightly concerning the Distance of an Object, we conceive it greater, the farther we imagine it to be from us. and *vice versâ*; because it requires a larger Object to exhibit the same Image upon the *Retina*, when it is situated at a great Distance, than when near. Thus we imagine the Sun and Moon to be farther off, when they are in the *Horizon*, than when they are near the *Meridian*, and accordingly think them

\* That the Position of the *optic Axes* is a Means whereby we judge of Distances, is evident from hence, *viz.* that they who have lost the Sight of one Eye, find it much more difficult to estimate the Distances of Objects, than they did, when they had the Use of both.

† We have a remarkable Instance of the Error of our Judgment concerning the Distances of very remote Bodies, in that we look upon the Sun, Moon, and Stars to be all at the same Distance, whereas some of them are a thousand Times farther from us than others.

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proportionably larger in one Situation than in the other, though they are found to exhibit the same Image upon the *Retina* in both Cases \*.

We are never able to see very distant Objects with Distinctness; this is not solely owing to the Pupil's not receiving into it a sufficient Number of Rays for that Purpose, or because they are not collected into *Foci* upon the *Retina*, but because the Object being very far off, the Rays which flow from Points of the Object that are contiguous fall too near each other upon the *Retina* to excite distinct Sensations in the Mind, so that the *Idea* of the Whole is confused.

#### C H A P. VII.

#### *Of the Appearance of Objects seen through Media of different Forms.*

**T**HAT what we shall say upon the Subject of this Chapter may more readily be understood, we shall premise the five following Particulars, which are all comprised in the foregoing Chapter, or follow immediately from what has been there laid down, *viz.*

1. That, as each Point of an Object, when viewed by the naked Eye, appears in its pro-

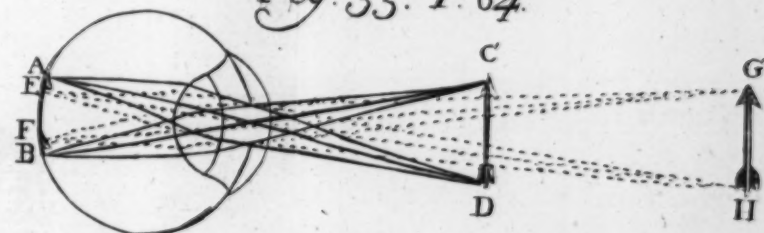
\* See the Dissertation on the *horizontal* Moon, annexed to this Part.



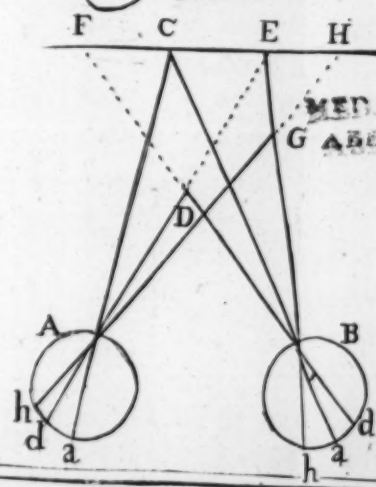
*Fig 32. P. 61.*



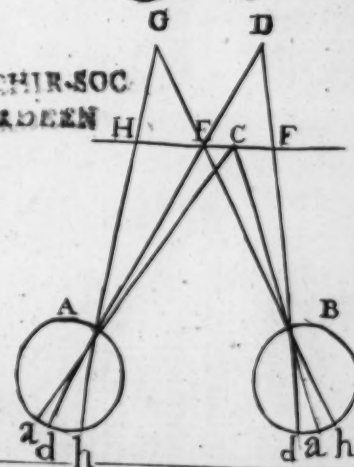
*Fig 33. P. 64.*



*Fig 34. P. 68.*



*Fig 35. P. 69.*



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per Place, and as that Place is always to be found in the Line in which the *Axis* of a Pencil of Rays flowing from it enters the Eye, we from hence acquire an Habit of judging the Point to be situated in that Line; and, because the Mind is unacquainted with what Refractions the Rays suffer before they enter the Eye, therefore, in Cases where they are diverted from their natural Course by passing through any *Medium*, it judges the Point to be in that Line produced back in which the *Axis* of a Pencil of Rays flowing from it is situated the Instant they enter the Eye, and not in that it was in before Refraction. We shall therefore in what follows, suppose the apparent Place of an Object, when seen through a refracting *Medium* to be somewhere in that Line produced back in which the *Axis* of a Pencil of Rays flowing from it proceeds after they have passed through the *Medium*.

2. That we are able to judge, though imperfectly, of the Distance of an Object by the Degree of Divergency, wherein the Rays flowing from the same Point of the Object enter the Pupil of the Eye, in Cases where that Divergency is considerable; but because in what follows, it will be necessary to suppose an Object, when seen through a *Medium* whereby its apparent Distance is altered, to appear in some determinate Situation; in those Cases where the Divergency of the Rays at their Entrance  
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into the Eye is considerable, we will suppose the Object to appear where those Lines which they describe in entering, if produced back, would cross each other; though it must not be asserted that this is the precise Distance; because the Brightness, Distinctness, and apparent Magnitude of the Object, on which its apparent Distance in some Measure depends, will also suffer an Alteration by the Refraction of the Rays in passing through that *Medium*.

3. That we estimate the *Magnitude* of an Object by that of the *optic Angle*.

4. That, Vision is the *brighter*, the greater the Number of Rays is which enter the Pupil. And,

5. That, in some Cases, the *apparent Brightness*, *Distinctness*, and *Magnitude* of an Object are the only Means whereby our Judgment is determined in estimating the Distance of it.

Prop. I. An Object placed within a *Medium* terminated by a plain Surface on that Side which is next the Eye, if the *Medium* be denser than that in which the Eye is (as we shall always suppose it to be, unless where the contrary is expressed) appears *nearer* to the Surface of the *Medium* than it is.

Thus, if A be a Point of an Object placed within the *Medium* BCDE (*Fig. 36*), and Ab, Ac be two Rays proceeding from thence, these Rays passing out of a denser into a rarer *Medium*, will be refracted from their respective

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Perpendiculars  $bd$ ,  $ce$ , and will enter the Eye at H, suppose in the Directions  $bf$ ,  $cg$ , let then these Lines be produced back till they meet in F; this will be the apparent Place of the Point A; and because the refracted Rays  $bf$ ,  $cg$  will diverge more than the incident ones  $Ab$ ,  $Ac$  (Chap. III. Prop. 3.), it will be nearer to the Points  $b$  and  $c$ , than the Point A; and as the same is true of each Point in the Object, the Whole will appear to an Eye at H, *nearer* to the Surface BC than it is \*.

Prop. II.

\* From hence it is, that when one End of a strait Stick is put under Water, and the Stick is held in an oblique Position, it appears bent at the Surface of the Water; *viz.* because each Point that is under Water appears nearer the Surface, and consequently higher than it is.

From hence likewise it is, that an Object at the Bottom of a Vessel may be seen when the Vessel is filled with Water, though it be so placed with Respect to the Eye, that it cannot be seen when the Vessel is empty. To explain this; let ABCD (Fig. 37.) represent a Vessel, and let E be an Object lying at the Bottom of it. This Object, when the Vessel is empty, will not be seen by an Eye at F, because HB the upper Part of the Vessel will obstruct the Ray EH; but when it is filled with Water to the Height GH, the Ray EK being refracted at the Surface of the Water into the Line KF, the Eye at F shall see the Object by Means of that.

In like Manner, an Object situated in the Horizon appears above its true Place, upon Account of the Refraction of the Rays which proceed from it in their Passage through the *Atmosphere* of the Earth. For first, if the Object be situated beyond the Limits of the *Atmosphere*, its Rays in entering it will be refracted towards the Perpendicular, that is, towards a Line drawn from the Point where they enter, to the Center of the Earth which is the Center of the *Atmosphere*, and as they pass on they will be continually refracted the same Way, because they are all along entering a denser Part, the Center of whose Convexity is still the same Point; upon which Account the Line they describe will be a Curve bending downwards; and there-

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Prop. II. An Object seen through a *Medium* terminated by plain and parallel Surfaces, appears *nearer, brighter, and larger*, than with the naked Eye.

For Instance, let AB (*Fig. 38.*) be the Object, CDEF the *Medium*, and GH the Pupil of an Eye, which is here drawn large to prevent Confusion in the Figure. And 1<sup>st</sup> let RK, RL be two Rays proceeding from the Point R, and entering the denser *Medium* at K and L; these Rays will here by Refraction be made to diverge less (*Chap. III. Prop. 2.*) and to pro-

fore none of the Rays that come from that Object can enter an Eye upon the Surface of the Earth, except what enter the *Atmosphere* higher than they need to do, if they could come in a right Line from the Object; consequently the Object must appear above its proper Place. Secondly, if the Object be placed within the *Atmosphere*, the Case is still the same; for the Rays which flow from it must continually enter a denser *Medium* whose Center is below the Eye, and therefore being refracted towards the Center, that is, downwards as before, those which enter the Eye must necessarily proceed as from some Point above the Object, wherefore the Object will appear above its proper Place.

From hence it is, that the Sun, Moon, and Stars appear above the *Horizon*, when they are just below it, and higher than they ought to do, when they are above it: Likewise distant Hills, Trees, &c. seem to be higher than they are.

Farther, the lower these Objects are in the *Horizon*, the greater is the Obliquity with which the Rays which flow from them, enter the *Atmosphere*, or pass from the rarer into the denser Parts of it, and therefore they appear to be the more elevated by Refraction; upon which Account the lower Parts of them are *apparently* more elevated than the other. This makes their upper and under Parts seem nearer together than they ought to do, as is evident in the Sun and Moon, which appear of an oval Form when they are in the *Horizon*, their *horizontal* Diameters appearing of the same Length they would do if the Rays suffered no Refraction, while their *vertical* ones are shortened thereby.

ceed

ceed afterwards, suppose in the Lines  $Ka$ ,  $Lb$ ; at  $a$  and  $b$  where they pass out of the denser *Medium*, they will be as much refracted the contrary Way, proceeding in the Lines  $ac$ ,  $bd$ , parallel to their first Directions (see Chap. IV.); produce these Lines back till they meet in  $e$ , this will be the apparent Place of the Point  $R$ , and it is evident from the Figure that it must be nearer the Eye than that Point; and because the same is true of all other Pencils flowing from the Object  $AB$ , the Whole will be seen in the Situation  $fg$ , nearer to the Eye than the Line  $AB$ . 2d, As the Rays  $RK$ ,  $RL$  would not have entered the Eye, but have passed by it in the Directions  $Kr$ ,  $Lt$ , had they not been refracted in passing through the *Medium*, the Object appears *brighter*. 3d, The Rays  $Ab$ ,  $Bi$ , will be refracted at  $b$  and  $i$  into the less converging Lines  $bk$ ,  $il$ , and at the other Surface into  $kM$ ,  $lM$  parallel to  $Ab$  and  $Bi$  produced (see Chap. 4.), so that the Extremities of the Object will appear in the Lines  $Mk$ ,  $Ml$  produced, *viz.* in  $f$  and  $g$ , and under as large an Angle  $fMg$ , as the Angle  $AqB$  under which an Eye at  $q$  would have seen it, had there been no *Medium* interposed to refract the Rays; and therefore it appears *larger* to the Eye at  $GH$ , being seen through the interposed *Medium*, than otherwise it would have done. But it is here to be observed, that the nearer the Point  $e$  appears to the Eye on



Account of the Refraction of the Rays RK, RL, the shorter is the Image *fg*, because it is terminated by the Lines *Mf* and *Mg*, upon which Account the Object is made to appear less; and therefore the apparent Magnitude of an Object is not much augmented by being seen through a *Medium* of this Form.

Farther, it is apparent from the Figure, that the Effect of a *Medium* of this Form depends wholly upon its Thickness; for the Distance between the Lines *Rr* and *ec*, and consequently the Distance between the Points *e* and *R* depends upon the Length of the Line *Ka*: Again, the Distance between the Lines *AM* and *fM*, depends on the Length of the Line *bk*; but both *Ka* and *bk* depend on the Distance between the Surfaces *CE* and *DF*, and therefore the Effect of this *Medium* depends upon its Thickness.

Prop. III. An Object seen through a convex *Lens*, appears *larger, brighter, and more distant*, than with the naked Eye.

To illustrate this, let *AB* (*Fig. 39.*) be the Object, *CD* the *Lens*, and *EF* the Eye. 1. From *A* and *B* the Extremities of the Object draw the right Lines *AYr*, *BXr* crossing each other in the Pupil of the Eye; the Angle *ArB* comprehended between these Lines, is the Angle under which the Object would be seen with the naked Eye. But by the Interposition of a *Lens* of this Form, whose Property it is to render converging Rays more so (see Chap. IV.) the Rays *AY* and *BX* will be made

to

to cross each other before they reach the Pupil. There the Eye at E, will not perceive the Extremities of the Object by means of these Rays (for they will pass it without entering), but by some others which must fall without the Points Y and X, or between them; but if they fall between them, they will be made to concur sooner than they themselves would have done, and therefore if the Extremities of the Object could not be seen by them, it will much less be seen by these. It remains therefore, that the Rays which will enter the Eye from the Points A and B after Refraction, must fall upon the *Lens* without the Points Y and X; let then the Rays AO and BP be such. These after Refraction entering the Eye at *r*, the Extremities of the Object will be seen in the Lines *rQ*, *rT* produced, and under the *optic Angle* *QrT* which is larger than *ArB*, and therefore the *apparent Magnitude of the Object will be increased*. 2. Let GHI be a Pencil of Rays flowing from the Point G; as it is the Property of this *Lens* to render diverging Rays less diverging, parallel or converging (see Chap. IV.) it is evident, that some of those Rays which would proceed on to M and N and miss the Eye, were they to suffer no Refraction in passing through the *Lens*, will now enter it; by which means the Object will appear *brighter*. 3. As to the apparent Distance of the Object, that will vary according to the Situation of

of it with respect to the *Focus* of parallel Rays of the *Lens*. 1. Then, let us suppose the Object placed so much nearer the *Lens* than its *Focus* of parallel Rays, that the refracted Rays KE and LF though rendered less diverging by passing through it, may yet have a considerable Degree of Divergency, so that we may be able to form a Judgment of the Distance of the Object thereby. In this Case, the Object ought to appear where EK, FL produced back concur, which, because they diverge less than the Rays GH, GI, will be beyond G, that is, at a greater Distance from the *Lens* than the Object is. But because both the Brightness and Magnitude of the Object will at the same Time be augmented, Prejudice will not permit us to judge it quite so far off as the Point wherethose Lines meet, but somewhere between that Point and its proper Place. 2. Let the Object be placed in the *Focus* of parallel Rays, then will the Rays KE and LF become parallel (see Chap. IV.) and though in this Case the Object would appear at an immense Distance, if that Distance were to be judged of by the Direction of the Rays KE and LF, yet upon Account of the Brightness and Magnitude of it, we shall not think it much farther from us, than if it were seen by the naked Eye. 3. If the Object be situated beyond the *Focus* of parallel Rays, as in AB (*Fig. 40.*) the Rays flowing from thence and falling upon the *Lens* CD,

CD, will be collected into their respective *Foci* at *a* and *b*, and the intermediate Points *m*, *n*, &c. and will there form an Image of the Object AB; and after crossing each other in the several Points of it, as expressed in the Figure, will pass on diverging as from a real Object. Now if an Eye be situated at *c*, where *Ac*, *Bc*, Rays proceeding from the extreme Points of the Object, make not a much larger Angle *AcB*, than they would do if there were no *Lens* interposed; and the Rays belonging to the same Pencil do not converge so much as those the Eye would receive, if it were placed nearer to *a* or *b*, the Object upon these Accounts appearing very little larger or brighter than with the naked Eye, is seen nearly in its proper Place; but if the Eye recedes a little Way towards *ab*, the Object then appearing both brighter and larger, seems to approach the *Lens* \*: which is an evident Proof of what has

\* That the Object should seem to approach the *Lens* in this Case, was a Difficulty that exceedingly puzzled the learned *Barrow*, and which he pronounces insuperable, and not to be accounted for by any Theory we have of Vision. *Molineux* also leaves it to the Solution of others, as that which will be inexplicable, till a more intimate Knowledge of the visive Faculty, as he expresses it, be obtained by Mortals.

They imagined, that seeing an Object appears *farther off*, the *less* the Rays *diverge* which fall upon the Eye; if they should proceed *parallel* to each other, it ought to appear *exceedingly remote*, and if they should *converge*, it should then appear *more distant still*: The Reason of this was, because they looked upon the apparent Place of an Object, as owing only to the Direction of the Rays whatever it was, and not at all to its apparent Magnitude or Splendour,

been

been so often asserted, *viz.* that we judge of the Distance of an Object in some Measure by its Brightness and Magnitude \*; for the Rays converge the more the farther the Eye recedes from the *Lens*; and therefore if we judged of the Distance of the Object by the Direction of the Rays which flow from it, we ought in this Case to conceive it at a greater Distance, than when the Rays were parallel, or diverged at their Entrance into the Eye.

Prop. IV. If an Object be placed farther from a convex *Lens*, than its *Focus* of parallel Rays, and the Eye be situated farther from it on the other Side, than the Place where the Rays of the several Pencils are collected into their respective *Foci*, the Object appears *inverted*, and *pendulous in the Air*, between the Eye and the *Lens*.

To explain this, let AB (*Fig. 40.*) represent the Object, CD the *Lens*, and let the Rays of the Pencil ACD be collected in *a*, and those of BCD in *b*, forming there an inverted Image of the Object AB, and let the Eye be placed in F: It is apparent from the Figure, that some of the refracted Rays which pass through each Point of the Image, will enter

\* Perhaps it may proceed from our judging of the Distance of an Object in some Measure by its Magnitude, that that Deception of Sight commonly observed by Travellers may arise; *viz.* that upon the first appearing of a Building larger than usual, as a *Cathedral Church*, or the like, it generally seems nearer to them, than they afterwards find it to be.



the Eye as from a real Object in that Place, and therefore the Object AB will appear there, as the Proposition asserts. But we are so little accustomed to see Objects in this Manner, that it is very difficult to perceive the Image with one Eye; but if both Eyes are situated in such a Manner, that Rays flowing from each Point of the Image may enter both, as at G and H, and we direct our optic Axes to the Image, 'tis easy to be perceived.

If the Eye be situated in *a* or *b*, or very near them on either Side, the Object appears exceedingly confused, *viz.* if at *d*, the Rays which proceed from the same Point of the Object converge so very much, and if at *e*, they diverge so much, that they cannot be collected together upon the *Retina*; but fall upon it as if they were the Axes of so many distinct Pencils coming through every Point of the *Lens*; wherefore little more than one single Point of the Object is seen at a Time, and that appears all over the *Lens*; from whence nothing but Confusion arises.

If the *Lens* be so large that both Eyes may be applied to it, as in *b* and *k*, the Object will appear double; for 'tis evident from the *Figure*, that the Rays which enter the Eye at *b* from either Extremity of the Object A or B, do not proceed as from the same Point with that from whence those which enter the other at *k* seem to flow; the Mind therefore is here deceived,



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and looks upon the Object as situated in two different Places, and therefore judges it to be double.

Prop. V. An Object seen through a concave *Lens* appears *nearer, smaller, and less bright*, than with the naked Eye.

Thus, let AB (*Fig. 41.*) be the Object, CD the Pupil of an Eye, and EF the *Lens*. Now, as it is the Property of a *Lens* of this Form, to render diverging Rays more so, and converging ones less so, the diverging Rays GH, GI, proceeding from the Point G, will be made to diverge more, and so to enter the Eye as from some nearer Point *g*; and the Rays AH, BI, which converge, will be made to converge less, and to enter the Eye as from the Points *a* and *b*; wherefore the Object will appear in the Situation *agb*, *less* and *nearer* than without the *Lens*. Farther, as the Rays which proceed from G, are rendered more diverging, some of them will be made to pass by the Pupil of the Eye, which otherwise would have entered it, and therefore each Point of the Object will appear *less bright* \*.

Prop. VI.

\* From what has been observed about the Properties of convex and concave *Lenses*, we may see the Reason why the former Sort are made Use of by *old People* to help their Sight; and the latter by those who are *purblind*. *Old People*, as was observed before, having the *Tunica Cornea* of their Eyes too flat, require that the Object be placed at a greater Distance from them, than other People whose Eyes are of a just Form, that the Rays which enter the Pupils of their Eyes from the same

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Prop. VI. An Object seen through a *polygonous* Glass, that is, such as is terminated by several plain Surfaces, is *multiplied* thereby.

For Instance, let A (*Fig. 42.*) be an Object, and BC a polygonous Glass terminated by the plain Surfaces BD, DE, &c. and let the Situation of the Eye F be such, that the Rays AB being refracted in passing through the Glass, may enter it in the Direction BF, and the Rays AC in the Direction CF. Then will the Eye by means of the former, see the Object in G, and by the latter in H; and by means of the Rays AI, the Object will appear also in its proper Situation A.

Thus much for the Principles of *Dioptrics*, and the Solution of some obvious *Phænomena* which tend to confirm the same: Those which yet remain to be accounted for, shall, according to the Method we have hitherto observed, be treated of in the Dissertations of this Part.

same Point of the Object, may not diverge too much. Now a convex *Lens* makes those Rays diverge less, as they would naturally do if the Object was placed farther off. Again, those who are *purblind*, having the *Tunica Cornea* too protuberant, require such a *Lens* as may render those Rays more diverging, lest they should be collected into their respective *Foci* before they fall upon the *Retina*; and therefore *Lenses* of the concave Sort are of Use to them.

## DISSERTATION I.

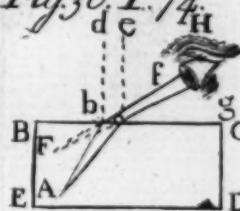
*Of the Horizontal Moon.*

**T**HE *Phænomenon* of the *horizontal* Moon is this: When the Moon is just above the Surface of the Earth, either immediately after she is risen, or just before she sets, she appears four or five Times greater in Diameter, than when she is in her Meridian Altitude: And yet her apparent Diameter, if taken by an Instrument, is found to subtend the same Angle in either Situation \*.

The Moon's apparent Diameter being found to subtend the same Angle, whether she be in the Horizon or Meridian, it is evident the Image of her projected upon the *Retina* of an Eye, is of the same Dimensions in either Case; and therefore that she should appear of a different Magnitude in one Situation from what

\* What is said here of the Moon's Diameter, as taken by an Instrument, must be understood of her *horizontal* Diameter, and not of her *vertical* one, for the Length of this is diminished by Refraction (as explained Chap. VII. Note the first) and, therefore, if it be taken by an Instrument, it will not be found to subtend the same Angle in the Horizon as in the Meridian: But notwithstanding this, it appears longer to the naked Eye when in the former, than in the latter Situation, as well as the horizontal Diameter.

Fig. 36. P. 74<sup>H</sup>  
d e



*Fig 37 P. 75.*

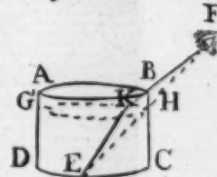
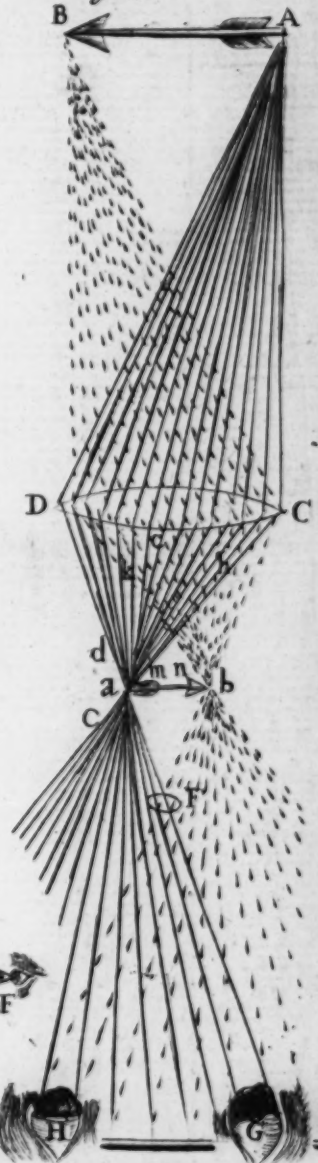


Fig. 40. P. 82



*Fig. 38. P. 76.*

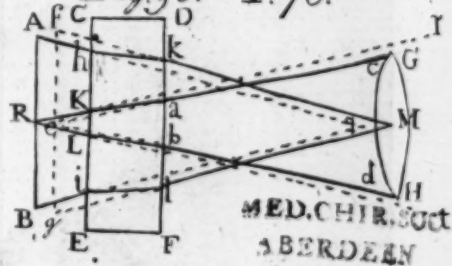
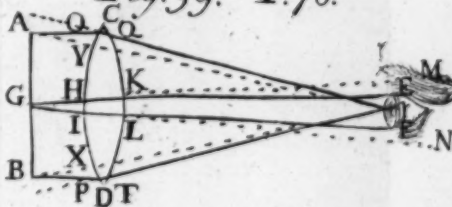
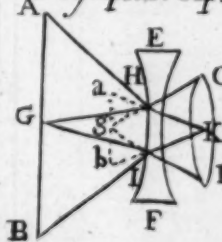


Fig. 39. P. 78.



*Fig. 41. P. 84.*



*Fig. 42. P. 85.*



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she does in the other, has always been Matter of great Speculation among the *Connoisseurs* both in *Optics* and *Astronomy*. *Des Cartes* was of Opinion, that we think the Moon larger when she is in the Horizon, than when she is in the Meridian, because in the former Case by comparing her Distance with that of interposed Objects, we imagine it greater than when she is elevated: And that as we judge her Distance greater in that Situation, we of Course think her Diameter longer, because it subtends the same Angle in either Case. But more of this by and by, when we come to the Explication Dr. *Wallis* has given of this Matter.

*Gassendus* was of Opinion, that because the Moon appears less bright when in the Horizon than in the Meridian, we view her in the former Situation with a larger Pupil, than we do in the latter; and from thence he concludes, that the Image of her upon the *Retina* must be larger. But this is contrary to the Laws of *Optics*; for if the refractive Power of the Humours of the Eye collects the Rays of the several Pencils into their respective *Foci* upon the *Retina* (and there is no Reason to suppose the contrary in this Case) the Breadth of the Pupil makes no Alteration in the Magnitude of the Image; because the Situation of those *Foci* is determined by the *Axes* of the several Pencils, which crossing each other in the Center



### 88 *Of the Horizontal Moon. Part III.*

ter of the Pupil (as was shewn Chap. VI. pag. 61. in the Note) pass on to the same Points of the *Retina*, whether the Pupil be broad or narrow.

*Molineux* in the Philosophical Transactions No. 187. tells us of a certain *French Abbé*, that revived the forementioned Supposition of *Gassendus*, and adding two others of his own, endeavoured to account for this *Phænomenon*. His Suppositions were these, *viz.* “ That this  
“ contracting and enlarging the Pupil (sup-  
“ posed by *Gassendus*) causeth a different  
“ Shape in the Eye; an open Pupil making  
“ the Crystalline flatter, and the Eye longer,  
“ and the narrower Pupil shortening the Eye,  
“ and making the Crystalline Humour more  
“ convex. The first attends our looking at  
“ Objects that are remote, or which we think  
“ so, the latter accompanies the viewing Ob-  
“ jects nigh at Hand. Likewise an open Pu-  
“ pil and flat Crystalline attends Objects of  
“ a more sedate Light, whilst Objects of more  
“ forcible Rays require a greater Convexity,  
“ and narrower Pupil. From these Positions,  
“ continues *Molineux*, the *Abbé* endeavoured  
“ to give an Account of our *Phænomenon*, as  
“ follows. When the Moon is nigh the Ho-  
“ rizon, by Comparison with interposed Ob-  
“ jects, we are apt to imagine her much far-  
“ ther from us than when more elevated, and  
“ therefore we order our Eyes as for viewing  
“ an

Differt. I. *Of the Horizontal Moon.* 89

“ an Object farther from us ; that is, we some-  
 “ thing enlarge the Pupil, and thereby make  
 “ the CrySTALLINE flatter ; moreover the Dus-  
 “ kiness of the Moon in that Posture does  
 “ not so much strain the Sight ; and conse-  
 “ quently the Pupil will be more large, and  
 “ the CrySTALLINE more flat ; hence a larger  
 “ Image shall be projected on the Fund of the  
 “ Eye, and, therefore, the Moon shall appear  
 “ larger. These two forementioned Accidents,  
 “ viz. the Moon’s imaginary Distance and  
 “ Duskiness gradually vanishing as she rises,  
 “ a different *Species* is hereby introduced in  
 “ the Eye, and consequently she seems gradu-  
 “ ally less and less, till again she approaches  
 “ nigh the Horizon.”

As to what is taken for granted in this So-  
 lution concerning a Change in the CrySTALLINE  
 Humour and Form of the Eye, upon viewing  
 an Object in a *dusky* or faint Light, that seems  
 to be very ill grounded. We know of no such  
 Connection between the Muscles of the *Iris*  
 and those of the *Ligamenta Ciliaria*, as is  
 necessary to produce this Effect. And the  
 Coats of the Eye are not so pliable, as easily  
 to admit of an Alteration in their Form \*.  
 Could the Author have made good his other  
 Supposition, viz. *That by Comparison with*

\* See what has been observed concerning the Power we have  
 of making an Alteration in the Eye, in order to see *distinctly*.  
 (Chap. VI. pag. 62.)

90 *Of the Horizontal Moon. Part III.*

*interposed Objects we are apt to imagine her much farther from us, than when more elevated, he need not have had Recourse to any other; this alone would have been sufficient; but hic Labor est.* This alone, I say, would have been sufficient; for if by comparing her Distance with that of interposed Objects, we imagine it greater when she is in the Horizon, than when she is in the Meridian; as she subtends an equal Angle in both Cases, we must in Consequence thereof (agreeably to *Des Cartes's* Notion above-mentioned) imagine her to be bigger in the former Situation than in the latter; because a distant Object cannot subtend the same Angle at the Eye that one which is nearer does, unless it be proportionably larger\*.

The famous *Hobbs* endeavoured at a Solution of this *Pænomenon*, but it is hardly worth mentioning: The Figure he has drawn to explain his Solution by, seems to have been the Occasion of his Error. He draws a Circle to represent that blue Surface commonly called the Sky, in which the heavenly Bodies seem to be fixed, and concentric to this, a lesser, to represent the Surface of the Earth, but vastly too big in Proportion; so that a Spectator upon the Surface of this Earth, is considerably nearer to the upper Part of the

\* See Chap. VI. pag. 71.

II. Dissert. 1. *Of the Horizontal Moon.* 91

other Circle than to the Sides of it : Wherefore an Object that subtends the same Angle at different Heights must necessarily hide a greater Portion of that Ark when, it is in the Horizon, than when it is in the Meridian ; because that Ark is farther behind the Object in the former than in the latter Situation ; from whence he concludes that the Moon must appear bigger in that Situation than in the Meridian. Had he drawn his Circles in any tolerable Proportion to that which he designed them to represent, he would easily have seen his Mistake.

A few Years ago Mr. *De Veil* published a Treatise upon the Subject of the horizontal Moon, which he dedicates to the Ladies of *Northampton*. If I remember right, his Solution of it was in the following Manner : 1. When an Object is placed beyond the *Focus* of parallel Rays of a convex *Lens*, the farther the Eye (situated on the other Side the *Lens*) recedes from it towards the *Focus* of the Rays which flow from that Object, the larger that Object appears. 2. Rays of Light flowing from the Moon, and passing thro' the *Atmosphere* of the Earth, are collected into a *Focus* on the other Side of it. 3. When the Moon is in the Horizon, we are nearer to this *Focus* by almost a *Semidiameter* of the Earth, than when she is in the Meridian : And therefore, the Moon ought to appear larger when in the former than in the latter Situation.

## 92 *Of the Horizontal Moon. Part III.*

The Propositions in this Solution are all true, but the Second is not applicable in the present Case; for unless we consider the Refraction that Rays of Light which flow from the Moon, and pass through the Atmosphere of the Earth, suffer in their Emerſion, that is, while they pass through the latter half of it, as well as that which they suffer in their Immerſion, or while they pass through the former half, we shall find that they will not be collected into their respective *Foci* on the other Side the Earth, as this Gentleman imagines: Which if it can be shewn, his Solution falls to the Ground of Course; for the Refraction which the Rays suffer in their Emerſion is not to be taken into Consideration, because they reach the Eye of a Spectator upon the Earth as soon as they have passed through the first half of the Atmosphere, when the Moon is in his Horizon; and before they have passed through that half, when she is in his Meridian.

Let us then imagine two Rays flowing from one and the same Point of the Surface of the Moon, it being necessary in order to constitute a *Focus* that such Rays should after Refraction meet in a Point; the Meeting of such as flow from different Points in the same Surface is not sufficient; if it were, we might then have *Foci* where we pleased, and that as well without refracting or reflecting Surfaces as with them. And let the first of those two Rays fall perpendicularly upon the Atmosphere of the Earth, and  
be



Differt. 1. *Of the Horizontal Moon.* 93

be supposed to pass through the Center of it; and let the other after Refraction pass by the Surface of the Earth. Now, the Moon's *Parallax*, that is, the Angle under which the Semidiameter of the Earth is seen from the Moon, being about one Degree, it is evident that these Rays must, before their Incidence upon the Earth's Atmosphere, diverge the one from the other by such an Angle. But it appears from Sir *Isaac Newton's* Table of Refractions published by Dr. *Halley* in the Philosophical Transactions, No. 368, that, when any of the heavenly Bodies appears in the Horizon, the Rays by which it is seen, are refracted but by an Angle of thirty-three Minutes and forty-five Seconds; and therefore the Ray which we have supposed after Refraction to pass by the Surface of the Earth will be refracted only by such an Angle; which falling considerably short of one Degree, the Angle by which it diverged from the perpendicular one before Refraction; it will be so far from being made to converge towards it thereby, that it will still be in a State of Divergency from it. And therefore Rays flowing from the Moon and refracted only in their Immersion into the Atmosphere of the Earth, will not be collected into their respective *Foci* on the other Side: Which was to be shewn.

Dr. *Wallis* in the Philosophical Transactions, No. 187, gives us a Solution of the horizontal Moon (or rather an Explication of what *Des Cartes* had given before) which is as follows:



94 *Of the Horizontal Moon. Part III.*

He ascribes this *Phænomenon* to the Deception of the Imagination, and accounts for that Deception in the following Manner. He observes: First, that the Imagination doth not estimate the Greatness of an Object seen, by the *optic Angle* only, but by this compared with the supposed Distance. So that if two Things are seen under the same or equal Angles, and if, upon any Account whatever, we apprehend one of these to be farther from us than the other, that which we apprehend to be farther from us, will to the Imagination appear greater. Secondly, That one great Advantage for estimating the apparent Distance of any Thing, is from the Variety of intermediate Objects between the Eye and the Thing seen; for then the Imagination must allow Room for all those Things.

“ Now, says he, when the Sun \* or Moon  
 “ is near the Horizon, the Prospect we have of  
 “ Hills and Vallies, Plains and Woods, &c.  
 “ represent to our Imagination a great Distance, capable of receiving all these: Or, if  
 “ it happens that these interposed Objects are  
 “ not actually seen, yet, having been accustomed to see them, the Memory suggests to  
 “ us a View as large as is the visible Horizon.  
 “ But when the Sun or Moon is in an higher Position, we see nothing between us and  
 “ them (unless, perhaps, some Clouds) and

\* For the Sun appears larger in the *Horizon*, as well as the Moon.

“ there-

Differt. 1. *Of the Horizontal Moon.* 95

“ therefore nothing that can present to our  
“ Imagination so great a Distance as the other  
“ is. And therefore, though both be seen un-  
“ der the same Angle, they do not appear (to  
“ the Imagination) of the same Bigness, be-  
“ cause not fancied at the same Distance : But  
“ that near the Horizon is judged bigger (be-  
“ cause supposed farther off) than the same,  
“ when at a greater Altitude.”

If I might be allowed to mention any Thing of my own, after these great *Genius's* have given their Opinions upon this Matter, it should be this, *viz.* That I have often thought, that he who would give a rational Account, why the Sun or Moon appears farther from us in the Horizon than in the Meridian (for that is all that is requisite, towards a Solution of the horizontal Moon, as has been already observed) should first show why that apparent azure Surface we call the Sky, does not seem to be an entire concave *Hemisphere*, but only a Portion of such an one: For our judging the Heavens to be no more than such a Portion, is undoubtedly the Cause why we judge both the Sun, Moon, and Stars to be farther from us when in the Horizon than in the Meridian; because we have nothing else we can refer their Places to, but that.

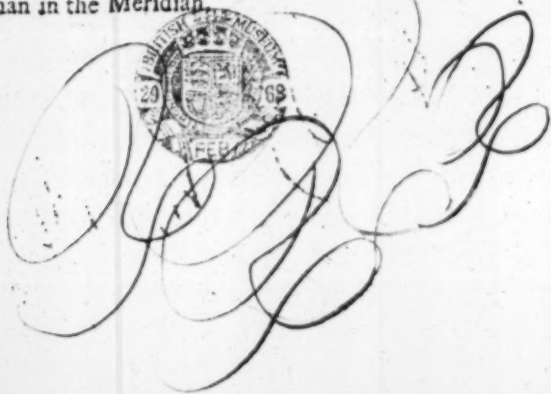
Now, possibly, the Cause why we think the Heavens of that Form may after all be only this, *viz.* That, as the Rays which come from the upper Parts of that imaginary Surface, the  
Sky,

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Sky, pass through a less Portion of the *Atmosphere* than such as come from the horizontal Parts of it, the Sky appears to us more *distinctly*, and generally more *bright* in those Parts than in the latter; and therefore, since we daily observe that those Objects which appear most *distinct* are generally such as are *nearest* to us, and also as *bright* Objects, when we have nothing but bare Imagination to determine us in estimating the Distance of them, appear *nearer* to us than the same Objects when less so \*, we think the upper Parts of the Sky *nearer* us than the lower. Wherefore, since we refer all the heavenly Bodies to this Surface, we necessarily imagine them farther from us, and consequently larger, and also more distant from each other †, when near the Horizon, than when they are arrived at their meridian Altitude.

\* See what has been said concerning the Brightness of an Object being a Means whereby it appears nearer us, under Prop. 3. of the 7th Chapter.

† The apparent horizontal Distance of two Stars from one another is observed to be greater, when they are in the Horizon, than in the Meridian.



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